

## THE STRUCTURE OF UV DIVERGENCES IN SS YM THEORIES

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Using the background-field method, covariant supergraphs and infrared subtraction techniques we analyse the structure of the higher-loop ultraviolet divergences in supersymmetric Yang–Mills theories. We show that in regularization by dimensional reduction they are given by a specific gauge invariant term constructed out of the vector connection and that they can be obtained by analyzing a special subset of all possible supergraphs. From the structure of the divergences we are able to conclude that in supersymmetric theories if the self-energies of the quantum fields in the presence of the background are finite up to  $L$  loops the YM background effective action is finite at  $(L + 1)$  loops.

Recently two of us have presented covariant D-algebra techniques for computing supergraphs [1]. These techniques make use of the superspace background-field method and organize the calculations so that *beyond one loop* individual supergraphs give contributions to the SSYM effective action that depend only on the *vector* connection  $\Gamma_a(x, \theta)$  and the field strengths  $W_\alpha, \bar{W}_{\dot{\alpha}}$ , but not on the spinor connections. In the present work we discuss the structure of the UV divergences of SSYM that follows from this feature of the background-field method.

At any loop order, after subtraction of UV *and*, if present, IR subdivergences, the infinite part of individual supergraph contributions to the effective action is local and, in the background-field method, their sum is gauge invariant under the (vector representation) gauge transformations of the background fields  $\delta\Gamma_a = \partial_a K + [K, \Gamma_a]$ ,  $\delta W_\alpha = [K, W_\alpha]$ . By superspace power counting (the effective action contains a  $d^4x d^4\theta$  integral of dimension 2, while  $\dim \Gamma_a = 1$ ,  $\dim W_\alpha = 3/2$ ) it follows that the only possible divergent structure is quadratic in the vector connections (but not their derivatives) and by gauge invariance must have the form

$$\Gamma_\infty = z(\epsilon) \text{Tr} \int d^4x d^4\theta \Gamma^a \Gamma_b (\delta_a^b - \hat{\delta}_a^b). \quad (1)$$

where the singular coefficient  $z(\epsilon)$  arises from momentum integration of divergent graphs. The  $n$ -dimensional  $\hat{\delta}_a^b$  arises from symmetric momentum integration while the four-dimensional  $\delta_a^b$  comes from terms such as  $\Gamma^a \Gamma_a$  in the expansion of the covariant d'alembertian or from covariant derivatives  $\nabla^a \dots \nabla_a$  produced in the course of covariant D-algebra. We are using the rules of superspace dimensional regularization (by dimensional reduction) with

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momenta continued to  $n < 4$  dimensions while the vector index on  $\Gamma^a$  is kept in four dimensions. The gauge invariance of (1) is a consequence of these rules. The other possible gauge-invariant structure of dimension two,  $\Gamma^a \Gamma_b (\delta_a^b - p_a p^b / 2p^2)$ , is excluded by locality.

In (1) the  $d^4\theta$  integral is a consequence of the superspace Feynman rules. However, the result can be cast in a form proportional to the classical action: We replace  $d^2\bar{\theta}$  by  $\bar{\nabla}^2$ , use the Bianchi identity

$$[\bar{\nabla}_{\dot{\gamma}}, \nabla^a] = \partial^a (i\Gamma_{\dot{\gamma}}) - i[\bar{\nabla}_{\dot{\gamma}}, \Gamma^a] = \delta_{\dot{\gamma}}^{\dot{\alpha}} W^\alpha \tag{2}$$

[the  $\partial^a$  term vanishes when substituted into (1)] and the rules of dimensional reduction

$$\delta_a^b \delta_{\dot{\beta}}^{\dot{\alpha}} = 2\delta_{\alpha}^{\beta}, \quad \hat{\delta}_a^b \delta_{\dot{\beta}}^{\dot{\alpha}} = \frac{1}{2} n \delta_{\alpha}^{\beta}, \tag{3}$$

and find

$$\text{Tr} \int d^4x d^4\theta \Gamma^a \Gamma_b (\delta_a^b - \hat{\delta}_a^b) = -\epsilon \text{Tr} \int d^4x d^2\theta W^\alpha W_\alpha. \tag{4}$$

Thus, the integral in (1) is of order  $\epsilon$ , and to obtain a divergence the coefficient  $z(\epsilon)$  must contain at least a  $1/\epsilon^2$  singularity. For an  $L$ -loop contribution this will be the case *only* if lower loop subdivergences are present. Therefore, *if the theory is finite up to  $L - 1$  loops* (finite quantum, and quantum-background Green functions) *then the background effective action will be finite at  $L$  loops.*

We will show below how the gauge-invariant, local UV divergent structure of (1) arises at the two-loop level in explicit examples. In order to avoid complications due to IR singularities we discuss first massive SQED. We then reexamine the two loop SSYM calculation of ref. [1]. In that reference a nonlocal gauge fixing term was used to cancel spurious infrared divergences present because radiative corrections take us out of Feynman gauge [2]. Here instead we use the IR subtraction procedure of Chetyrkin and Tkachov [3]. It is based on the observation that IR divergences correspond to "short-distance" singularities in momentum space and can be subtractively removed by counterterms *local* in momentum space, i.e. proportional to Dirac  $\delta$ -functions of momenta. The corresponding  $\bar{R}$ -operation, when combined with the usual R-operation [4] defines a  $R^*$ -operation that leads to graphs which are free of both UV and IR divergences.

*Supersymmetric QED.* After quantum-background splitting for the vector multiplet, and gauge-fixing for the quantum  $V$  field, the SQED action is

$$S = \int d^4x d^4\theta [\tilde{\eta}_+ \eta_+ + \tilde{\eta}_- \eta_- + V(\tilde{\eta}_+ \eta_+ - \tilde{\eta}_- \eta_-) - \frac{1}{2} V \square_0 V] - m \left( \int d^4x d^2\theta \eta_+ \eta_- + \text{h.c.} \right), \tag{5}$$

where  $\eta$  is background covariantly chiral. We have expanded the action in powers of  $V$  and kept only terms relevant for a two-loop calculation of the background effective action.

According to the rules of ref. [1] we start with the vacuum graphs of fig. 1, with covariant propagators and  $\nabla^2, \bar{\nabla}^2$  factors at vertices involving chiral superfields. The propagators are [5]

$$\langle \tilde{\eta} \eta \rangle = -[\square_+ - m^2]^{-1}, \quad \langle \eta_+ \eta_- \rangle = -m \nabla^2 [\square_+ (\square_+ - m^2)]^{-1}, \quad \langle V V \rangle = \square_0^{-1}, \tag{6}$$

with

$$\square_+ = \square_0 - i\Gamma^a \partial_a - \frac{1}{2} i(\partial^a \Gamma_a) - \frac{1}{2} \Gamma^a \Gamma_a - \frac{1}{2} i(\nabla^a W_\alpha) - iW^\alpha \nabla_\alpha. \tag{7}$$

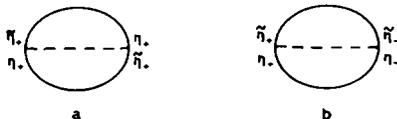


Fig. 1. Two-loop covariant supergraphs for SQED.

It is easy to check that the diagram in fig. 1b will not contribute to the two-point function. (Using  $\bar{\nabla}^2 \nabla^2 \bar{\nabla}^2 = \square_+ \bar{\nabla}^2$  we see that there are not enough D's in the loops.) From fig. 1a we obtain, expanding the propagators and doing trivial D-algebra [cf. ref. [1], eqs. (4.4)–(4.9)], the contributions

$$\begin{aligned}
 I_1 &= 2W^\alpha(-p) \bar{W}^{\dot{\alpha}}(p) \int \frac{d^n q \, d^n k}{(2\pi)^{2n}} \frac{(p-k)_{\alpha\dot{\alpha}}}{(q^2+m^2)(q+k)^2(k^2+m^2)^2[(k-p)^2+m^2]}, \\
 I_2 &= \frac{1}{2} \nabla^\alpha W_\alpha(-p) \nabla^\beta W_\beta(p) \int \frac{d^n q \, d^n k}{(2\pi)^{2n}} \frac{1}{(q^2+m^2)(q+k)^2(k^2+m^2)^2[(k-p)^2+m^2]}, \\
 I_3 &= -\frac{1}{2} \Gamma^a(-p) \Gamma^b(p) \int \frac{d^n q \, d^n k}{(2\pi)^{2n}} \frac{(2k-p)_a(2k-p)_b}{(q^2+m^2)(q+k)^2(k^2+m^2)^2[(k-p)^2+m^2]}, \\
 I_4 &= \Gamma^a(-p) \Gamma_a(p) \int \frac{d^n q \, d^n k}{(2\pi)^{2n}} \frac{1}{(q^2+m^2)(q+k)^2(k^2+m^2)^2}, \\
 I_5 &= \frac{1}{4} \Gamma^a(-p) \Gamma^b(p) \int \frac{d^n q \, d^n k}{(2\pi)^{2n}} \frac{(2k+p)_a(2q-p)_b}{(q+k)^2(k^2+m^2)[(k+p)^2+m^2](q^2+m^2)[(q-p)^2+m^2]}, \\
 I_6 &= \frac{1}{4} \nabla^\alpha W_\alpha(-p) \nabla^\beta W_\beta(p) \int \frac{d^n q \, d^n k}{(2\pi)^{2n}} \frac{1}{(q+k)^2(k^2+m^2)[(k+p)^2+m^2](q^2+m^2)[(q-p)^2+m^2]}, \tag{8}
 \end{aligned}$$

where the first three terms correspond to graphs as in fig. 2a,  $I_4$  is the tadpole in fig. 2b, and the last two terms correspond to fig. 2c.

The integral  $I_6$  is convergent and need not be discussed further. We subtract out subdivergences in  $I_1, \dots, I_4$  by the replacement

$$\Sigma(k) = \int \frac{d^n q}{(2\pi)^n} \frac{1}{(q^2+m^2)(q+k)^2} \rightarrow \Sigma(k) + Z^{(1)}, \tag{9}$$

with

$$Z^{(1)} = -1/\epsilon, \tag{10}$$

which leads to additional contributions  $I'_1, \dots, I'_4$ . (For the sake of clarity we omit here and below factors  $(4\pi)^{-2}$  produced at each loop order. They will be reinstated in the final answer.) It is obvious, but can be checked explicitly, that  $I_1 + I'_1, I_2 + I'_2$  are finite and need not concern us further, as are the parts in  $I_3, I_5$ , that contain external momentum factors. From  $I_3 + I'_3, I_5$  we obtain the divergence

$$\frac{1}{2} \Gamma^a \Gamma_b (1/\epsilon^2 - 1/\epsilon) \delta_a^b, \tag{11}$$

while  $I_4 + I'_4$  give

$$-\frac{1}{2} \Gamma^a \Gamma_a (1/\epsilon^2 - 1/\epsilon), \tag{12}$$

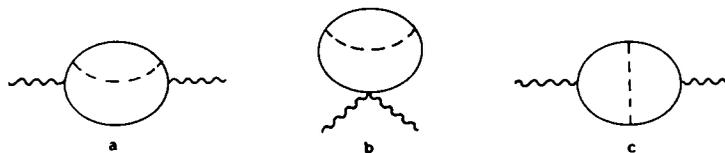


Fig. 2. Two-loop contributions to the gauge-field self-energy.

so that the complete two-loop divergence is given by

$$-\frac{1}{2}(1/\epsilon^2 - 1/\epsilon)(\delta_a^b - \hat{\delta}_a^b) \Gamma^a(-p) \Gamma_b(p). \tag{13}$$

Using (4) we obtain

$$\Gamma_\infty^{(2)} = \frac{1}{2\epsilon} \frac{1}{(4\pi)^4} \int d^4x d^2\theta W^\alpha W_\alpha. \tag{14}$$

Some comments are in order: Had we used a standard (nonsupersymmetric) dimensional regularization, replacing in particular  $\delta_a^b$  by  $\hat{\delta}_a^b$  everywhere, we would have obtained a zero result. This is the correct result but for a different theory: Exactly the same contributions are obtained in an ordinary, *nonsupersymmetric* theory of a charged scalar field (represented by the solid lines in fig. 2) interacting with a neutral scalar field (represented by the dashed lines) in a background photon field (represented by the wavy lines) where the effective action is precisely given by the sum  $I_3 + I_4 + I_5$ . Here power counting (the charged-neutral coupling constant has dimensions of mass) and gauge invariance imply that no local divergence is possible. In fact, in such a theory, using dimensional reduction would lead to trouble. The expression in (13) can be interpreted as a two-loop contribution to the effective action for  $\epsilon$ -scalars and it would lead to violations of unitarity similar to the ones observed by 't Hooft and van Damme [6].

It is clear that in our case the complete result can be obtained by calculating the tadpole term and then "covariantizing" it by the replacement  $\delta_a^b \rightarrow \delta_a^b - \hat{\delta}_a^b$ . Furthermore we need only look at the  $1/\epsilon^2$  contribution in (13) which arises typically from the integral containing the renormalized (subtracted) self-energy insertion

$$\int \frac{d^n k}{(k^2 + m^2)^2} [\Sigma(k) - 1/\epsilon] = -1/2\epsilon^2, \quad \Sigma(k) \equiv k^{-2\epsilon}/\epsilon. \tag{15}$$

Note that the  $1/2\epsilon^2$  comes from an incomplete cancellation between the two terms in the integral above even though  $\Sigma_R(k) = \Sigma(k) - 1/\epsilon$  is finite.

*Supersymmetric Yang-Mills.* The two-loop contributions to the self-energy in  $N = 1$  SSYM were given in ref. [1]. (But the tadpole contribution was not written down since it vanishes in dimensional regularization. Here we must include it because of our treatment of IR divergences.) They can be obtained from the ones of SQED in eq. (8) by setting  $m = 0$ , omitting  $I_2$  and  $I_6$ , and multiplying the remaining terms by a factor of  $-3/2$ . (We omit factors of  $g^2 C_A$  and  $(4\pi)^{-2}$  produced by the loop integrals, and restore such factors only in the final result.) Some of the resulting integrals have now infrared divergences due to the presence of  $k^{-4}$  factors. In this section we show how to remove these divergences using the  $R^*$  subtraction procedure of ref. [2], so that we can unambiguously identify the ultraviolet divergences. However we note that the final result can be read immediately from (14) since the mass is a good IR regulator.

We begin with the contribution  $I_1$  and first of all subtract the UV self-energy subdivergence, replacing therefore  $I_1$  by (we have performed the  $q$  integration and used  $\overline{MS}$ )

$$I_1 + I_1' = 3W_\alpha \bar{W}_{\dot{\alpha}} \int \frac{d^n k}{(2\pi)^n} \frac{(k-p)^{\alpha\dot{\alpha}}}{k^4 (p-k)^2} \epsilon^{-1} [(1+2\epsilon)/(k^2)^\epsilon - 1]. \tag{16}$$

We subtract now the infrared divergence. We add to the subdiagram which is IR divergent a counterterm  $X\delta(k)$  such that the resulting expression is finite when integrated with a test function  $\Psi(k)$  which is regular at the origin and vanishes as  $k \rightarrow \infty$ . The IR divergent subdiagram consists of the two  $k^{-2}$  propagators and the (subtracted) self-energy insertion. Therefore we choose  $X$  so that

$$\int d^n k \Psi(k) \{k^{-4} \epsilon^{-1} [(1+2\epsilon)/(k^2)^\epsilon - 1] + X\delta(k)\} < \infty. \tag{17}$$

To determine  $X$  we choose any convenient  $\Psi(k)$  e.g.  $\Psi(k) = (p-k)^{-2}$ . This determines, up to finite terms,

$$X = -(1 - \epsilon)/2\epsilon^2. \quad (18)$$

Returning to the original integral  $I_1 + I_1'$  with the additional subtraction  $X\delta(k)$  in the integrand, it is trivial to see, because of our choice of  $\Psi(k)$ , that in fact no divergence is left; the subtracted contribution is UV finite. Actually even this calculation was not necessary since for dimensional reasons (and Lorentz covariance) the result of the integration must be proportional to  $p_{\alpha\dot{\alpha}}/p^2$  which is nonlocal and therefore the divergent part must vanish after subtractions.

The same argument applies to  $I_3$  except for the part containing two internal momenta  $k_a k^b$ . This part, which is only UV divergent, produces a local contribution proportional to the  $n$ -dimensional  $\hat{\delta}_a^b$

$$\int \frac{d^n k}{(2\pi)^n} \frac{4k_a k^b}{(p-k)^2 k^4} \epsilon^{-1} [(1+2\epsilon)/(k^2)^\epsilon - 1] = \hat{\delta}_a^b (\epsilon - 2)/2\epsilon^2. \quad (19)$$

All the other divergent terms in  $I_3$  vanish after ultraviolet and infrared subtraction of the subdivergences, as expected.  $I_5$  contains neither UV nor IR subdivergences and contributes a term

$$(1/2\epsilon) \hat{\delta}_a^b. \quad (20)$$

The massless tadpole  $I_4$  contains the same kind of infrared subgraph we encountered above. After IR subtraction it gives

$$-2\hat{\delta}_a^b \int \frac{d^n k}{(2\pi)^n} \{k^{-4} \epsilon^{-1} [(1+2\epsilon)/(k^2)^\epsilon - 1] - [(1-\epsilon)/2\epsilon^2] \delta(k)\} = [(1-\epsilon)/\epsilon^2] \hat{\delta}_a^b, \quad (21)$$

where, in dimensional regularization, the whole contribution comes from the IR counterterm. Adding together the results and using (4) we obtain for the divergent part of the two-loop effective action the familiar result

$$-\frac{3}{4} [g^2 C_A / (4\pi)^4] \text{Tr} \int d^4 x d^2 \theta W^\alpha W_\alpha. \quad (22)$$

As in the SQED case the result can be obtained completely from a knowledge of the UV tadpole contribution proportional to  $\hat{\delta}_a^b$ . As we pointed out in ref. [1] the ghosts give no contribution at the two-loop level.

*Higher-loop divergences.* Once we realize that the only possible gauge invariant local term must have the structure in (1), we are led at any loop order to a simple recipe for computing the UV divergences of the background effective action:

(a) Start with vacuum graphs with covariant propagators and vertices for  $V$ , ghost, and, if present, chiral matter superfields. Since we are not interested in background  $W_\alpha, \bar{W}_{\dot{\alpha}}$  terms we can drop them from interactions, covariant propagators, or commutators such as  $[\nabla_\alpha, \nabla_{\beta\dot{\beta}}] = C_{\alpha\beta} \bar{W}_{\dot{\beta}}$ . In particular this last statement implies that the algebra of the covariant derivatives  $\nabla_A = (\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}, \nabla_{\alpha\dot{\alpha}})$  is the same as that of the ordinary derivatives  $D_A = (D_\alpha, \bar{D}_{\dot{\alpha}}, \partial_{\alpha\dot{\alpha}})$ . We can therefore do covariant D-algebra on vacuum graphs with propagators  $\square^{-1} = (\frac{1}{2} \nabla^{\alpha\dot{\alpha}} \nabla_{\alpha\dot{\alpha}})^{-1}$  and the usual D-algebra rules, the only nontrivial (anti)commutator being  $\{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\} = i\nabla_{\alpha\dot{\alpha}}$ .

(b) Since the UV structure we are looking for has the form of eq. (1), it is sufficient to compute the coefficient of the  $\Gamma^a \Gamma_b \delta_a^b = \Gamma^a \Gamma_a$  terms. Contributions which can give rise to such terms are tadpole graphs with  $\Gamma^a \Gamma_a$  from the expansion of  $\square^{-1}$ , and graphs where the D-algebra produces factors  $\nabla^{\alpha\dot{\alpha}} \dots \nabla_{\alpha\dot{\alpha}}$  acting on different lines of the graph which cannot be brought together by integration by parts (so as to cancel a propagator) and from which we can separate a  $\Gamma^a \Gamma_a$  term (or more general structures such as  $\nabla_{\alpha\beta} \dots \nabla^{\beta\dot{\beta}} \dots \nabla_{\beta\dot{\gamma}} \dots \nabla^{\alpha\dot{\gamma}} = \Gamma_{\alpha\beta} k^{\beta\dot{\beta}} k_{\beta\dot{\gamma}} \Gamma^{\alpha\dot{\gamma}} + \dots \sim \Gamma_{\alpha\beta} \Gamma^{\alpha\dot{\gamma}} \delta_{\dot{\gamma}}^{\beta} k^2 + \dots$ ). Any term where a  $\Gamma_a$  ends up fully contracted with a momentum can be dropped.

(c) Since we are interested only in contributions with a  $1/\epsilon^2$  or higher singularity we can drop any graphs which do not contain subdivergences since they only lead to at most  $1/\epsilon$  singularities. Therefore the structure of the  $(L+1)$ -loop background field divergences is completely controlled by the  $L$ -loop quantum field divergences in the presence of the background.

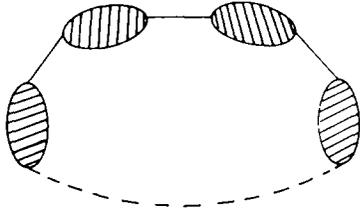


Fig. 3. Higher-loop vacuum graphs.

To determine the divergences of the YM background effective action at the  $(L + 1)$ -loop level it is sufficient in fact to know the two-point quantum Green functions up to  $L$  loops in the presence of the background. The vacuum amplitudes that we start with can be represented as the sum (with a factor of  $1/N$ ) of diagrams as shown in fig. 3, containing  $N$  1PI *renormalized* self-energy insertions  $\Sigma_R$  corresponding to  $V$ , ghost and, if present, chiral matter fields. Both the self-energies and the propagators are background dependent through  $\Gamma_a$  (for our purpose  $W_a$  dependence can be ignored). The two-loop examples of the previous sections can be cast in this form.

After performing D-algebra in the loop of fig. 3, which will lead in general to cancellation of  $N - 1$  of the propagators, we are left with the evaluation of an expression of the form

$$\int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 - \frac{1}{2}\Gamma^a \Gamma_a} [\Sigma_R(k^2, \Gamma^a \Gamma_a)]^N. \tag{23}$$

(We have dropped pieces where  $\Gamma_a$  is contracted with a momentum.) We keep only terms that add up to a total of  $L$  loops and work to first order in  $\Gamma^a \Gamma_a$ . Note that the infrared divergences of the previous example are generated when we expand the denominator in the integral above.

In the absence of the background (and ignoring masses) the *renormalized*  $L$ -loop self-energy has the form

$$\Sigma_R(k^2) = \sum_{s=0}^L a_s(\epsilon)/(k^2)^{s\epsilon}. \tag{24}$$

This expression converges in the limit  $\epsilon \rightarrow 0$ , i.e.

$$\lim_{\epsilon \rightarrow 0} \sum_s (-s\epsilon)^m a_s(\epsilon) < \infty, \quad m = 0, 1, 2, \dots, L, \tag{25}$$

but the individual  $a_s(\epsilon)$  may be singular.  $a_0(\epsilon)$  represents the  $L$ -loop counterterm while singularities in the other  $a_s(\epsilon)$  reflect lower loop subdivergences.

The momentum integration in (23) will produce one factor of  $1/\epsilon$  which is cancelled when we use (4), and a sum of terms involving products of the coefficients  $a_s(\epsilon)$ . In general this sum is singular in the  $\epsilon \rightarrow 0$  limit and gives the  $(L + 1)$ -loop divergence of the background effective action as a function of the divergences of the  $L$ -loop self-energy coefficients  $a_s(\epsilon)$ . However, if none of the coefficients  $a_s(\epsilon)$  is singular there will be no contribution to the divergence of the background effective action. We have therefore the following theorem:

*If for a supersymmetric system of  $N = 1$  Yang–Mills and scalar multiplets the self-energies of the quantum fields in the presence of the Yang–Mills background are completely finite at  $L$ -loops, then the Yang–Mills background effective action is finite at  $(L + 1)$ -loops and the  $(L + 1)$ -loop gauge  $\beta$ -function is zero.*

This theorem generalizes the result that one-loop finite theories are two-loop finite [7]. However we cannot draw any conclusions about finiteness in the chiral sector.

By *complete* finiteness we understand finiteness in the Feynman gauge of all the  $a_s(\epsilon)$  coefficients in the  $L$ -loop self-energies, a condition that means that the self-energies are finite and contain no subdivergences. This will cer-

tainly be the case if all the Green functions of the theory (including mixed quantum-background Green functions) are finite up to  $L$  loops but in general this is too strong a requirement since even finite theories may have divergent Green functions. For example  $N = 4$  SSYM has a divergent one-loop, four- $V$  Green function [8] and this could show up as a subdivergence in a three-loop self-energy unless further cancellations occur (and they must occur). Therefore, to draw conclusions about  $(L + 1)$ -loop finiteness one should examine directly the  $L$ -loop self-energies.

The same remark applies to other examples such as  $N = 1$  one- and two-loop finite theories [7]. It is obvious, using our methods, that one-loop finiteness implies two-loop background YM finiteness. To establish the three-loop finiteness of these theories we must know the two-loop *quantum* self-energies in the presence of the background. However, what has been shown so far is that two-loop chiral self-energies and the *background* YM effective action are finite and to settle the issue of three-loop finiteness more work is required.

The results of this paper are based on the gauge invariance of the background effective action (in particular, if chiral multiplets are present we must assume that they are in anomaly-free, or anomaly-cancelling representations) and the existence of the local, gauge invariant structure  $\Gamma^a \Gamma_b (\delta_a^b - \hat{\delta}_a^b)$ . This structure is very specific to superspace regularization by dimensional reduction and we must assume that this form of regularization can be carried through in some consistent manner even at high-loop order. In principle our covariant rules and higher-loop manipulations hold in any other background-covariant, supersymmetric regularization scheme, e.g. higher-derivative regularization. However, in regularization schemes that stay in four dimensions no local, gauge invariant structure is available and it would seem that no divergence occurs beyond one loop [cf. our remarks after eq. (14)]. The implications of this remark are under investigation.

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