

ISOBARIC-SPIN SPLITTING OF SINGLE-PARTICLE RESONANCES

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Abstract: When a single proton is added to, or a single neutron removed from, a definite shell-model orbit in a target nucleus, the total strength can in general be divided into two parts, each part being characterized by a definite value of the isobaric spin T . This separation is pertinent even when the target isobaric spin is a redundant quantum number determined simply by the neutron excess, and probably in many cases when the isobaric spin is not conserved in the individual states which contribute to the strength. General expressions are given for the total strengths and for the strengths of the T -components. The practical problem of observing this effect as a T -splitting of giant resonances is discussed briefly.

1. Introduction

Consider the states produced when a nucleon is added in a definite shell-model orbit j (more completely nlj) to a nucleus with a positive neutron excess †† $(N-Z) = 2M$ and isobaric spin $T_{\text{target}} \equiv T_t$. Since in all cases of practical interest to us here, $T_t = M$, it is clear that addition of a neutron ($m = +\frac{1}{2}$) can lead to only one final isobaric spin $T_t + \frac{1}{2}$. On the other hand, the addition of a proton can excite states with isobaric spin $T_t \pm \frac{1}{2}$. From the preceding paper ¹⁾ it follows that even when a proton is added, states of only one isobaric spin ($T_t - \frac{1}{2}$) exist if the j -neutron shell is filled. In other cases single-particle states of both isobaric spins exist and can be excited by addition of a proton.

If we deal with heavier nuclei or with levels of sufficiently high excitation, these single-particle states can be observed as giant resonances in stripping ^{2,3)} or in elastic scattering of nucleons ^{4,5)}. The possibility of exciting states of two

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†† The notation M, m is used for the z -components of isobaric spin; M, N and Z refer to the target (in either pick-up or stripping reactions) and m refers to the transferred particle ($m = +\frac{1}{2}$ for a neutron).

different isobaric spins in proton reactions may then lead to a T -splitting of the giant resonances. In this paper we examine the possibility of observing such a splitting and consider also the strengths of the giant resonances and of their isobaric-spin components. A parallel discussion is given of the single-hole states produced in nucleon pick-up reactions. In this case a splitting is expected when a neutron is removed from an orbit which contains some protons.

One might at first object to the idea of T splitting on the ground that the isobaric-spin quantum number may not be good for excited states in heavier nuclei. It seems quite clear that T should be a rather good quantum number for the lowest states of even fairly heavy nuclei (say $A \approx 100$) but the situation for highly excited states is quite obscure. However, the whole question of the goodness of T is probably irrelevant. The essential point would seem to be that, when we add a proton to a shell which is not filled for neutrons, the system so formed has two single-particle modes of oscillation. If we had precise charge-independence these modes would be characterized by T ; but if the single-particle components are fairly far apart (say 1 MeV or more) it seems altogether probable that, while the Coulomb effects could make a large change in T , they can scarcely make any major change in the description of the two *single-particle* modes of oscillation. In this case we may as well continue to characterize them by T . The single-particle-level strength is now spread out among neighbouring states to form a giant resonance. It is possible, though not at all obviously true, that the isobaric spin is not preserved in the spreading. In this case the giant resonances (always assuming that the T splitting is large enough) could still be characterized well enough by T even though the major components of some of the contributing states could not.

As discussed earlier¹⁾ it is difficult at present to estimate the magnitude of the splitting to be expected in various cases. Just as in the energy-level case, the isobaric spin of the *final-state equivalent group* is significant also when we add a nucleon to a nucleus. In principle then one can distinguish four isobaric components of a resonance (two components with $T = T_t + \frac{1}{2}$, two with $T = T_t - \frac{1}{2}$) and splitting could occur in various ways. The final outcome would probably then depend strongly on the number of particles in the equivalent group. In order to be specific we consider in the following the cases in which the splitting is that of the total isobaric spin. More detailed results are included in the Appendix. It should be noted that the question of equivalent-group isobaric-spin conservation does not at all affect the *validity* of the equations given later (unlike the energy-level case) but instead the possibility of testing and using them. To be easily observable the splitting should be larger than the widths of the individual components[†] which for s-wave resonances near $A = 50$ are about 1–2 MeV.

[†] Components with a definite T ! If the target spin is different from 0, there is also a J -splitting but we do not consider separately the components which arise in this way.

There are not many cases of interest in which we can at present identify a T -splitting. One such involves the $2s_{\frac{1}{2}}$ single-particle levels in N^{14} discussed in the previous paper ¹). And of course, to the extent that they are describable as $d_{\frac{3}{2}}^2$, the low-lying levels of Cl^{34} could be regarded as an example. This and other cases like it are not of much interest in the present context.

2. Resonance Strengths

The strength of a level in a nucleon-transfer reaction is characterized by its "relative reduced width" $\mathcal{S} = \theta^2/\theta_0^2$, the reduced width in units of the single-particle width ⁶). Including spin statistical factors and the isobaric-spin coupling factor [†] $(C)^2$, we define the strength G of a group of levels as

$$G = \sum \frac{2J_t+1}{2J_i+1} (C)^2 \mathcal{S} \quad (1)$$

for capture reactions (elastic scattering or stripping) and

$$G = \sum (C)^2 \mathcal{S} \quad (2)$$

for pick-up. The summations extend over all levels we wish to consider but must include only one l -value since we have divided out the l -dependent quantity θ_0^2 . Except for purely dynamical factors, G characterizes ^{††} the experimental cross section.

We consider the target isobaric spin to be decomposed into two parts

$$\mathbf{T}_t = \mathbf{T}_e + \mathbf{T}_1, \quad (3)$$

where \mathbf{T}_e refers to the entire group of j -nucleons ^{†††} in the target and \mathbf{T}_1 to all the others (inequivalent to j); of course T_e , T_1 will not in general be good quantum numbers.

As shown in the Appendix an entirely general expression for the total strength and its separate T components in a stripping or elastic scattering

[†] See eqs. II.19 and II.21 of ref. ⁶).

^{††} In other words, the 'formal' strength defined by eq. (1) or eq. (2) is a correct measure of the true strength of a resonance. This is obvious in the case of high-resolution experiments. To describe the strength of a single resonance in a low-resolution study, following Schiffer *et al* ²) we first plot the area under the resonance as a function of angle. If the main contributions are of one l -value, the hybrid angular distribution so obtained can be approximately treated as if it arose from a single resolved level. The 'reduced width' obtained in the usual fashion is then simply $G\theta_0^2(l)$, where G is the formal strength defined above. Of course, it may happen that the composite angular distribution does not uniquely identify a value of l (perhaps because several l -values contribute significantly to the resonance). A better resolution experiment would then be needed to clarify the situation and in any case comparisons of low- and high-resolution experiments with the same target are needed to establish the reliability of the poor-resolution technique.

^{†††} The physical meaning of eq. (3) is obvious but there are some formal complications, discussed in the Appendix.

experiment is

$$\begin{aligned}
 G_m &= \frac{1}{2} \langle \text{holes} \rangle_j - 2mM \langle \mathcal{L} \rangle, \\
 G_m(T_>) &= \left(\frac{T_t + 2mM + 1}{2T_t + 1} \right) \left\{ \frac{1}{2} \langle \text{holes} \rangle_j - T_t \langle \mathcal{L} \rangle \right\}, \\
 G_m(T_<) &= \left(\frac{T_t - 2mM}{2T_t + 1} \right) \left\{ \frac{1}{2} \langle \text{holes} \rangle_j + (T_t + 1) \langle \mathcal{L} \rangle \right\},
 \end{aligned} \tag{4}$$

where $\langle \text{holes} \rangle_j$ is the average number [†] of holes in the complete j -shell (neutrons and protons) and $\langle \mathcal{L} \rangle$ is the average value of the Landé factor

$$\mathcal{L} = \frac{T_t(T_t + 1) + T_e(T_e + 1) - T_1(T_1 + 1)}{2T_t(T_t + 1)}. \tag{5}$$

In the usual case where $T_t = M$ we have, writing neutron and proton reactions separately,

$$\begin{aligned}
 G_n &= G_n(T_>) = \left\{ \frac{1}{2} \langle \text{holes} \rangle_j - T_t \langle \mathcal{L} \rangle \right\}, \\
 G_p &= \left\{ \frac{1}{2} \langle \text{holes} \rangle_j + T_t \langle \mathcal{L} \rangle \right\}, \\
 G_p(T_>) &= \frac{1}{2T_t + 1} \left\{ \frac{1}{2} \langle \text{holes} \rangle_j - T_t \langle \mathcal{L} \rangle \right\}, \\
 G_p(T_<) &= \frac{2T_t}{2T_t + 1} \left\{ \frac{1}{2} \langle \text{holes} \rangle_j + (T_t + 1) \langle \mathcal{L} \rangle \right\}.
 \end{aligned} \tag{6}$$

As shown also in the Appendix these may be written in terms of neutron and proton holes as

$$\begin{aligned}
 G_n &= \langle \text{neutron holes} \rangle_j, \\
 G_p &= \langle \text{proton holes} \rangle_j, \\
 G_p(T_<) &= \frac{1}{(N - Z + 1)} \langle \text{neutron holes} \rangle_j, \\
 G_p(T_>) &= \langle \text{proton holes} \rangle_j - \frac{1}{(N - Z + 1)} \langle \text{neutron holes} \rangle_j.
 \end{aligned} \tag{7}$$

For the pick-up experiments the strengths are given once again by eqs. (4), (6), (7) with the modifications

$$\langle \text{holes} \rangle_j \rightarrow \langle \text{particles} \rangle_j, \quad m \rightarrow -m, \quad n \leftrightarrow p, \tag{8}$$

the Landé factor \mathcal{L} being still given by eq. (5). Eqs. (7) and their pickup analogues are unsymmetrical in neutrons and protons because we assume a *neutron* excess in the target.

[†] If the target wave function is given in the form $\sum_k A_k \phi_k$ and if in ϕ_k the number of holes in the j shell is n_k then $\langle \text{holes} \rangle_j = \sum_k A_k^2 n_k$. The other averages are defined in the same way.

3. Discussion

We have stressed the obvious, but often ignored, fact that isobaric spin is significant in experiments which add a proton or remove a neutron even when the target quantum number T is redundant, as in the case of heavier nuclei. We have evaluated reaction strengths in a general and compact form.

If we deal with reactions in which all the active nucleons are in the same shell, the present results add little to those which have been given previously ⁶⁾. But in other cases, and particularly when we have particle or hole giant resonances, they seem to have considerable value.

It is clear from eqs. (7), (8) that, if the neutron excess is at all large, most of the resonant strength when we add a proton or take away a neutron will belong to the smaller-isobaric-spin component. Thus we shall have a good chance of observing both T -components in a giant resonance only when the target neutron excess is small and hence, say, $A < 70$.

We can always locate the $T_>$ particle resonance by adding a neutron and the $T_<$ resonance by adding a proton to the same target. The position of the $T_>$ resonance in the proton experiment might in some cases be inferred from binding energies and Q values. The same procedure could be followed for hole resonances. Finally, although little is known about the systematics of single-hole states, recent (p, 2p) studies ⁷⁾ suggest that single-hole states are rather more widely spaced than single-particle states. If this is indeed the case, it may be easier to detect the T -splitting of hole resonances.

Appendix

The expressions for G given in the text (eqs. (4)—(8)) can be deduced from the sum rules quoted without proof as eqs. (III.140') and (III.141') in Appendix 2 of ref. ⁶⁾. We derive them here, not by summing explicit expressions for the reduced widths, but by a projection-operator technique which is much simpler and could be used for other sum rules as well.

We consider first the pick-up sum rules (their derivation is simpler) and indicate only the isobaric-spin quantum numbers; it will be understood that all other final-state quantum numbers must be summed over. We expand the target wave function in a representation which specifies the number of j -nucleons and, noting that in a sum rule there is no interference between orthogonal components, we treat each term separately.

A characteristic term has the structure

$$\{T_1 \times T_e\}_{T_1, M},$$

where \times describes vector coupling, $\{\}$ indicates antisymmetry in all nucleons, T_e refers to the j^n group and T_1 to the remaining particles (eq. (3)). We shall

also use the notation

$$(T_1 \times T_e)_{T_t, M}$$

for a state in which the antisymmetry is confined to the separate groups; it will be understood that the nucleons in the T_1 group are numbered $1, 2 \dots (A-n)$, those of the T_e group having the numbers $(A-n+1) \dots A$.

By definition (see eq. (2) and section III.7 of ref. ⁶), the total strength for given m is

$$G_m = A \sum_{T'_e T_t} (C_{M-m, m}^{T_t T'_e})^2 |\langle \{T_1 \times T_e\}_{T_t, M} | \{ \{T_1 \times T'_e\}_{T_t} \times t(A) \}_{T_t, M} \rangle|^2, \quad (9)$$

where m specifies whether the transferred nucleon is neutron or proton and T'_e refers to the group j^{n-1} in the residual nucleus.

Now $\{T_1 \times T_e\}_{T_t, M}$ can be expanded into $A!/n!(A-n)!$ terms differing from $(T_1 \times T_e)_{T_t, M}$ only in the distribution of nucleon numbers $1, 2 \dots A$ between the two groups, and similarly for $(\{T_1 \times T'_e\}_{T_t} \times t(A))$. The overlap integral in (9) is clearly diagonal in distributions of the nucleon numbers $1, 2 \dots A-1$ (A being fixed in the T_1 group). Thus each term in the expansion of $(\{T_1 \times T'_e\}_{T_t} \times t(A))$ gives a non-vanishing contribution to the overlap integral with one and only one term in the expansion of $\{T_1 \times T_e\}$; furthermore, all such contributions are equal. Remembering normalization factors, simple counting yields

$$G_m = n \sum_{T'_e T_t} (C)^2 |\langle (T_1 \times T_e)_{T_t, M} | ((T_1 \times T'_e)_{T_t} \times t(A))_{T_t, M} \rangle|^2. \quad (10)$$

and summing this over m gives

$$\sum_m G_m = n \sum_{T'_e T_t} |\langle (T_1 \times T_e)_{T_t, M} | ((T_1 \times T'_e)_{T_t} \times t(A))_{T_t, M} \rangle|^2. \quad (11)$$

A more detailed discussion of the steps leading from (9) to (10) is given in ref. ⁶ (p. 608). When applied to the states appearing in (10), the formal significance of the separation $\mathbf{T}_t = \mathbf{T}_1 + \mathbf{T}_e$ (eq. (3)) is immediately clear.

Since the states with the separated particle form a vector space which contains all the states $(T_1 \times T_e)_{T_t, M}$ we can use completeness for the sum in (11) and obtain

$$\sum_m G_m = n. \quad (12)$$

But also by inserting a projection operator

$$P(T_t) \propto \mathbf{T}_t^2 - T'_t(T'_t + 1),$$

we can restrict the sum in (11) to states of definite final isobaric spin T_t (T'_t being then the other possible value of final-state isobaric spin). Using $\mathbf{T}_t = \mathbf{T}_1 - \mathbf{t}(A)$ and normalizing the P operators we have

$$P(T_t >) = 1 - P(T_t <) = \frac{1}{(2T_t + 1)} \{ (T_t + 1) - 2\mathbf{T}_t \cdot \mathbf{t}(A) \}. \quad (13)$$

Inserting this in eq. (11) and summing gives then

$$\sum_m G_m(T_t >) = n - \sum_m G_m(T_t <) = \frac{(T_t + 1)}{(2T_t + 1)} \{n - 2T_t \mathcal{L}\}, \quad (14)$$

where we have used the fact that, in the target state (antisymmetrical in the j -nucleons), $t(4) \equiv T_e/n$, rewritten $T_t \cdot T_e$ by using eq. (3), and used eq. (5) in writing the final result (14). Note that $T_t \cdot T_e \equiv \mathcal{L} T_t^2$ and also that $T_{e,z} \equiv \mathcal{L} T_{t,z}$.

Our real interest of course is in reactions for a transferred neutron or proton. But we can undo the sum in (14) by noting that (for fixed m) $(C)^2$ is the same for all terms in any sum in which T_t is specified. Then we have

$$\begin{aligned} G_m(T_t >) &= (C_{M-m, m}^{T_t+1, \frac{1}{2}, T_t})^2 \sum_{m'} G_{m'}(T_t >) \\ &= (C_{M, -m}^{T_t+1, T_t+1})^2 \{\frac{1}{2}n - T_t \mathcal{L}\}, \\ G_m(T_t <) &= (C_{M, -m}^{T_t, T_t-1})^2 \{\frac{1}{2}n + (T_t + 1) \mathcal{L}\}. \end{aligned} \quad (15)$$

Now when we sum over the various components of the target function, $n \rightarrow \langle \text{particles} \rangle_j$, $\mathcal{L} \rightarrow \langle \mathcal{L} \rangle$ and eqs. (4), (8) follow on inserting explicit forms for the Clebsch-Gordan coefficients.

We have now derived the pick-up sum rules. A formal complication is encountered with the stripping or elastic-scattering sum rules. The same overlap integral enters as in the pickup case but now the states of the final nucleus do not span as large a vector space as do the states with a separated nucleon. We therefore cannot directly use completeness to carry out the summations. This difficulty disappears when we make a j -shell hole \leftrightarrow particle transformation as described several times in ref. ⁶) (see section III.10). We need not repeat the arguments here but the results is to produce eq. (8) which relates the pickup and stripping strength functions.

To rewrite our sum rules in terms of neutrons and protons we use

$$\begin{aligned} \langle \text{particles} \rangle_j &= \langle \text{protons} \rangle_j + \langle \text{neutrons} \rangle_j, \\ \langle \text{holes} \rangle_j &= \langle \text{proton holes} \rangle_j + \langle \text{neutron holes} \rangle_j, \\ \langle \mathcal{L} \rangle &= \frac{\langle \text{neutrons} \rangle_j - \langle \text{protons} \rangle_j}{2M} = \frac{\langle \text{proton holes} \rangle_j - \langle \text{neutron holes} \rangle_j}{2M}, \end{aligned} \quad (16)$$

and, for the special case $T_t = M$, eqs. (6), (8) then follow.

If we add the strength functions for (d, p) and (d, t) experiments on the same target we find simply $(2j+1)$. This result, which has an obvious physical significance, could be useful, for example, in indicating that an identification of a group of levels has been wrongly made [†]. And of course if for any reaction we are unable to separate the two j values for given l a further summation over j can be performed.

[†] Or in some cases that nucleons may be transferred with two different *principal* quantum numbers but the same (lj) .

Finally, as discussed in the text, it may be of interest to derive expressions for the strengths for fixed values of the isobaric spin T'_e of the equivalent group in the residual nucleus. We shall evaluate the four quantities $\sum_m G_m(T'_e, T_f)$ from which the strengths for fixed m follow immediately as in (15). To do this we recognize that the two different values of T_f in a summation for fixed T'_e are weighted in proportion to the square of the Racah coefficient

$$U(T_1 T'_e T_{\frac{1}{2}} : T_f T_e) \equiv U(T'_e T_f),$$

which carries out the recoupling transformation from a scheme which specifies T'_e to one which specifies T_f (see eq. (9)). In other words, we have for both values of T'_e

$$\frac{\sum_m G_m(T'_e, T_f >)}{U(T'_e, T_f >)^2} = \frac{\sum_m G_m(T'_e, T_f <)}{U(T'_e, T_f <)^2}. \quad (17)$$

We could now solve the four independent linear equations (14) and (17) for $\sum_m G_m(T'_e, T_f)$. However, we obtain an alternative derivation which is much simpler algebraically if we notice that (17) can be rewritten in the form

$$\sum_m G_m(T'_e, T_f) = U(T'_e, T_f)^2 \sum_{T'_f} \sum_m G_m(T'_e, T'_f). \quad (18)$$

But the sums on the right side of (18) are simply the strengths for fixed T'_e summed over T_f and can be obtained directly from (11) by inserting the projection operators

$$P(T'_e >) = 1 - P(T'_e <) = \frac{1}{(2T_e + 1)} \{ (T_e + 1) - 2\mathbf{T}_e \cdot \mathbf{t}(A) \}, \quad (19)$$

constructed in obvious analogy to (13). Repetition of the operations which led from (13) to (14) now yields

$$\sum_{T_f} \sum_m G_m(T'_e >, T_f) = n - \sum_{T_f} \sum_m G_m(T'_e <, T_f) = \frac{(T_e + 1)}{(2T_e + 1)} \{ n - 2T_e \}. \quad (20)$$

Substituting from eqs. (20) in eqs. (18), inserting explicit forms for the Racah coefficients, and extracting the strengths for fixed m as in eqs. (15), we have

$$\begin{aligned} G_m(T'_e >, T_f >) &= \left\{ \frac{T_t - 2mM + 1}{2(T_t + 1)(2T_t + 1)} \right\} \left\{ \frac{(T_1 + T_e + T_t + 2)(-T_1 + T_e + T_t + 1)}{(2T_e + 1)} \right\} \left\{ \frac{1}{2}n - T_e \right\}, \\ G_m(T'_e >, T_f <) &= \left\{ \frac{T_t + 2mM}{2T_t(2T_t + 1)} \right\} \left\{ \frac{(T_1 + T_e - T_t + 1)(T_1 - T_e + T_t)}{(2T_e + 1)} \right\} \left\{ \frac{1}{2}n - T_e \right\}, \\ G_m(T'_e <, T_f >) &= \left\{ \frac{T_t - 2mM + 1}{2(T_t + 1)(2T_t + 1)} \right\} \left\{ \frac{(T_1 - T_e + T_t + 1)(T_1 + T_e - T_t)}{(2T_e + 1)} \right\} \left\{ \frac{1}{2}n + (T_e + 1) \right\}, \\ G_m(T'_e <, T_f <) &= \left\{ \frac{T_t + 2mM}{2T_t(2T_t + 1)} \right\} \left\{ \frac{(T_1 + T_e + T_t + 1)(-T_1 + T_e + T_t)}{(2T_e + 1)} \right\} \left\{ \frac{1}{2}n + (T_e + 1) \right\}. \end{aligned} \quad (21)$$

We then obtain the strength equations for pick-up in their most general form by averaging eqs. (21) over the various components in the target wave function; expressions appropriate to stripping and elastic scattering can then be written down by reversing the transformation of eqs. (8).

We take this opportunity to correct an error in the first Appendix of ref. ⁶). It was stated there that, in a transition between two $(n-p)_T$ states, the isobaric-spin coupling factor $(C)^2$ is necessarily unity. But in the exceptional case that a proton is added to (or removed from) a shell filled for neutrons, $(C)^2$ is instead $(2T_0/2T_0+1)$, in the notation of ref. ⁶). An example of such a case is in fact discussed on p. 618 of ref. ⁶). The reduced widths θ^2 and the relative reduced widths \mathcal{S} can now be quoted in either a T or an $n-p$ formalism and the results will not be the same, but instead $(C)^2 \mathcal{S}^{(T)} = \mathcal{S}^{(n-p)}$. In these cases $\mathcal{S}^{(n-p)}$ will be a physically simpler quantity than $\mathcal{S}^{(T)}$ but, to avoid confusion, it is still preferable to extract widths using an equation in which the $(C)^2$ factor explicitly enters.

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