Cox, J. A. M.
Groot, S. R. de,
Hartogh, Chr. D.
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NUMERICAL CALCULATIONS FOR THE ANGULAR DISTRIBUTION OF GAMMA RADIATION EMITTED BY ORIENTED ^{58}Co NUCLEI

by J. A. M. COX *), S. R. DE GROOT and CHR. D. HARTOGH

Instituut voor theoretische natuurkunde, Universiteit, Utrecht, Nederland

Synopsis

In this note the theoretical results for the angular distribution of γ-radiation emitted by oriented radioactive nuclei are applied to the case of ^{58}Co nuclei. The angular distribution function of the γ-radiation has been calculated for an arbitrary degree of nuclear orientation and in dependence of a parameter, which describes the character of the β^+-transition or the K-capture preceding the γ-transition.

§ 1. *Introduction*. When the quantisation axis η is an axis of rotational symmetry of the initial ^{58}Co nuclei, the state of orientation of the nuclei can be described by $2j_0$ independent parameters $f_k(j_0)$ (cf. [1]) formula 25 and 32). j_0 is the spin quantum number of the nuclei. These parameters $f_k(j_0)$ are completely determined by j_0 and by the relative populations a_{m_0} of the magnetic sublevels m_0

$$f_k(j_0) = w_k(j_0)\ \Sigma_{m_0} < m_0|\varrho|\ m_0 > (-1)^{j_0-m_0} < j_0 m_0, j_0 - m_0\ |(j_0 j_0)\ k\ 0 >. \quad (1)$$

In the special case under consideration the nuclei are contained in a tutton salt [2]) [3]) [4]) [5]) where due to the orientation mechanism the levels m_0 and $-m_0$ are equally favoured. This is expressed also in the formula for a_{m_0} by

$$a_{m_0} = C \cosh (\beta m_0), \quad (2)$$

where C is determined by the normalization condition

$$\Sigma_{m_0} a_{m_0} = 1. \quad (3)$$

In (2) β is considered as a parameter which can vary from 0 (random orientation) to ∞ (total orientation). We assume the ^{58}Co-nuclei to decay according to the scheme [6]) shown in fig. 1.

*) Present address: Van der Waals laboratorium, Universiteit, Amsterdam, Nederland

For the calculation of the angular distribution of the γ quadrupole radiation we can use the formula (cf. [1]) formula 93)

$$W(\vartheta)=2[1-(\tfrac{15}{7})N_2(j_i)\, f_2(j_i)\, P_2(\cos\vartheta)-5N_4(j_i)\, f_4(j_i)\, P_4(\cos\vartheta)], \quad (4)$$

where ϑ is the angle between the direction of emission $\mathbf{k}$ of the γ-quantum and $\boldsymbol{\eta}$. The functions P_2 ($\cos\vartheta$) and P_4 ($\cos\vartheta$) are normalized Legendre polynomials. The distribution function $W(\vartheta)$ is normalized to

$$\int W(\vartheta)\, d\Omega = 8\pi. \quad (5)$$

In order to find the distribution function (4) we have calculated the orientation parameters $f_k(j_i)$ from the initial parameters $f_k(j_0)$ before the β^+-transition or K-capture. The relation between them is given by (cf. [7]) formulae 28, 29)

$$f_2(j_i) = \tfrac{1}{2}\,(1+\lambda)\, f_2(j_0), \quad (6)$$

$$f_4(j_i) = \tfrac{1}{3}\,(-2+5\lambda)\, f_4(j_0). \quad (7)$$

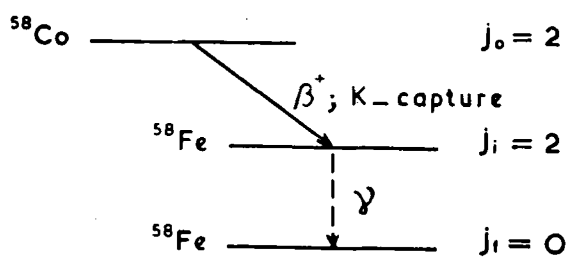

Fig. 1. Decay-scheme of ^{58}Co.

Here λ ($0 \leqslant \lambda \leqslant 1$) is a parameter, which describes the character of the transition $j_0 \to j_i$. When this process is determined by the squared nuclear matrix element $|\int 1|^2$ the parameter $\lambda = 1$ (Fermi type interaction). For Gamow-Teller interaction only $|\int \boldsymbol{\sigma}|^2$ occurs and then $\lambda = 0$.

Ultimately we obtain an expression for $W(\vartheta)$ which depends on the parameters β and λ. The procedure of the calculations and the numerical results will be given in section 2.

§ 2. *Procedure of the calculations and numerical results.* Since only f_k with k even are needed we can use instead of (2) the Boltzmann distribution

$$a_{m_0} = C \exp(\beta m_0). \quad (8)$$

We have calculated a_{m_0} for several values of β ($0 \to \infty$). With these values $f_2(j_0)$ and $f_4(j_0)$ are calculated from (1) with the quantum number $j_0 = 2$ for ^{58}Co (fig. 1). With (6) and (7) this leads to the parameters $f_2(j_i)$ and $f_4(j_i)$. If we write formula (4) as

$$W(\vartheta) = 2\,[1 - C_2(\lambda, \beta)\; P_2\,(\cos \vartheta) - C_4(\lambda, \beta)\; P_4\,(\cos \vartheta)] \qquad (9)$$

it is now possible to evaluate the coefficients $C_2(\lambda, \beta)$ and $C_4(\lambda, \beta)$. In the table the results for these coefficients have been given as a function of β for values of $\lambda = 0$, $\frac{1}{2}$ and 1. On account of the strong dependence on λ it might be possible to determine λ experimentally as has been indicated before (cf. [4]) pages 8 and 9).

TABLE I

β	$C_2(0,\beta)$	$C_4(0,\beta)$	$C_2(\frac{1}{2},\beta)$	$C_4(\frac{1}{2},\beta)$	$C_2(1,\beta)$	$C_4(1,\beta)$
0	0,0000	—0,0000	0,0000	0,0000	0,0000	0,0000
0,04	0,0004	—0,0000	0,0006	0,0000	0,0008	0,0000
0,05	0,0006	—0,0000	0,0009	0,0000	0,0012	0,0000
0,10	0,0025	—0,0000	0,0037	0,0000	0,0050	0,0000
0,15	0,0055	—0,0000	0,0083	0,0000	0,0111	0,0000
0,20	0,0098	—0,0001	0,0146	0,0000	0,0195	0,0001
0,25	0,0150	—0,0001	0,0225	0,0000	0,0301	0,0002
0,30	0,0213	—0,0003	0,0319	0,0001	0,0426	0,0004
0,35	0,0284	—0,0005	0,0426	0,0001	0,0569	0,0008
0,40	0,0363	—0,0009	0,0545	0,0002	0,0727	0,0013
0,45	0,0449	—0,0013	0,0673	0,0003	0,0898	0,0020
0,50	0,0540	—0,0020	0,0810	0,0005	0,1080	0,0029
0,55	0,0635	—0,0028	0,0953	0,0007	0,1270	0,0041
0,60	0,0733	—0,0038	0,1100	0,0010	0,1467	0,0058
0,70	0,0936	—0,0064	0,1403	0,0016	0,1871	0,0095
0,80	0,1140	—0,0098	0,1709	0,0025	0,2279	0,0147
0,90	0,1340	—0,0142	0,2010	0,0035	0,2680	0,0213
1,00	0,1532	—0,0193	0,2298	0,0048	0,3065	0,0290
1,25	0,1965	—0,0353	0,2947	0,0088	0,3930	0,0529
1,50	0,2320	—0,0539	0,3479	0,0135	0,4639	0,0809
1,75	0,2601	—0,0734	0,3901	0,0183	0,5201	0,1100
2,0	0,2820	—0,0921	0,4230	0,0230	0,5640	0,1381
2,5	0,3121	—0,1241	0,4681	0,0310	0,6242	0,1862
3,0	0,3301	—0,1475	0,4951	0,0369	0,6601	0,2213
4,0	0,3473	—0,1737	0,5209	0,0434	0,6945	0,2605
5,0	0,3535	—0,1841	0,5303	0,0460	0,7070	0,2762
∞	0,3571	—0,1905	0,5357	0,0476	0,7143	0,2857

Maximum error 1 unit of the last decimal.

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