

NEUTRON-DEUTERON SCATTERING WITH LOCAL POTENTIALS

W. M. KLOET and J. A. TJON

Institute for Theoretical Physics, University of Utrecht, Utrecht, the Netherlands

Received 17 November 1971

Padé technique is applied to the multiple scattering series of elastic n-d scattering above breakup threshold, using local two-particle interactions. As an example the phase parameters and differential cross-section are calculated at 14.1 MeV.

In the theoretical investigations of n-d scattering extensive use has been made of separable potentials, mainly because the equations simplify considerably [1,2]. The singularities which arise in the kernel of the resulting integral equations above breakup threshold were in general circumvented by making a contour deformation and the equations were solved by direct matrix inversion. However, by changing the integration path from the real axis into the complex plane, the presence of additional singularities in the T -matrix may spoil the method, which therefore remains somewhat unsatisfactory. Moreover, in the case of local two-particle potentials the integral equations become two-dimensional and are therefore extremely difficult to solve by matrix inversion technique.

Several attempts have been made recently to solve the Faddeev equations in the case of local potentials. One possible way consists of making a separable expansion of the local potential [3]. It is expected to be a practical method only if the number of separable terms needed is small. Another method has been suggested by one of the authors [4] and consists of applying Padé techniques [5] to the multiple scattering series. It was shown that below breakup threshold the diagonal Padé approximants for the scattering amplitude converge rapidly enough to be a useful method. In this note we present some results on the rate of convergence of the Padé approximants for the phase parameters above breakup threshold.

The Faddeev equations for elastic n-d scat-

tering generate a multiple scattering series. In diagrammatic form the S -matrix element can be represented as in fig.1. The whole series may be written as

$$M(\lambda) = \sum_{n=0}^{\infty} \lambda^n M_n \quad (1)$$

where M_n stands for the diagram which contains n times the two-particle T -matrix. From eq.(1) the physical S -matrix element for elastic n-d scattering can simply be found by setting $\lambda = 1$. As is well known the multiple scattering series $\sum_n M_n$ is in general divergent [6]. The method proposed here consists of computing the Padé approximants $M_{[N,N]}$ of this series at $\lambda = 1$. These are known to converge to the correct value as $N \rightarrow \infty$, since $M(\lambda)$ is meromorphic in λ [7].

The potentials for the two-nucleon interaction used in this calculation are restricted to only s-waves and are given in ref. [8] (potentials I and III for the singlet and triplet s-waves respectively). They are purely central and of the Yukawa form

$$V(r) = -\lambda_A \frac{\exp(-\mu_A r)}{r} + \lambda_R \frac{\exp(-\mu_R r)}{r} \quad (2)$$

In order to obtain the coefficients M_n , the Faddeev equations are iterated numerically. The integrations one has to carry out, contain two types of singularities. In the first place, the two-particle T -matrix has a pole at the deuteron

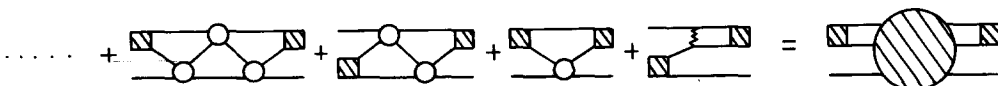


Fig.1. Diagrams for n-d scattering. Here round and shaded square blobs represent the two-particle T -matrix and the deuteron wave function respectively. The first term on the right hand side represents the Born term.

Table 1
Diagonal Padé approximants for doublet and quartet phase shifts and inelasticity parameters of neutron-deuteron scattering at $E_{\text{lab}}^n = 14.1$ MeV. Phase shifts are in degrees.

	${}^2\delta_0$	${}^2\eta_0$	${}^2\delta_1$	${}^2\eta_1$	${}^2\delta_2$	${}^2\eta_2$	${}^2\delta_3$	${}^2\eta_3$
[0,0]	31.3	1.000	-12.7	1.000	4.43	1.000	-1.56	1.000
[1,1]	78.0	0.704	4.8	0.584	6.78	0.947	-1.29	0.989
[2,2]	90.9	1.094	13.8	0.690	6.93	0.948	-1.29	0.989
[3,3]	45.0	0.253	14.4	0.687	6.93	0.948		
[4,4]	106.3	0.489	14.4	0.687				
[5,5]	106.3	0.495						
[6,6]	106.3	0.495						
	${}^4\delta_0$	${}^4\eta_0$	${}^4\delta_1$	${}^4\eta_1$	${}^4\delta_2$	${}^4\eta_2$	${}^4\delta_3$	${}^4\eta_3$
[0,0]	129.5	1.000	24.2	1.000	-8.80	1.000	3.13	1.000
[1,1]	126.8	0.675	29.3	0.999	-8.87	0.980	3.13	0.997
[2,2]	79.0	0.979	32.8	0.912	-8.93	0.976	3.13	0.997
[3,3]	73.2	0.943	32.9	0.910	-8.93	0.976		
[4,4]	73.2	0.943	32.9	0.910				

boundstate energy. Secondly, there is a singularity from the Green function. It is this second singularity and the fact that the two-particle T -matrix becomes complex, that distinguishes the problems above and below breakup threshold. Both singularities are removed by making subtractions in the integrals. To find the complete

off-shell two-particle T -matrix for positive energy arguments, use has been made of the Kowalsky-Noyes trick [9].

The calculations have been performed up to 50 MeV laboratory energy. Over this wide range of energies at most a [6,6] approximant was needed to find a convergent result. There was a tendency that lower approximants were needed at higher energies. This is to be expected, since at very high energy the impulse approximation should be reasonable. As a typical example of the rate of convergence, the various Padé approximants are given in table 1 for the phase shifts and inelasticity parameters at 14.1 MeV. The [0,0] approximant is equivalent to the Born term. From this table we see that the convergence rate in the quartet case is in general faster than in the doublet case. Also for higher l values lower approximants are sufficient which is in accordance with the fact that the multiple scattering series becomes less divergent. For $l \geq 5$ the Born term already gives the correct value within two percent. The resulting differential cross-section taking into account the first twelve partial waves, is shown in fig.2. Also the quartet and doublet contributions are given separately. From this figure we see that there is a remarkable agreement with the experimental data. The main difference with the separable result is that in the latter case the differential cross-section is too low in the forward direction [1,2].

To summarize, we have shown that the Padé approximants converge rapidly below and above the breakup threshold and therefore this method is a powerful alternative to obtain the amplitude for three-particle scattering with local potentials.

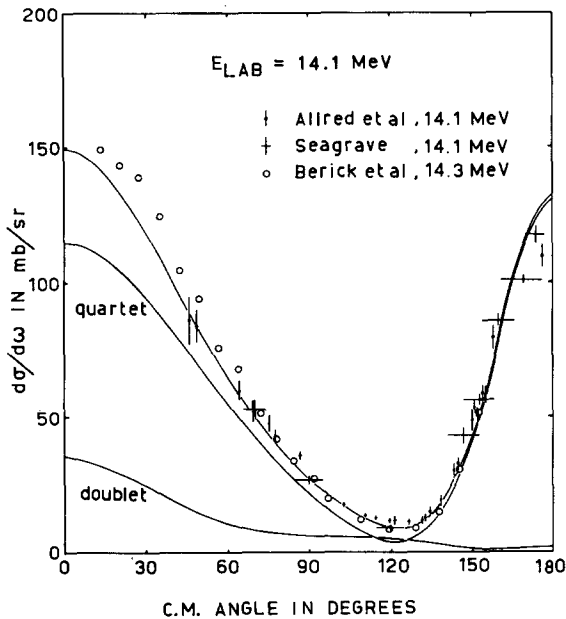


Fig.2. Experimental and theoretical angular distribution for neutron-deuteron scattering at 14.1 MeV. For the experimental values see ref. [10]. The quartet and doublet contributions are given separately.

A more detailed account of the method and results at other energies will be published elsewhere.

This investigation is part of the research program of the 'Stichting voor Fundamenteel Onderzoek der Materie' and was made possible by financial support from the 'Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek'.

References

- [1] R. Aaron, R. D. Amado and Y. Y. Yam, Phys. Rev. 140B (1965) 1291;
R. Aaron and R. D. Amado, Phys. Rev. 150 (1966) 857.
- [2] R. T. Cahill and I. H. Sloan, Phys. Lett. 31B (1970) 353; 33B (1970) 195;
I. H. Sloan, Nucl. Phys. A168 (1971) 211.
- [3] J. S. Levinger, Proc. Symp. on the Nuclear three-body problem and related topics, Budapest (1971), Acta Physica Hungarica, to be published.
- [4] J. A. Tjon, Phys. Rev. D1 (1970) 2109.
- [5] For a review of the theory of Padé approximants, see The Padé approximant in theoretical physics, eds. G. A. Baker Jr. and J. L. Gammel (Academic Press, New York, 1970).
- [6] I. H. Sloan, Phys. Rev. 185 (1969) 1361.
- [7] R. Chisholm, J. Math. Phys. 4 (1963) 1506;
J. Nuttall, J. Math. Anal. Appl., to be published.
- [8] R. A. Malfliet and J. A. Tjon, Nucl. Phys. A127 (1969) 161; Ann. Phys. 61 (1970) 425.
- [9] K. L. Kowalski, Phys. Rev. Lett. 15 (1965) 798;
H. P. Noyes, Phys. Rev. Lett. 15 (1965) 538.
- [10] J. C. Allred et al., Phys. Rev. 91 (1953) 90;
J. D. Seagrave, Phys. Rev. 97 (1955) 757;
A. C. Berick et al., Phys. Rev. 174 (1968) 1105.

* * * * *