Tectonophysics, 115 (1985) 275–295
Elsevier Science Publishers B.V., Amsterdam – Printed in The Netherlands

INTERLAYER MATERIAL TRANSPORT DURING LAYER-NORMAL SHORTENING. PART I. THE MODEL

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(Received December 28, 1983; revised version accepted November 29, 1984)

ABSTRACT

Van der Molen, I., 1985. Interlayer material transport during layer-normal shortening. Part I. The model. *Tectonophysics*, 115: 275–295.

To analyse mass-transfer during deformation, the case is considered of a multilayer experiencing a layer-normal shortening that is volume constant on the scale of many layers. Strain rate is homogeneously distributed on the layer-scale if diffusion is absent; when transport of matter between the layers occurs, strain rate and dilatation rate are found to vary with distance from the contacts between layers. An instantaneous model for differentiation is developed on the assumption that the sole driving force for diffusion is the difference in the hydrostatic component of stress caused by flow of rheologically contrasting layers. Physical properties incorporated in the model are: viscosity, η, mobility of the migrating component, M, and a factor f for (an)isotropy in removal or addition of material. Each is found to influence flow behaviour and diffusion to a degree depending on scale. The magnitude of the *characteristic diffusion length*, $B=\sqrt{2v_0\eta Mf}$, where v_0 is the specific volume of the diffusing component, is essential in this respect. Mass-removal and mass-addition contribute significantly to deformation of relatively thin layers. In thick layers, beyond about six times the characteristic diffusion length away from a contact, deformation proceeds as in the absence of diffusion. The theory further predicts a dynamically maintained stable layer-thickness to which thicker and thinner layers converge during prolonged deformation. The question how (periodically) layered gneiss may develop by deformation of directionless igneous rock is discussed in the light of this conclusion.

INTRODUCTION

There is evidence in metamorphic rocks of all grades that heterogeneities in composition can grow from their surroundings during deformation. The process is called syntectonic differentiation and its products include layered axial plane foliations in folds, pressure shadows around porphyroblasts, infillings between boudins and a variety of tectonic veins. The typical layering in high-grade terranes may well be sedimentary or igneous in origin, or it can be derived from a non-layered arrangement of compositionally distinct volumes that became reoriented

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and flattened into a layered structure by high strain (Fig. 1). Whichever the case, the compositional contrast in gneissic layering has generally been intensified by the growth of some layers at the expense of others while straining was in progress.

A theory for differentiation during layer-normal shortening is presented here in its own right to prevent confusion with the complications of geological reality. A viscous fluid in the model is meant to be just that. The reader should temporarily suspend his mechanical notion of rock as a polymineralic crystalline aggregate permeated by pore fluid and having a finite tensional strength. These concepts will be needed in Part II which deals with observed boudinage, pinch-and-swell and related differentiation phenomena. In the discussions of both papers it will be shown that the viscous model for differentiation can advantageously guide our thinking about the processes leading to common structures in high-grade gneiss terranes.

Previous models

Our understanding of syntectonic interlayer transport of material is limited. Ramberg (1949, 1955, 1956) first described the phenomenon and suggested the dynamic explanation that has been adhered to since in some form or other.

Fig. 1. Typical layering in amphibolite facies Precambrian gneisses from the Nagssugtoqidian of West Greenland, near Søndre Strømfjord Airport. a. Quartz-feldspar veins in a finer grained, more mica-rich medium. Light veins have dark margins of biotite. b. Periodic gneissic layering cross-cut by a mafic dyke.

Strömgård (1973) improved Ramberg's mechanical model for boudinage in a manner of relevance for the subject under discussion. The role of syntectonic differentiation was reassessed by Robin (1979) in a stimulating paper summarising and expanding the insights of earlier work. To date most authors will agree that rheological differences from one layer to the next imply differences in the state of stress during deformation. Where possible, material is dissolved from areas of high pressure, or high mean stress, and after transport it is deposited in areas of low pressure or low mean stress. Deposition occurs preferably at fracture sites opening up in the direction of maximum tensional stress, or, if tensional strength is negligible, in the direction of zero effective normal stress (cf. Robin, 1979; Fletcher, 1982).

The existing theory is held to be valid but not specific enough. It explains qualitatively, for example, that a homogeneous igneous rock can deform into layered gneiss, but fails to throw light on the question why many gneisses of equal descent have no layering at all. Quantitative interpretation of the length-scale of differentiation has become possible through the work of Fletcher (1982). The material property *characteristic diffusion length* introduced by Fletcher plays an important role in the model for differentiation worked out below.

THEORY

Deformation of a multilayer without diffusion between the layers

Two contrasting fluids are arranged in an infinite multilayer of alternating thicknesses $2a$ and $2b$. A coordinate system is defined with the x-direction parallel to layering, the z-direction normal to layering and $z = 0$ in the centre of a layer (Fig. 2). This multilayer is subject to a pure shear with the direction of maximum elongation parallel to x and such that every element experiences the same deformation as the total system (Fig. 3a, b). The principal *strain rates of the total system* are denoted by $\dot{\varepsilon}^{\mathrm{T}}_{xx}$ and $\dot{\varepsilon}^{\mathrm{T}}_{zz}$ for the x- and z-direction. The *contributions of flow* to the system strain rates have symbols $\dot{\not{\varepsilon}}^{\mathrm{T}}_{xx}$ and $\dot{\not{\varepsilon}}^{\mathrm{T}}_{zz}$ respectively. In this section total strain rate and strain rate due to flow are the same. Later, when diffusion between the layers is allowed, the distinction becomes essential. The deformation is volume constant and independent of coordinates. The boundary conditions are simple:

$$\left.\begin{aligned} \dot{\varepsilon}^{\mathrm{T}}_{xx} &= \dot{\not{\varepsilon}}^{a}_{xx} = \dot{\not{\varepsilon}}^{b}_{xx} \\ \dot{\varepsilon}^{\mathrm{T}}_{zz} &= \dot{\not{\varepsilon}}^{a}_{zz} = \dot{\not{\varepsilon}}^{b}_{zz} \\ \dot{\varepsilon}^{\mathrm{T}}_{xx} &= -\dot{\varepsilon}^{\mathrm{T}}_{zz} \end{aligned}\right| \qquad (1)$$

where superscripts a and b refer to the two fluids.

What stresses belong to this deformation? Firstly note that for a balance of forces the normal stresses in the z-direction need to be equal. Superscripts may be dropped:

$$\sigma^{a}_{zz} = \sigma^{b}_{zz} = \sigma_{zz} \qquad (2)$$

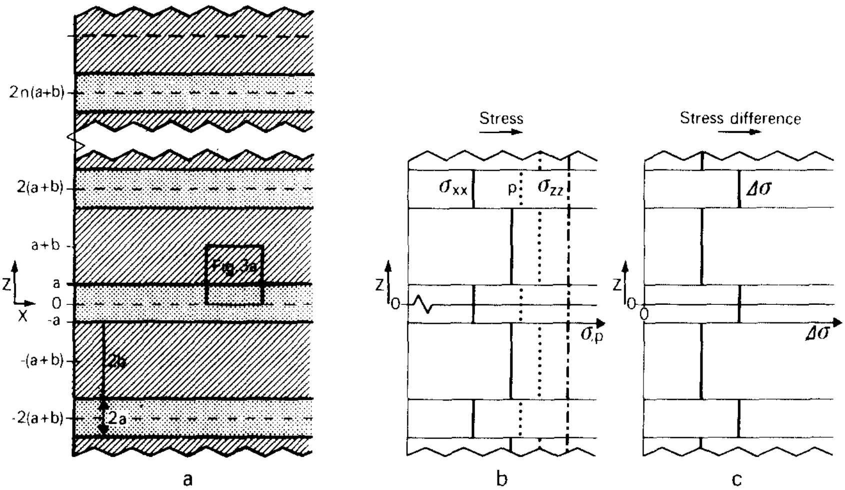

Fig. 2. a. Multilayer with definition of the coordinate system. b. Hydrostatic component of the stress and normal stresses in the x- and z-directions during layer-normal shortening. There is no diffusion between the layers and $\eta^a > \eta^b$. c. Differential stress for the same case (after Robin, 1979).

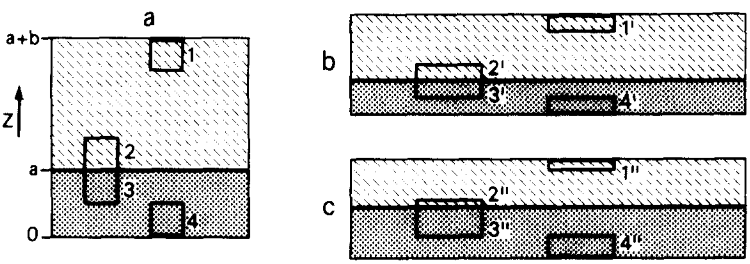

Fig. 3. a. Element of the multilayer consisting of two half-layers. Four passive markers are shown at time t. b. The same element at $t + dt$. There is no diffusion between the layers. c. The same element at $t + dt$ when diffusion is allowed. Material transport across the interface has occurred. Note that the volume enclosed by passive marker *1* has been less reduced than that enclosed by marker *2* closer to the contact. The reverse applies for *3* and *4*. Strains and volume changes have been exaggerated for clarity. The model itself is instantaneous.

Constitutive relations for the fluids in the layers are needed to establish stress components in the x-direction. In the simple case of an incompressible Newtonian viscous fluid the relation is:

$$\sigma_{ij} = p\delta_{ij} - 2\eta\dot{\gamma}_{ij} \quad * \qquad (3)$$

where δ is the Kronecker delta: $\delta_{ij} = 1$ for $i = j$; $\delta_{ij} = 0$ for $i \neq j$, p is the hydrostatic component of the stress, η is viscosity and $\dot{\gamma}_{ij}$ the rate of viscous flow.

* Sign convention: normal stress and hydrostatic component of stress are positive for compression; the strain rate is positive during elongation.

For the pure shear geometry considered $\dot{\gamma}_{ij} = 0$ and $\sigma_{ij} = 0$ for $i \neq j$ and:

$$p = \frac{\sigma_{xx} + \sigma_{zz}}{2} \tag{4}$$

From (3) and (4) we find that the *differential stress*:

$$\Delta\sigma \equiv \sigma_{zz} - \sigma_{xx} = 4\eta\dot{\gamma}_{xx} \tag{5}$$

and the hydrostatic component of stress may be rewritten as:

$$p = \sigma_{zz} - \frac{\Delta\sigma}{2} \tag{6}$$

One can now express the stresses belonging to deformation in terms of the strain rate imposed on the total system and the viscosities of the two fluids:

$$\Delta\sigma^{a,b} = 4\eta^{a,b}\dot{\gamma}^{a,b}_{xx} = 4\eta^{a,b}\dot{\varepsilon}^{\mathrm{T}}_{xx} \tag{7}$$

$$p^{a,b} = \sigma_{zz} - \frac{\Delta\sigma^{a,b}}{2} \tag{8}$$

Note that the absolute values of the stresses are not determined, but differences in normal stress or in the hydrostatic components of the stress are fixed. Figure 2b and 2c illustrate two points. Stress is constant within a layer and discontinuous at the contact of two layers. The low value for the hydrostatic component of stress is in the layer where the viscosity is high. The latter result has led to predictions that in similar deformations with possible diffusion a tendency would exist for material to migrate from layers with low viscosity to layers with high viscosity (Ramberg, 1949; Robin, 1979).

Deformation of a multilayer with diffusion between the layers

Because of the symmetry and the periodicity of a regular two component multilayer the analysis can be restricted to an element consisting of a half-layer *a* in contact with a half-layer *b* (Figs. 2a, 3a). Diffusion is allowed within, but not between such elements. Now consider the following. A component diffuses from half-layer *b* to half-layer *a* with higher viscosity $\eta^a > \eta^b$. While in *b* this component has all material properties of the fluid in *b*, once in *a* the component takes on the material properties of the fluid in *a*. This may seem complicated at first. A very small amount of material that has left *b* does not change the properties of the material left behind; a very small amount added to *a* does not alter the properties there.

The model developed here is *instantaneous*. Integrated over time interlayer diffusion is likely to affect material properties, but not so at once. Half-layer *a* gains mass and volume at the detriment of half-layer *b* (Fig. 3a, c). It will be assumed that the *sole driving force* for this diffusion is the difference in the hydrostatic component of the stress caused by viscous flow in the two materials during deformation. The imposed deformation is again described by contact-parallel elongation and contact-normal shortening. As in the previous section normal stress σ_{zz} is constant and there will be no gradients in stress in the x-direction. We will be dealing with diffusion in the z-direction only.

The driving force for diffusion is a difference in the hydrostatic component of stress; so let the *chemical potential* be given by:

$$\mu = \mu_0 + v_0 \cdot p \tag{9}$$

where μ_0 and v_0 are the chemical potential and the specific volume in a reference state. Substitution of (7) and (8) yields the chemical potential of the diffusing component in both layers:

$$\mu^a = \left(\mu_0^a + v_0^a\right)\sigma_{zz} - v_0^a \frac{\Delta\sigma^a}{2}$$

$$\mu^b = \left(\mu_0^b + v_0^b\right)\sigma_{zz} - v_0^b \frac{\Delta\sigma^b}{2} \tag{10}$$

Diffusion requires *continuity of chemical potential*. This is granted if $\Delta\sigma^a$ and $\Delta\sigma^b$ are continuous within their respective half-layers and if the chemical potentials are equal at the contact:

$$\mu^a\langle a\rangle = \mu^b\langle a\rangle \tag{11}$$

where the value between brackets refers to the z-coordinate. There may well be a discontinuity in stress difference at the contact, but to simplify further calculations we will make the additional assumption that the chemical potential in the reference state and the specific volume in the reference state are equal for the diffusing component in both materials:

$$\mu_0^a = \mu_0^b$$

$$v_0^a = v_0^b = v_0 \tag{12}$$

Now, continuity of chemical potential is ensured if there is continuity of the stresses for every value of z. At the contact:

$$\Delta\sigma^a\langle a\rangle = \Delta\sigma^b\langle a\rangle \tag{13}$$

The effects of diffusion will be less felt at larger distances from the contact and material cannot diffuse through the boundaries of the element. Intuitively one therefore expects a variation of the stress as sketched in Figs. 4a and b. The problem is to transform this hunch into equations for stress as a function of the z-coordinate, layer thicknesses, material properties and the strain rate imposed on the system.

The stress differences drawn result in a diffusion of material in the z-direction according to our assumptions. The *flux* of material within half-layer a depends on z:

$$J^a\langle z\rangle = -M^a \frac{\mathrm{d}}{\mathrm{d}z}\left(\mu^a\right) \tag{14}$$

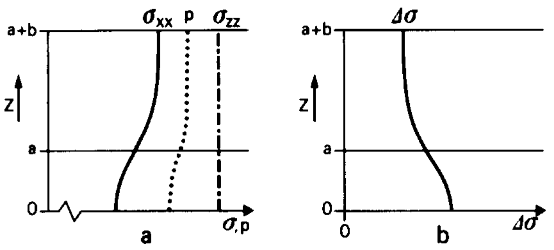

Fig. 4. a. Hydrostatic component of the stress and normal stresses in the x- and z-directions during layer-normal shortening with diffusion between the layers. As in Fig. 2: $\eta^a > \eta^b$. b. Differential stress for the same case.

where $M^a > 0$ represents the *mobility* of the migrating component in a; the minus sign in the equation ensures a positive flux in the direction of decreasing chemical potential. Differentiating (10) with respect to z and substituting the result in (14) we find:

$$J^a\langle z\rangle = +\frac{M^a v_0}{2}\frac{\mathrm{d}}{\mathrm{d}z}(\Delta\sigma^a)$$

and similarly:

$$J^b\langle z\rangle = +\frac{M^b v_0}{2}\frac{\mathrm{d}}{\mathrm{d}z}(\Delta\sigma^b) \qquad (15)$$

where $M^b > 0$ is the mobility of the migrating component in b.

Continuity of flux is the second requirement for diffusion. To prevent the appearance or disappearance of matter in a surface normal to z we make sure that, per unit time, the same amount of matter is entering the surface on one side as is leaving on the other. The surfaces at $z = 0$ and at $z = a + b$ are special because no material moves through them at all. Therefore, by (15), the product of mobility and the first derivative of the stress difference has to vary continuously within and between the half-spaces, and reduce to zero at the extremities of the element. This gives the following conditions:

$$M^a\frac{\mathrm{d}}{\mathrm{d}z}(\Delta\sigma^a)\langle a\rangle = M^b\frac{\mathrm{d}}{\mathrm{d}z}(\Delta\sigma^b)$$

$$\frac{\mathrm{d}}{\mathrm{d}z}(\Delta\sigma^a)\langle 0\rangle = \frac{\mathrm{d}}{\mathrm{d}z}(\Delta\sigma^b)\langle a+b\rangle = 0 \qquad (16)$$

Addition and removal of material are caused by *changes in the flux.* If $(\mathrm{d}/\mathrm{d}z)(J) < 0$ then less material moves per unit time through a surface at $z + \mathrm{d}z$ than through one at z and the layer in between has gained mass. If $(\mathrm{d}/\mathrm{d}z)(J) > 0$ then mass has flown out of the level between z and $z + \mathrm{d}z$. Removal and addition of mass may be described as a volumetric strain rate or *rate of dilatation*, $\dot{V}$. During removal $\dot{V} < 0$ and during addition $\dot{V} > 0$, so:

$$\dot{V}^{a,b}\langle z\rangle = -\frac{\mathrm{d}}{\mathrm{d}z}(J^{a,b})\langle z\rangle = -\frac{M^{a,b}v_0}{2}\frac{\mathrm{d}^2}{\mathrm{d}z^2}(\Delta\sigma^{a,b}) \tag{17}$$

The local rate of dilatation varies with the second derivative of the stress difference with respect to z. There is no requirement for continuity of dilatation. The rates of dilatation $\dot{V}^a$ and $\dot{V}^b$ are not independent variables, however; they are related to $\Delta\sigma^a$ and $\Delta\sigma^b$ via the viscous strain rates $\dot{\varepsilon}_\gamma^a$ and $\dot{\varepsilon}_\gamma^b$. These relationships will be elaborated below.

The boundary conditions for deformation are different from (1) when diffusion is allowed. Thus a *fraction*, f^a, of the positive dilatation rate $\dot{V}^a$ will contribute to $\dot{\varepsilon}_{xx}^a = \dot{\varepsilon}_{xx}^{\mathrm{T}}$, the strain rate in the x-direction of half-layer a. The contribution of viscous flow, $\dot{\varepsilon}_{\gamma xx}^a$, can now be less than the total strain rate. Similarly a fraction, f^b, of the negative dilatation rate $\dot{V}^b$ will contribute to $\dot{\varepsilon}_{xx}^b = \dot{\varepsilon}_{xx}^{\mathrm{T}}$, the strain rate of half-layer b in the x-direction. Loss of material may be compared to shortening, so the rate of viscous flow in half-space b now has to be faster than the total system strain rate to make up for the difference. Diffusion occurs only between the half layers and not in or out of the element, $\dot{V}^{\mathrm{T}} = 0$. The boundary conditions are:

$$\dot{\varepsilon}_{xx}^a = \dot{\varepsilon}_{\gamma xx}^a\langle z\rangle + f^a\dot{V}^a\langle z\rangle = \dot{\varepsilon}_{xx}^{\mathrm{T}} \tag{18a}$$

$$\dot{\varepsilon}_{zz}^a\langle z\rangle = -\dot{\varepsilon}_{\gamma xx}^a\langle z\rangle + (1-f^a)\dot{V}^a\langle z\rangle = -\dot{\varepsilon}_{xx}^{\mathrm{T}} + \dot{V}^a\langle z\rangle \tag{18b}$$

$$\dot{\varepsilon}_{xx}^b = \dot{\varepsilon}_{\gamma xx}^b\langle z\rangle + f^b\dot{V}^b\langle z\rangle = \dot{\varepsilon}_{xx}^{\mathrm{T}} \tag{18c}$$

$$\dot{\varepsilon}_{zz}^b\langle z\rangle = -\dot{\varepsilon}_{\gamma xx}^b\langle z\rangle + (1-f^b)\dot{V}^b\langle z\rangle = -\dot{\varepsilon}_{xx}^{\mathrm{T}} + \dot{V}^b\langle z\rangle \tag{18d}$$

$$\dot{V}^{\mathrm{T}} = \int_0^a \dot{V}^a\mathrm{d}z + \int_a^{a+b}\dot{V}^b\mathrm{d}z = 0 \tag{18e}$$

where the addition $\langle z\rangle$ is absent for quantities that are independent of the z-coordinate. The fractions $0 < f^a < 1$ and $0 < f^b < 1$ have been introduced as material properties for evaluation of the effects of anisotropy in removal or addition. Isotropic dilatations are obtained by putting $f^a = f^b = 0.5$.

To determine the stresses associated with the present deformation we recall that stresses are only generated by viscous flow: (7) applies. The relation between rate of dilatation and stress difference is given in (17). With these equations (18a) is rewritten:

$$\dot{\varepsilon}_{xx}^a = \dot{\varepsilon}_{xx}^{\mathrm{T}} = \frac{\Delta\sigma^a\langle z\rangle}{4\eta^a} + \left(\frac{-f^aM^av_0}{2}\right)\frac{\mathrm{d}^2}{\mathrm{d}z^2}(\Delta\sigma^a)$$

or:

$$\Delta\sigma^a\langle z\rangle = 4\eta^a\,\dot{\varepsilon}_{xx}^{\mathrm{T}} + (B^a)^2\frac{\mathrm{d}^2}{\mathrm{d}z^2}(\Delta\sigma^a) \qquad \text{for } 0 \leqslant z \leqslant a \tag{19}$$

where $B^a = \sqrt{2\eta^a f^a M^a v_0}$. This is a second order linear homogeneous differential equation with the general solution:

$$\Delta\sigma^a\langle z\rangle = 4\eta^a \dot{\varepsilon}^{\mathrm{T}}_{xx} + \left[C_1 \exp\left(\frac{z}{B^a}\right) + C_2 \exp\left(\frac{-z}{B^a}\right)\right] \qquad \text{for } 0 \leqslant z \leqslant a \tag{20}$$

where C_1 and C_2 are constants. The term between square brackets represents the influence that half-layer b has on the stress differences in half-layer a. Similarly:

$$\Delta\sigma^b\langle z\rangle = 4\eta^b \dot{\varepsilon}^{\mathrm{T}}_{xx} + \left[C_3 \exp\left(\frac{z}{B^b}\right) + C_4 \exp\left(\frac{-z}{B^b}\right)\right] \qquad \text{for } a \leqslant z \leqslant a+b \tag{21}$$

where $B^b = \sqrt{2\eta^b f^b M^b v_0}$ and C_3 and C_4 are constants to be determined. Note that B^a and B^b have dimension L. This length is a material property called *characteristic diffusion length* (Fletcher, 1982).

The conditions for continuity of chemical potential at the contact (13), for continuity of flux at the contact and zero flux at the extremities of the element (16) and for constant volume (18e) are sufficient to solve the unknown constants in (20) and (21). The persistent reader will verify that the final result can be written as:

$$\left.\begin{aligned} \frac{\Delta\sigma^a\langle z\rangle}{4\eta^a} &= \dot{\xi}^a_{xx}\langle z\rangle = \dot{\varepsilon}^{\mathrm{T}}_{xx}\left[1 - P\cosh\left(\frac{z}{B^a}\right)\right] && \text{for } 0 \leqslant z \leqslant a \\ \frac{\Delta\sigma^b\langle z\rangle}{4\eta^b} &= \dot{\xi}^b_{xx}\langle z\rangle = \dot{\varepsilon}^{\mathrm{T}}_{xx}\left[1 - Q\cosh\left(\frac{z-(a+b)}{B^b}\right)\right] && \text{for } a \leqslant z \leqslant a+b \end{aligned}\right| \tag{22}$$

where:

$$P = \left(1 - \frac{\eta^b}{\eta^a}\right)\left[\frac{B^b M^a}{B^a M^b}\sinh\left(\frac{a}{B^a}\right)\coth\left(\frac{b}{B^b}\right) + \cosh\left(\frac{a}{B^a}\right)\right]^{-1}$$

$$Q = \left(1 - \frac{\eta^a}{\eta^b}\right)\left[\frac{B^a M^b}{B^b M^a}\sinh\left(\frac{b}{B^b}\right)\coth\left(\frac{a}{B^a}\right) + \cosh\left(\frac{b}{B^b}\right)\right]^{-1}$$

From these both the differential stress and the viscous strain rate may be determined as a function of the z-coordinate, the imposed strain rate, the layer-thicknesses and the physical properties of the two materials. Substitution in (18a) and (18c) yields the local rates of dilatation:

$$\left.\begin{aligned} \dot{V}^a\langle z\rangle &= \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{P}{f^a}\cosh\left(\frac{z}{B^a}\right) && \text{for } 0 \leqslant z \leqslant a \\ \dot{V}^b\langle z\rangle &= \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{Q}{f^b}\cosh\left(\frac{z-(a+b)}{B^b}\right) && \text{for } a \leqslant z \leqslant a+b \end{aligned}\right| \tag{23}$$

Equations (22) and (23) have been derived for $\eta^a > \eta^b$. It is easily seen that they are also valid for the reverse case. To attain periodic equations for the full multilayer, we prescribe that the results valid for $0 \leqslant z \leqslant a$ are also valid for z-values such that

$0 \leqslant |z| - 2n(a+b) \leqslant a$, where n is 0, 1, 2,... and that the results valid for $a \leqslant z \leqslant a+b$ are also valid for z values such that $a \leqslant |z| - 2n(a+b) \leqslant a+b$ (Fig. 2a).

The rate at which volume is removed from one layer-half and added to the neighbouring layer-half of contrasting material is:

$$\int_0^a \dot{V}^a \mathrm{d}z = -\int_a^{a+b} \dot{V}^b \mathrm{d}z = \dot{\varepsilon}^{\mathrm{T}}_{xx} \frac{2v_0(\eta^a - \eta^b)}{\dfrac{B^a}{M^a}\coth\left(\dfrac{a}{B^a}\right) + \dfrac{B^b}{M^b}\coth\left(\dfrac{b}{B^b}\right)} \tag{24}$$

In the regular multilayer under consideration *double* the absolute value of (24) is removed per unit time from each layer with lesser viscosity and added to each layer with higher viscosity.

To summarize the results of this section: for every z there exist linear relationships between the strain rate imposed on the system, $\dot{\varepsilon}^{\mathrm{T}}_{xx}$, the differential stresses in the materials (22), the contribution of viscous flow to total strain rate (22) and the local rate of dilatation (23). The rate of transport from one layer to the next is also linearly related to strain rate (24).

Special cases

The general solutions for an element of a regular multilayer (22)–(24) are used in this section to derive relations for selected special cases.

(a) *No diffusion.* If the mobilities of the diffusing components M^a and M^b and, by consequence, the characteristic diffusion lengths B^a and B^b are reduced to zero, then, as would be expected, eqn. (22) is reduced to eqn. (7).

(b) *Very easy diffusion.* If the characteristic diffusion lengths are much longer than the corresponding half-layer thicknesses, i.e. if $B^a \gg a$ and $B^b \gg b$, then differential stress will be almost constant throughout the multilayer. Such a situation arises, for example, when the mobilities M^a and M^b are very high and if the layers are very thin. In the limits $\lim(a/B^a)\downarrow 0$ and $\lim(b/B^b)\downarrow 0$, the differential stresses $\Delta\sigma^a$ and $\Delta\sigma^b$ are equal for all values of z, because then:

$$\left.\begin{aligned} \frac{\Delta\sigma^a}{4\eta^a} &= \dot{\varepsilon}^a_{\mathrm{f}xx} = \dot{\varepsilon}^{\mathrm{T}}_{xx}\eta^b\left(\frac{af^b + bf^a}{af^b\eta^b + bf^a\eta^a}\right) && \text{for } |z| \leqslant a \\ \frac{\Delta\sigma^b}{4\eta^b} &= \dot{\varepsilon}^b_{\mathrm{f}xx} = \dot{\varepsilon}^{\mathrm{T}}_{xx}\eta^a\left(\frac{af^b + bf^a}{af^b\eta^b + bf^a\eta^a}\right) && \text{for } a \leqslant |z| \leqslant a+b \end{aligned}\right| \tag{25}$$

Also, the highest rates of addition and removal possible in the present model are attained:

$$\left.\begin{aligned}\dot{V}^a &= \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{(\eta^a-\eta^b)b}{af^b\eta^b+bf^a\eta^a} && \text{for } |z|\leqslant a\\ \dot{V}^b &= \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{(\eta^b-\eta^a)a}{af^b\eta^b+bf^a\eta^a} && \text{for } a\leqslant|z|\leqslant a+b\end{aligned}\right\} \tag{26}$$

$$\int_0^a \dot{V}^a \mathrm{d}z = -\int_a^{a+b}\dot{V}^b\mathrm{d}z = \frac{\dot{\varepsilon}^{\mathrm{T}}_{xx}(\eta^a-\eta^b)a\cdot b}{af^b\eta^b+bf^a\eta^a} \tag{27}$$

(c) *The single layer embedded in an infinite medium*. This important case is derived from the general element by letting half-layer thickness b increase to infinity (Fig. 3a). Differential stress and strain rate are now:

$$\left.\begin{aligned}\frac{\Delta\sigma^a\langle z\rangle}{4\eta^a} &= \dot{\gamma}^a_{xx}\langle z\rangle = \dot{\varepsilon}^{\mathrm{T}}_{xx}\left[1-R\cosh\left(\frac{z}{B^a}\right)\right] && \text{for } |z|\leqslant a\\ \frac{\Delta\sigma^b\langle z\rangle}{4\eta^b} &= \dot{\gamma}^b_{xx}\langle z\rangle = \dot{\varepsilon}^{\mathrm{T}}_{xx}\left[1-S\exp\left(\frac{-|z|+a}{B^b}\right)\right] && \text{for } |z|\geqslant a\end{aligned}\right\} \tag{28}$$

where:

$$R = \left(1-\frac{\eta^b}{\eta^a}\right)\left[\frac{B^bM^a}{B^aM^b}\sinh\left(\frac{a}{B^a}\right)+\cosh\left(\frac{a}{B^a}\right)\right]^{-1}$$

$$S = \left(1-\frac{\eta^a}{\eta^b}\right)\left[\frac{B^aM^b}{B^bM^a}\coth\left(\frac{a}{B^a}\right)+1\right]^{-1}$$

The local rate of dilatation is:

$$\left.\begin{aligned}\dot{V}^a\langle z\rangle &= \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{R}{f^a}\cosh\left(\frac{z}{B^a}\right) && \text{for } |z|\leqslant a\\ \dot{V}^b\langle z\rangle &= \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{S}{f^b}\exp\left(\frac{-|z|+a}{B^b}\right) && \text{for } |z|\geqslant a\end{aligned}\right\} \tag{29}$$

and the rate at which material is transported through a layer-medium contact is:

$$\int_0^a \dot{V}^a\mathrm{d}z = -\int_a^{\infty}\dot{V}^b\mathrm{d}z = \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{2v_0(\eta^a-\eta^b)}{\dfrac{B^a}{M^a}\coth\left(\dfrac{a}{B^a}\right)+\dfrac{B^b}{M^b}} \tag{30}$$

(d) *Contacting half-spaces*. This is a limit of the general element in which both half-layers a and b have infinite thickness. Since we are interested in the effects of diffusion in the vicinity of the contact between half-spaces (there are no effects far away from the contact), it is profitable to perform the transformation:

$$z' = z - a \tag{31}$$

whereby positive z' values lie within the half-space with b properties and negative z' values within the half-space with a properties. Now, recovering results obtained by Fletcher (1982):

$$\left.\begin{aligned} \frac{\Delta\sigma^{a}\langle z'\rangle}{4\eta^{a}} &= \dot{\varepsilon}^{a}_{xx}\langle z'\rangle = \dot{\varepsilon}^{\mathrm{T}}_{xx}\left[1 - T\exp\left(\frac{z'}{B^{a}}\right)\right] \qquad \text{for } z' \leqslant 0 \\ \frac{\Delta\sigma^{b}\langle z'\rangle}{4\eta^{b}} &= \dot{\varepsilon}^{b}_{xx}\langle z'\rangle = \dot{\varepsilon}^{\mathrm{T}}_{xx}\left[1 - U\exp\frac{-z'}{B^{b}}\right] \qquad \text{for } z' \geqslant 0 \end{aligned}\right\} \tag{32}$$

where:

$$T = \left(1 - \frac{\eta^{b}}{\eta^{a}}\right)\left[\frac{B^{b}M^{a}}{B^{a}M^{b}} + 1\right]^{-1}$$

$$U = \left(1 - \frac{\eta^{a}}{\eta^{b}}\right)\left[\frac{B^{a}M^{b}}{B^{b}M^{a}} + 1\right]^{-1}$$

$$\left.\begin{aligned} \dot{V}^{a}\langle z'\rangle &= \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{T}{f^{a}}\exp\left(\frac{z'}{B^{a}}\right) \qquad \text{for } z' \leqslant 0 \\ \dot{V}^{b}\langle z'\rangle &= \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{U}{f^{b}}\exp\left(\frac{-z'}{B^{b}}\right) \qquad \text{for } z' \geqslant 0 \end{aligned}\right\} \tag{33}$$

$$\int_{-\infty}^{0} \dot{V}^{a}\mathrm{d}z' = -\int_{0}^{\infty} \dot{V}^{b}\mathrm{d}z' = \dot{\varepsilon}^{\mathrm{T}}_{xx}\frac{2v_{0}(\eta^{a} - \eta^{b})}{\dfrac{B^{a}}{M^{a}} + \dfrac{B^{b}}{M^{b}}} \tag{34}$$

EVALUATION

Layer-thickness, viscosity, mobility of the migrating component and anisotropy factor all affect stress difference, rate of viscous flow and rate of dilatation in a deforming multilayer. This combined influence of dimension and physical parameters is best illustrated for a single layer embedded in an infinite medium (eqns. 28–30).

An arbitrarily chosen reference case is such that: (1) the layer has a viscosity ten times that of the medium; (2) the diffusing component mobilities are the same in layer and medium; (3) there is no anisotropy in removal or addition of material; (4) layer-thickness is equal to the characteristic diffusion length of the layer material in the reference state, i.e. $2a$ (ref) $= B^{a}$ (ref) $= \sqrt{10}\ B^{b}$ (ref). In the comparisons to be made one property of the layer is varied while the others and those of the medium are described as remaining constant. Any change in viscosity, mobility or anisotropy factor of the layer also alters its characteristic diffusion length, B^a (eqn. 19).

The influence of layer-thickness (Fig. 5)

Consider the reference case first. Figure 5b illustrates the effect of diffusion on stress difference. The ratio of local stress difference to the stress difference in the medium infinitely far away from the layer has been plotted against the ratio of the z-coordinate to the half-layer thickness a. In the absence of diffusion the ratio of differential stress would be 10 in the layer and 1 in the medium for the case $\eta^a = 10\eta^b$, as is indicated by dotted lines. With diffusion differential stresses are lower in the layer and higher in the medium, particularly near the contact where they become equal. Differential stress in the center of the layer, $z/a = 0$, is 30% of what it would have been without diffusion. The viscous strain rate is lowered correspondingly. This is shown in Fig. 5e by the graph marked $\dot{\varepsilon}_l/\dot{\varepsilon}^{\mathrm{T}}$ representing the ratio of the local viscous strain rate in the x-direction to the viscous strain rate infinitely far away from the layer, i.e. to the strain rate imposed on the system as a whole. It is crucial to the model, however, that the layer extension in the x-direction equals the rate of extension of the total system. What is not achieved by viscous flow is made up by material addition. The dilatation being isotropic in the present case, $f^a = 0.5$, only half of the addition of volume contributes to extension in the x-direction. In the centre of the layer the rate of dilatation therefore has to be 1.4 times the imposed total strain rate. The viscous contribution and the dilatational contribution match to obtain the total strain rate: $0.3\dot{\varepsilon}^{\mathrm{T}} + \frac{1}{2} \times 1.4\dot{\varepsilon}^{\mathrm{T}} = \dot{\varepsilon}^{\mathrm{T}}$. The local rate of dilatation is represented in Fig. 5e by the graph marked $\dot{V}/\dot{\varepsilon}^{\mathrm{T}}$. The model assumption that the system as a whole remains constant in volume at any time t is brought out by equality of the areas under the dilatation curves for layer and medium.

In layers ten times thicker than the reference case (Fig. 5a, d) virtually all strain in the center of the layer is achieved by viscous flow. In a layer ten times thinner than the reference case (Fig. 5c, f) most of the deformation is achieved by material transport from the medium to the layer, and differential stress in the layer is almost as low as in the medium at infinity.

Figure 5 illustrates another important point. The length of diffusional influence on the medium caused by the presence of a layer in it does not depend on layer-thickness. In the cases considered it is seen to extend to about 0.4, 4 and 40 half-layer thicknesses away from the contact. Figures 5a and 5d show that the medium exerts an influence on what happens in the layer to much larger absolute distances. This is not because there is more medium than layer material but because of the characteristic diffusion lengths for the layer and medium materials. B^a and B^b are indicated by arrows in Fig. 5a, b, c. Beyond about six times the characteristic diffusion length away from the contact with another material, deformation proceeds as if there were no such material present.

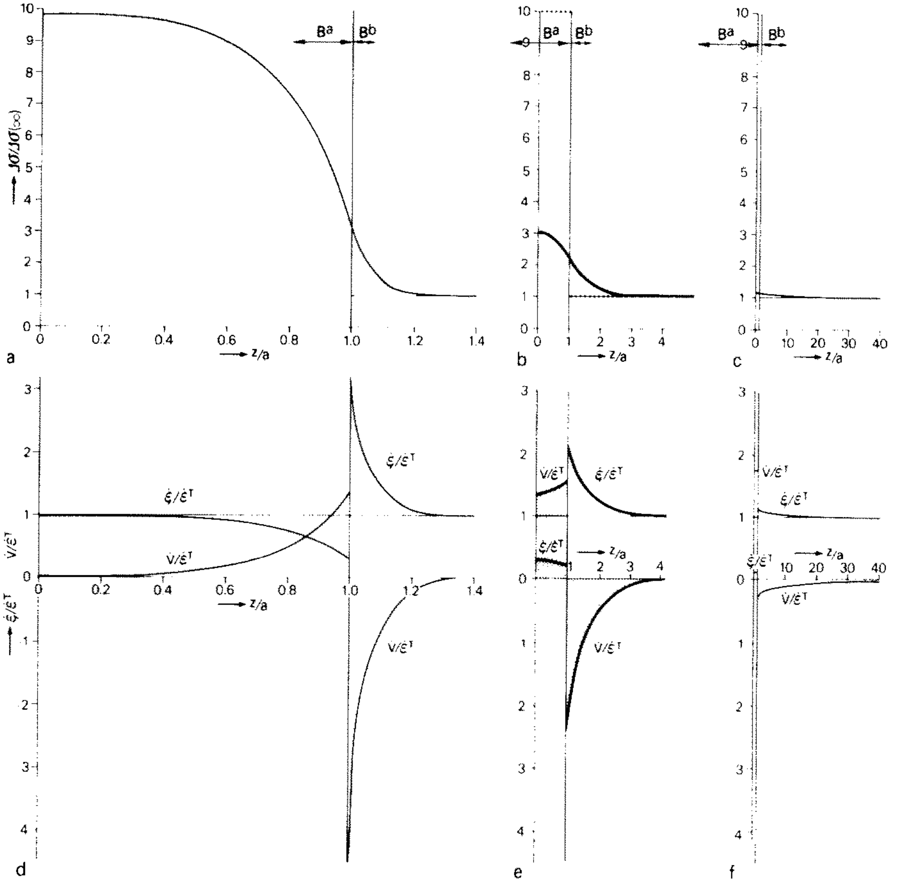

Fig. 5. The effect of layer-thickness. The reference case is shown in the centre; layers ten times thicker and ten times thinner to the left and right respectively.

a–c. Differential stress expressed as multiples of the differential stress infinitely far away from the layer and plotted against multiples of the distance from the centre of the layer to the contact with the medium. Stress levels in the absence of diffusion are shown by dotted lines in (b).

d–f. Similar plots for viscous strain-rate and rate of dilatation compared to the strain rate imposed on the system. Dotted areas in (e) represent volumes removed and added per unit time. They are equal in magnitudes: deformation is volume constant.

The influence of viscosity (Fig. 6)

Making the layer more viscous than in the reference case, of course, increases the stress difference, but not by much. If the viscosity ratio increases from 10 to 100, the stress ratio will increase from 10 to 100 in the absence of diffusion, with diffusion only from 3 to 3.75 in the centre of the layer and less than that at the contact. This is mainly due to the increase of B^a by a factor $\sqrt{10}$ (Fig. 6b, eqn. 19). Now the

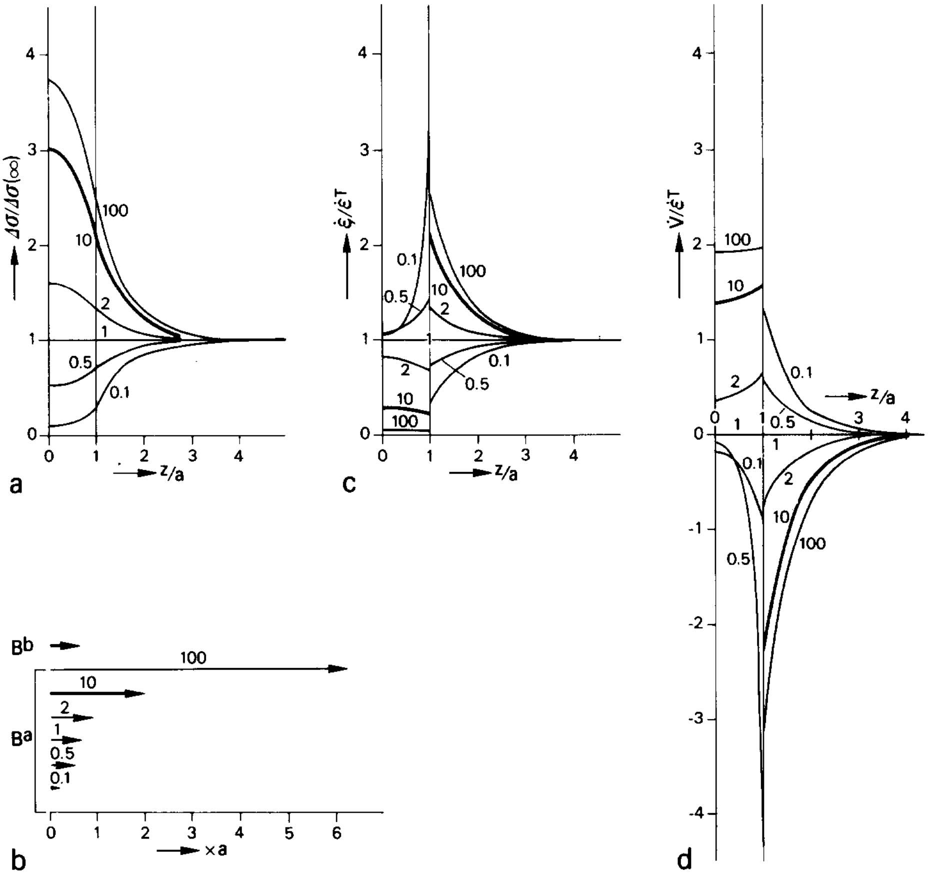

Fig. 6. The effect of viscosity. Only the viscosity of the layer is changed with respect to the reference case (heavy line). Numbers on curves and arrows represent the ratio between layer and medium viscosities. a. Stress difference. b. Characteristic diffusion lengths. c. Viscous strain rate. d. Rate of dilatation.

thickness of the layer is small compared to the characteristic diffusion length and the case of very easy diffusion, described in eqns. (25), (26) and (27) is approached: almost all elongation of the layer in the x-direction is achieved by dilatation (Fig. 6c, d).

For viscosity ratios less than unity the roles are reversed, material is dissolved from the layer and deposited in the medium on either side. The reduction of the characteristic diffusion length effectively restricts diffusional effects in the layer to the margin with the medium. In the centre of the layer, differential stress ratios are near the viscosity ratio (Fig. 6a) and the rate of dilatation is very small (Fig. 6c, d).

Comparison of the areas under the dilatational curves in Fig. 6d indicates that the nearer the viscosity ratio comes to unity, the smaller the amount of material diffusing per unit time between the medium and the layer. This applies whether the layer is less or more viscous than the medium. All physical properties of the medium remain constant in Fig. 6. Clearly, in the medium near the layer stress difference, viscous strain rate and rate of dilation depend on the magnitude of the physical properties of the layer, but the distance over which diffusional effects are felt remains unchanged at about six times the characteristic diffusion length of the medium material.

The influence of mobility (Fig. 7)

It might appear from the foregoing that the characteristic lengths are paramount in determining the amount and nature of the diffusion across contacts as well as the length-scale of diffusion. This is not so, as may be seen by comparing results for different combinations of physical properties that yield the same characteristic diffusion length. Figures 7a, c and d show the effect of changing the mobility of the diffusing component in the layer material. Compare, for example, the curves marked *10* in Fig. 7 (for a mobility ratio of 10 at a standard viscosity ratio of 10) with the curves marked *100* in Fig. 6 (for a viscosity ratio of 100 at standard mobility ratio 1). Figures 7b and 6b show these cases to have equal characteristic diffusion lengths. The stress difference ratio varies appreciably through the layer in Fig. 6a, but it remains nearly constant in Fig. 7a. For the highly viscous layer more than 90% of its elongation is achieved by mass-addition (Fig. 6c, d); for the high-mobility, lower viscosity layer of Fig. 7c, d viscous flow accounts for almost one quarter of the total strain rate. Comparison of the case without viscosity contrast in Fig. 6 ($\eta^a/\eta^b = 1$) with that for a mobility ratio of 0.1 in Fig. 7 brings out this point even more.

Four orders of mobility ratio are shown in Fig. 7. Obviously, variation of the mobility can—but need not—cause major changes in the stress difference and the rates of dilatation and viscous flow. For the layer as a whole, these effects are most marked when the change of mobility ratio is such that the characteristic diffusion length is altered toward or away from being close to the thickness of the layer. Finally, it takes two to diffuse. Low mobility in one material cuts down mass-transport no matter how high the characteristic diffusion length in the second material may be.

The influence of anisotropy (Fig. 8)

Thus far, we have considered dilatations in which removal and addition of material are independent of direction. The model's assumption that gradients in the (non-directional) hydrostatic component of stress govern diffusion does, however, by no means exclude anisotropic volume change. The extent of anisotropy is varied in the model by an independent physical property f. A mass transport that has the highest rate of take-away from the z-direction and the highest rate of addition in the x-direction allows the imposed total strain rate to be reached at lower differential

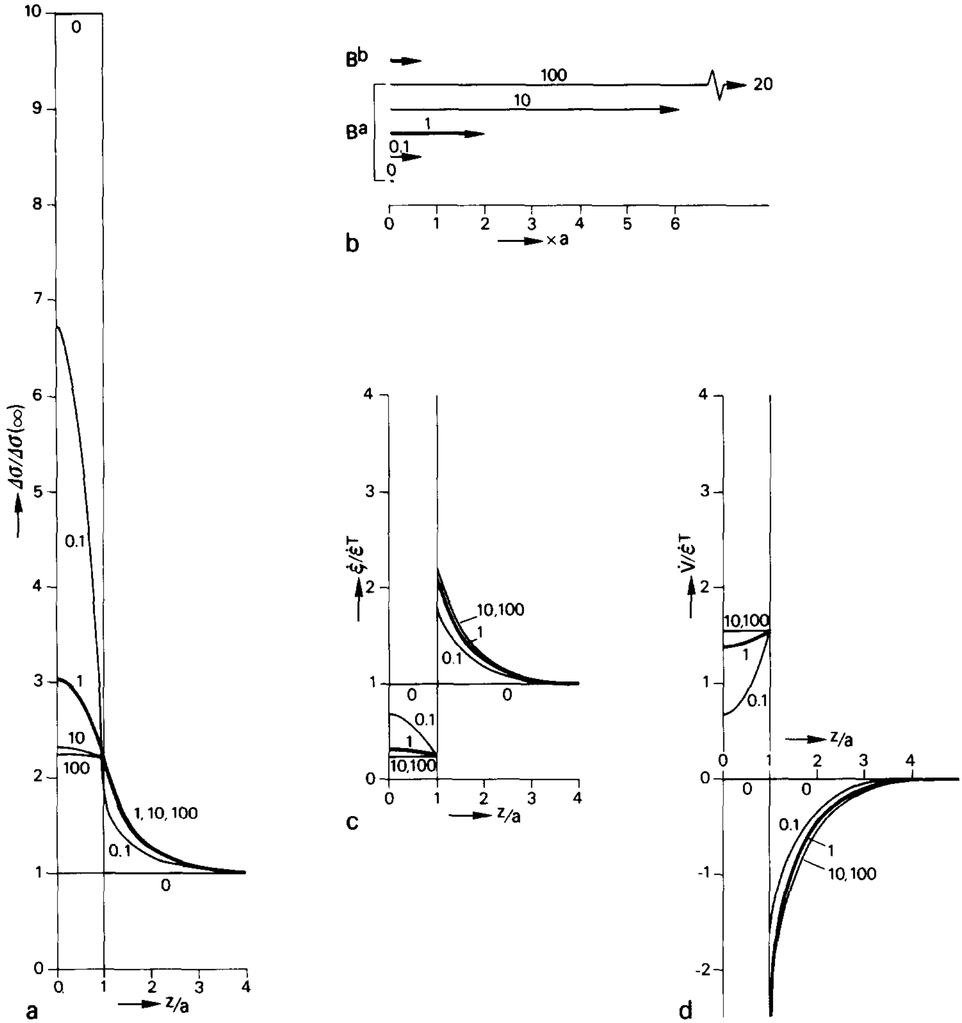

Fig. 7. The effect of mobility. The mobility of the diffusing component is changed for the layer only with respect to the reference case (heavy line). Numbers on curves represent the ratio between mobilities in layer and medium. a–d as in Fig. 6.

stresses than in the isotropic case. This is shown in Fig. 8a. The stress ratio curve marked 0.9, 0.5 represents a layer in which 90% of the added material is added in the x-direction; the mass-removal in the medium is still isotropic. On the right hand of Fig. 8a it is shown how differential stress diminishes further if 90% of the mass removed is taken away from the z-direction.

The right-hand sides of Figs. 8a, c, d represent the first cases in this evaluation in which a physical property of the medium has been varied. The medium's characteris-

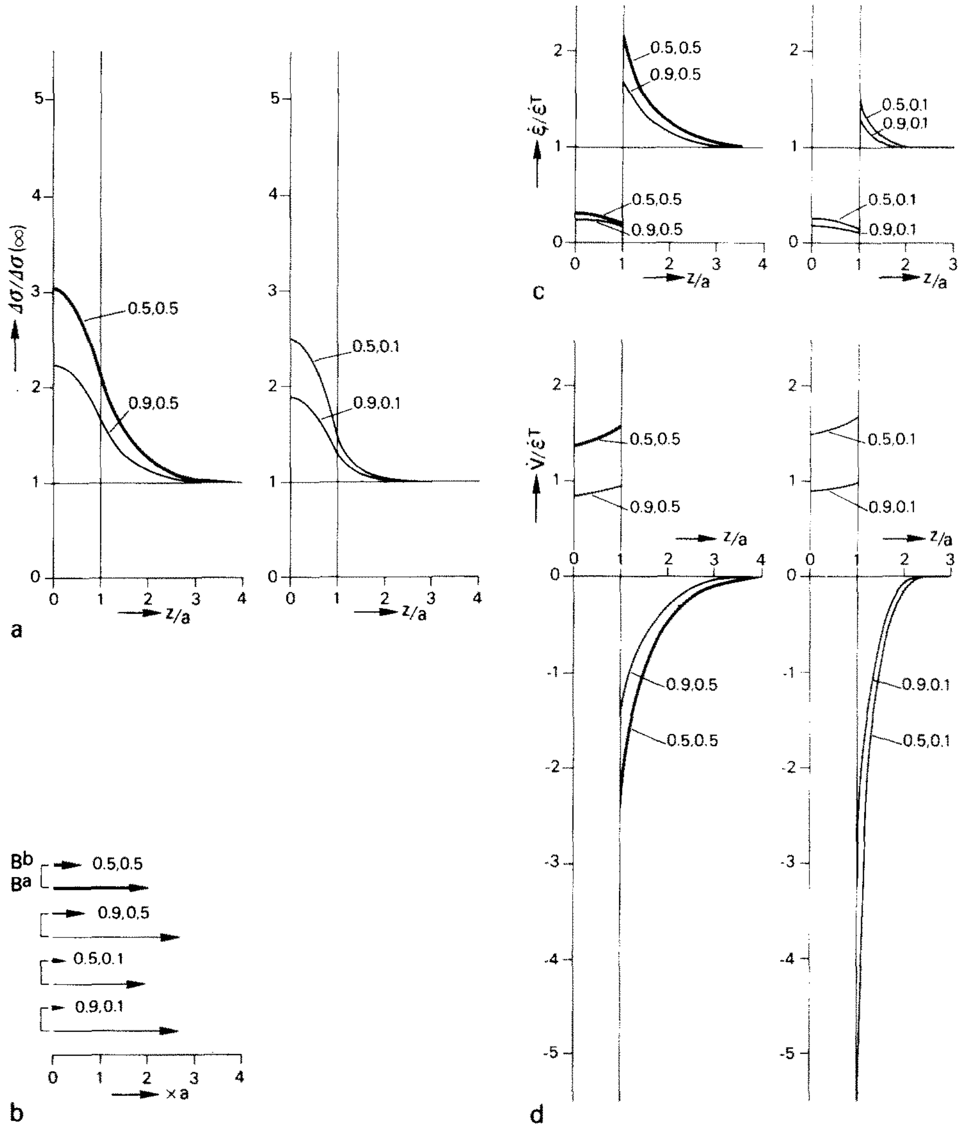

Fig. 8. The effect of anisotropy. First number on curves is the anisotropy factor of the layer, second number that of the medium. a–d as in Fig. 6.

tic diffusion length has been reduced by a factor $\sqrt{5}$ (Fig. 8b). The depth into the medium from which material is removed to contribute to extension of the layer has, therefore, been roughly halved. Anisotropy of the type described also reduces the amount of material that has to be transported per unit time across a layer/medium contact. This becomes apparent when in Fig. 8d the areas under the curves marked 0.5, 0.5 are compared with those marked 0.9, 0.1.

Fig. 9. Gneiss with irregularly spaced differentiation packets. From the same area as the examples in Fig. 1.

DISCUSSION

It has been shown that layer thickness, viscosity, mobility of a migrating component and anisotropy in removal or addition of material all affect the interplay of diffusion and deformation of a layered structure. In spite of the lengthiness of the derived equations the model is straightforward. It deals with a very simple instantaneous case only.

When mass transfer proceeds for some time the physical properties of contacting materials could become dependent on the z-coordinate. Viscosity and anisotropy, for instance, might vary with time and with position as strain increases and more and more material is added or removed. One could also theoretically investigate the effect of non-linear rheology and make that dependent or independent of time (strain) and position. At present, however, such sophistication would tend to obscure rather than clarify the problem.

Physical properties are expected to vary during the syntectonic differentiation from a more or less homogeneous rock into layered gneiss (Robin, 1979). Application of the simple theory can nevertheless be illustrative when considering this process. In outcrop, layered gneisses often show remarkable periodicity (Fig. 1b). If such regularity is absent one notes a rather limited range in the average thickness of the layers that have grown from their surroundings (Fig. 9). In the latter case there are three phases: a medium that is anisotropic but undifferentiated and, at irregular intervals, differentiation "packets" that consist of two marginal layers depleted in some component and a central layer that is enriched in it (for ease of discussion a closed system is assumed to apply to this natural example). I propose that the differentiation in Fig. 9 is a forerunner to periodic layering. Increasing the *number* of such packets rather than increasing their individual thickness will result in periodic layering. In this respect it is relevant that the simple theory predicts the existence of a *stable layer-thickness* for the more viscous material. Both thicker and thinner more viscous layers converge to it during deformation. This is elaborated below.

The rate of layer-normal shortening of an isolated layer embedded in an infinite medium (or at least distant enough from other layers to be considered as such) can be obtained from eqns. (18b) and (26) for every value of z between 0 and a. The average strain rate of the half-layer in the z-direction is given by:

$$\overline{\dot{\varepsilon}^a_{zz}} = \frac{1}{a}\int_0^a \dot{\varepsilon}^a_{zz} \mathrm{d}z = -\dot{\varepsilon}^{\mathrm{T}}_{xx} + \frac{1}{a}\int_0^a \dot{V}^a \mathrm{d}z \tag{35}$$

Depending on the magnitude of the second term on the right-hand side the half-layer may grow in thickness (positive average strain rate) or become thinner (negative average strain rate); constant thickness is maintained if:

$$\overline{\dot{\varepsilon}^a_{zz}} = 0 \tag{36}$$

The *stable half-layer thickness* is given by:

$$a^* = \frac{\int_0^{a^*} \dot{V}^a \mathrm{d}z}{\dot{\varepsilon}^{\mathrm{T}}_{xx}} \tag{37}$$

Inspect Fig. 5 once more. The upper dotted area in Fig. 5e represents the volume added per unit time to the growing half-layer. This volume is derived from medium to the right. By symmetry the same amount will be added to the left-hand layer-half from medium to the left (not shown). Equation (37) states that the average level of the $\dot{V}/\dot{\varepsilon}^{\mathrm{T}}$ curves within the viscous layer on such a graph would have to lie at 1 for stable layer thickness. For the thin layers of Fig. 5e and f this level is exceeded. The layers will grow, the thinner the layer the faster the rate of its growth. In the thick layer of Fig. 5d, on the other hand, the average level of the $\dot{V}/\varepsilon^{\mathrm{T}}$ curve lies well below 1. Layer-thickness will therefore be reduced during deformation. Between the thickness of Fig. 5d and that of Fig. 5e lies the stable value that will be dynamically maintained during deformation.

The value of a^* is found from (37), (30) and the formula for characteristic diffusion length (19) to be:

$$\frac{a^*}{B^a} = \frac{1}{f^a}\left(1 - \frac{\eta^a}{\eta^b}\right) \Big/ \left[\coth\frac{a^*}{B^a} + \sqrt{\frac{\eta^b}{\eta^a} \cdot \frac{M^a}{M^b} \cdot \frac{f^b}{f^a}}\right] \tag{38}$$

To solve analytically for a^*/B^a is cumbersome. Numerically, however, solutions are readily found with a pocket calculator or tables for hyperbolic functions. For the physical properties used in Fig. 5 ($\eta^a/\eta^b = 10$; $M^a/M^b = 1$; $f^a = f^b = 0.5$) it is found that $1.17\ B^a < a^* < 1.19\ B$. The corresponding stable layer-thickness, $2a^*$, is thus approximately 2.36 times the characteristic diffusion length B^a. As seen from eqn. (38) stable layer-thickness for more viscous layers is a physical property of the system, it depends on the properties of the medium as well as on those of the layer material itself.

A similar analysis could be done for the more complicated case of differentiation packets that are near to each other such that they may no longer be considered as being isolated in an infinite medium. The essential elements of the argument would not be changed. In natural cases, where model assumptions do not apply, one also expects the existence of a stable layer-thickness, or of a scale of periodicity that is dependent on the physical properties of the constituent materials only. If this length scale is of the same magnitude or less than the average grain-size in the rock (thought to be recrystallizing during deformation) one should not expect significant differentiation to occur. If the stable length is greater than grain size—for instance because of large characteristic diffusion lengths in the presence of a pore fluid enhancing effective mobility—differentiation will result in a layered structure. Instabilities that form at different times and on different scales will eventually converge to a regular layered pattern.

It is a large step from an instantaneous analysis for viscous fluids to the natural structures shown in Figs. 1 and 9. These are the products of long and varied histories of diffusion and deformation in real rocks. The complications that may arise by a superposition of changing conditions are innumerable (some of these are discussed in part II) and in general one will not be able fully to deduce a growth-history from its final product. Enumerating observed and theoretically possible complexities is one thing, trying to understand the underlying principles can be more rewarding.

REFERENCES

Fletcher, R.C., 1982. Coupling of diffusional mass transport and deformation in a tight rock. Tectonophysics, 83: 275–291.

Ramberg, H., 1949. The facies classification of rocks: a clue to the origin of quartzo-feldspathic massifs and veins. J. Geol., 57: 18–54.

Ramberg, H., 1955. Natural and experimental boudinage and pinch-and-swell structures. J. Geol., 63: 512–526.

Ramberg, H., 1956. Pegmatites in West Greenland. Geol. Soc. Am. Bull., 67: 185–214.

Robin, P.-Y.F., 1979. Theory of metamorphic segregation and related processes. Geochim. Cosmochim. Acta, 43: 1587–1600.

Strömgård, K.-E., 1973. Stress distribution during formation of boudinage and pressure shadows. Tectonophysics, 16: 215–248.