

ELECTRON ENERGIES

Using an iterative procedure known as the ω -technique^{6,7} "self-consistent" values for charges, bond orders and bond lengths were calculated for symmetrical (D_{2h}) and asymmetrical (C_{2h}) heptalene. This method proceeds as follows. After having solved a first determinant—e.g. of the Hückel type—a new determinant is set up in which the Coulomb integrals (α) and resonance integrals (β) have been corrected with the aid of charges q and bond orders p obtained from the solution of the first one. The relations used here are:

$$\alpha_i' = \alpha_i^\circ + \omega(1 - q_i)\beta^\circ$$

$$\beta = \beta^\circ \exp\left(\frac{+1.40 - l}{0.3106}\right)$$

$$l = 1.51 - 0.16p$$

The proportionality coefficient ω is taken to be unity.⁸ The expression for the exponential dependence of the resonance integral on the bond length l is taken from a paper by Longuet-Higgins and Salem.³

The figures considered to be final for charges, bond orders and free valence numbers are summarized in Table 1. For comparison the values obtained by solving the simple Hückel type equations are tabulated as well. The corresponding energies and symmetry assignments for the molecular orbitals are collected in Table 2. From these data the total π -electron energy for the ground states of the correspondingly treated models can be obtained. For the model with molecular symmetry D_{2h} —which entails the presence of the twofold axes C_2^x and C_2^y —a value of $(12\alpha^\circ + 14.90\beta^\circ)$ is found. If an asymmetrical distortion, leading to symmetry C_{2h} , is introduced, the structure possesses only a centre of symmetry. In that case a more favourable π -electron results namely $(12\alpha^\circ + 15.22\beta^\circ)$.

TABLE 1. CHARGES, BOND ORDERS AND FREE VALENCE NUMBERS IN HEPTALENE

charges			bond orders			free valence numbers					
atom	<i>a</i>	<i>b</i>	<i>c</i>	bond	<i>a</i>	<i>b</i>	<i>c</i>	atom	<i>a</i>	<i>b</i>	<i>c</i>
11	0.867	0.896	0.960	11-1	0.536	0.515	0.751	11	0.158	0.160	0.207
1	1.125	1.076	1.074	1-2	0.672	0.690	0.457	1	0.524	0.527	0.524
2	0.882	0.935	0.973	2-3	0.619	0.609	0.814	2	0.441	0.433	0.461
3	1.119	1.081	1.027	3-4			0.409	3	0.494	0.514	0.509
4			0.969	4-5			0.847	4			0.476
5			0.998	5-12			0.362	5			0.523
				11-12	0.502	0.542	0.412				

^a Hückel scheme. ^b ω -technique molecular symmetry D_{2h} . ^c idem C_{2h} .

It should be remarked that compared with the Hückel method, the iterative procedure—as any method that takes into account electron repulsion—smoothes out the charge densities. This tendency is most pronounced where bond lengths alternate.

So far the π -electrons have been considered, though in fact the σ -electrons enter

⁶ R. Daudel, R. Lefebvre and C. Moser, *Quantum Chemistry* p. 78. Interscience, New York (1959).

⁷ A. Streitwieser, *Molecular Orbital Theory for Organic Chemists* p. 115. Wiley, London (1961).

⁸ G. W. Wheland and D. E. Mann, *J. Chem. Phys.* **17**, 264 (1949).

into the calculations through the order-length relation. A comparison of stabilities should of course be based upon the total electron energies. It has been derived⁸ for polyenes with an even charge distribution that the total energy v of a bond in equilibrium is given by:

$$v = -2aP' + \text{constant}$$

where $P' = -(0.16)^{-1}$ and $a = 0.3106 \text{ \AA}$. So one may compare the sums of the total energies V of all bonds in a symmetrical and asymmetrical (subscripts s and a respectively) nonalternant hydrocarbon through:

$$V_{(s)} - V_{(a)} = -2aP' \sum_{i < j} [\beta_{ij(s)} - \beta_{ij(a)}] + (\sum_i q_i \alpha_i)_{(s)} - (\sum_i q_i \alpha_i)_{(a)}$$

Though the figure obtained $-0.08\beta^\circ$ —is rather small and may be subject to a large error, it indicates that a “deformation” of heptalene may be favoured by a stabilization energy of a few kcal/mole.

DISCUSSION

Similar to what has been found for pentalene,⁵ the distortion of the nuclear framework of heptalene has the effect that in the ground state the highest occupied—i.e. the sixth—and the lowest unoccupied level do acquire the same symmetry. The resulting interaction of these orbitals is demonstrated by the data of Table 2. The top occupied orbital is lowered by the deformation, in contrast to the seventh that rises. So, in passing from the symmetrical to the asymmetrical molecule, the lowest electronic transition—a forbidden one—increases from $0.17\beta^\circ$ to $0.43\beta^\circ$, which is equivalent to a large shift to the blue. In a qualitative way this is in agreement with the observations of Dauben and Bertelli.¹

TABLE 2. ENERGIES AND SYMMETRY ASSIGNMENTS OF MOLECULAR ORBITALS OF HEPTALENE*

Hückel	D _{2h}	C _{2h}	
-2.170 b _{2g}	-1.995 b _{2g}	-2.014 b _g	unoccupied in
-1.891 b _{1u}	-1.837 b _{1u}	-1.840 a _u	
-1.732 b _{3g}	-1.696 b _{3g}	-1.688 b _g	ground state
-1.000 a _{1u}	-1.009 a _{1u}	-1.002 a _u	
-0.705 b _{1u}	-0.662 b _{1u}	-0.705 a _u	
-0.311 b _{2g}	-0.252 b _{2g}	-0.363 b _g	
0 b _{3g}	-0.082 b _{3g}	+0.067 b _g	occupied in
+1.000 a _{1u}	+0.999 a _{1u}	+1.003 a _u	
+1.317 b _{1u}	+1.265 b _{1u}	+1.259 a _u	ground state
+1.481 b _{2g}	+1.415 b _{2g}	+1.440 b _g	
+1.732 b _{3g}	+1.683 b _{3g}	+1.691 b _g	
+2.278 b _{1u}	+2.171 b _{1u}	+2.152 a _u	

* The energies are characterized by the coefficients c in $E = \alpha + c\beta$.

Predictions for the chemical reactivity may be based upon ground state properties and as such the free valence numbers or the electronic charges are relevant. Inspection of Table 1 suggests that the asymmetrical structure accentuates clearly a charge difference at positions 1 and 3 which does not emerge from the symmetrical model. The free valence numbers in all three cases indicate that the highest reactivity

is to be expected at positions 1 and 5. So, in agreement with experiment, the 1-position should be the most reactive. It should be remarked that in the asymmetrical molecule the positions 1,5,6, and 10 are of course no longer equivalent.

For nonalternants a prediction for the chemical reactivity from charge densities and free valence numbers is not always in agreement with conclusions drawn from the computation of localization energies.⁹ Therefore the three types of localization energy were calculated according to a Hückel scheme in which all the resonance integrals are taken equal, and are shown in Table 3 in units of the resonance integral β° . Obviously for these calculations the phenomenon of bond alternation is no longer of interest. Except for nucleophilic substituents where position 2 should be favoured, again the 1-position is predicted to be the most reactive.

TABLE 3. LOCALIZATION ENERGIES FOR
HEPTALENE

position	Ae	An	Ar
1	1.659	2.193	1.926
2	2.032	2.032	2.032
3	1.715	2.337	2.026

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⁹ See also *Dictionary of Values of Molecular Constants* (Edited by C. A. Coulson and R. Daudel).