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SU(3) × SU(3) SYMMETRY BREAKING IN A SIMPLE MODEL

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Abstract: A field-theoretical model, due to Lévy, is studied. It contains a triplet of quarks and a pseudoscalar and a scalar meson nonet. The original SU(3) × SU(3) symmetry is broken by terms linear in the scalar meson fields. A renormalization and regularization procedure is defined in order to remove the ultra-violet divergences. The possibility of a spontaneous breakdown of the symmetry is described and the Goldstone theorem is verified in the one-loop approximation.
Moreover, the Ademollo-Gatto theorem is reproduced. The axial vector coupling constants differ in first order of the SU(3) symmetry breaking.

1. INTRODUCTION

Some years ago Gell-Mann [1] and Gell-Mann, Oakes and Renner [2] proposed a model for the strong interactions with two important features. In the first place the Hamiltonian density was given by

$$\mathcal{H} = \mathcal{H}_0 - c_0 u_0 - c_8 u_8 ,$$

where $\mathcal{H}_0$ is invariant under SU(3) × SU(3) symmetry and the scalar densities u_i together with the pseudoscalar densities v_i ($i = 0, 1, \ldots, 8$) transform according to the $(3, \overline{3}) + (\overline{3}, 3)$ representation of SU(3) × SU(3). Further the equal-time commutation relations of the vector and axial vector charge operators were assumed to remain valid also in the presence of the SU(3) × SU(3) breaking interaction.

One may be interested to what extend the symmetry breaking manifests itself in observable quantities. For example, according to the Ademollo-Gatto theorem no first order SU(3) symmetry breaking effects occur in the weak vector coupling constant. It is an interesting question as to what the symmetry breaking effects may be for the axial vector current. Experimentally no large breaking of SU(3) symmetry is seen, and one may be tempted to speculate that in certain models an Ademollo-Gatto-like theorem holds for the axial vector currents. In fact, recently Matsuda and Oneda [3] forwarded an argument concerning this possibility.

In this article we intend to study the effects of symmetry breaking in a field-theoretical example of Gell-Mann's proposal. This model was suggested by Lévy [4] as an extension of the well-known sigma model. The densities u_i and v_i are provided by the scalar and pseudoscalar meson fields. Besides the eighteen scalar and pseudoscalar mesons the model contains a triplet of quarks. Lévy investigated the effect of the symmetry breaking terms in the tree approximation. However, if one wants to study the effects of symmetry breaking in a wider sense one must compute higher order corrections. Fortunately the model is renormalizable and a procedure is available which conserves the symmetries of the model. Lee [5] gave a demonstration of this procedure for the sigma model and his arguments can easily be generalized for Lévy's model. The essential part of Lee's arguments is that it is possible to make the theory finite by using only the counterterms of the symmetric theory. Moreover he describes how the handle the terms linear in the scalar fields correctly in perturbation theory.

The first terms of a perturbation expansion including the effects of one closed loop are computed in this paper. Sect. 2 gives the explicit form of the model and its symmetry properties. An important point is the existence of some Ward-Takahashi identities, which serve as a test for the calculations.

In sect. 3 the renormalization and regularization scheme is presented. Sects. 4 and 5 give the calculation of the propagators and the verification of the Goldstone theorem [7]. Moreover the coupling constants of the vector and axial vector currents are determined.

In sect. 6 some SU(3) symmetry breaking effects are considered. The coupling constant of the strangeness changing vector current turns out to be unrenormalized to first order in SU(3) symmetry breaking. This is in accordance with the Ademollo-Gatto theorem [8]. In the case of a spontaneously broken solution this theorem remains valid. The difference of the strangeness changing and strangeness conserving axial vector coupling constants is of first order in SU(3) symmetry breaking. This is in contradiction with the suggestions of ref. [3].

2. THE MODEL AND ITS SYMMETRY STRUCTURE

The Lagrangian of the model is

$$\begin{aligned}\mathcal{L} = &- \bar{q}\gamma_\mu \partial_\mu q - \tfrac{1}{2}\mathrm{Tr}\,\{(\partial_\mu P)^2 + (\partial_\mu \Sigma)^2 + \mu^2(P^2+\Sigma^2)\} \\ &- g\bar{q}(i\gamma_5 P + \Sigma)q - \lambda W_1(P,\Sigma) - \rho W_2(P,\Sigma) - \nu W_3(P,\Sigma) + c_0\Sigma_0 + c_8\Sigma_8\,, \end{aligned} \tag{2.1}$$

where q is a quark triplet of four-component fermion fields and Σ and P are nonets of scalar and pseudoscalar meson fields, which are represented by three-by-three matrices in the following way:

$$P = \frac{1}{\sqrt{2}} P_i \lambda_i \quad \text{and} \quad \Sigma = \frac{1}{\sqrt{2}} \Sigma_i \lambda_i \,.$$

The λ_i are the usual Gell-Mann matrices [1], $(i = 0, 1, \ldots, 8)$. $W_1(P,\Sigma)$, $W_2(P,\Sigma)$ and $W_3(P,\Sigma)$ are meson-meson interactions of the following type:

$$W_1(P,\Sigma) = \mathrm{Tr}\{P^4 + \Sigma^4 + 4\Sigma^2 P^2 - 2P\Sigma P\Sigma\}\ ,$$

$$W_2(P,\Sigma) = [\mathrm{Tr}\{P^2 + \Sigma^2\}]^2\ ,$$

$$\begin{aligned} W_3(P,\Sigma) = {} & 2\,\mathrm{Tr}\{\Sigma^3\} - 3\,\mathrm{Tr}\{\Sigma\}\ \mathrm{Tr}\{\Sigma^2\} + (\mathrm{Tr}\{\Sigma\})^3 \\ & - 6\,\mathrm{Tr}\{\Sigma P^2\} + 6\,\mathrm{Tr}\{\Sigma P\}\ \mathrm{Tr}\{P\} + 3\,\mathrm{Tr}\{\Sigma\}\ \mathrm{Tr}\{P^2\} \\ & - 3\,\mathrm{Tr}\{\Sigma\}\ (\mathrm{Tr}\{P\})^2\ . \end{aligned} \tag{2.2}$$

Apart from the terms linear in the Σ fields, the Lagrangian is invariant under the following infinitesimal SU(3) × SU(3) transformations [4]:

$$\delta q = i\alpha_i^{\mathrm{V}}\ \tfrac{1}{2}\lambda_i q\ , \qquad \delta\bar{q} = -i\alpha_i^{\mathrm{V}}\ \bar{q}\tfrac{1}{2}\lambda_i\ ,$$

$$\delta P = i\alpha_i^{\mathrm{V}}\ (\tfrac{1}{2}\lambda_i P - P\tfrac{1}{2}\lambda_i)\ ,$$

$$\delta\Sigma = i\alpha_i^{\mathrm{V}}\ (\tfrac{1}{2}\lambda_i\Sigma - \Sigma\tfrac{1}{2}\lambda_i)\ , \qquad i = 1,\ldots,8 \tag{2.3}$$

and

$$\delta q = i\alpha_i^{\mathrm{A}}\ \tfrac{1}{2}\lambda_i\gamma_5 q\ , \qquad \delta\bar{q} = i\alpha_i^{\mathrm{A}}\ \bar{q}\gamma_5\tfrac{1}{2}\lambda_i\ ,$$

$$\delta P = -\alpha_i^{\mathrm{A}}\ (\tfrac{1}{2}\lambda_i\Sigma + \Sigma\tfrac{1}{2}\lambda_i)\ ,$$

$$\delta\Sigma = \alpha_i^{\mathrm{A}}\ (\tfrac{1}{2}\lambda_i P + P\tfrac{1}{2}\lambda_i)\ , \qquad i = 1,\ldots,8\ . \tag{2.4}$$

Note that the P_i and Σ_i transform according to the $(3,\bar{3}) + (\bar{3},3)$ representation of SU(3) × SU(3).

The transformations (2.3) generate an octet of vector currents.

$$\begin{aligned} V_{\mu i} &= -\frac{\delta\mathcal{L}}{\delta\partial_\mu\alpha_i^{\mathrm{V}}} \\ &= i\bar{q}\gamma_\mu\tfrac{1}{2}\lambda_i q + i\,\mathrm{Tr}\{\tfrac{1}{2}\lambda_i[P,\partial_\mu P] + \tfrac{1}{2}\lambda_i[\Sigma,\partial_\mu\Sigma]\}\ . \end{aligned} \tag{2.5}$$

An octet of axial vector currects is generated by the transformations (2.4).

$$A_{\mu i} = -\frac{\delta\mathcal{L}}{\delta\partial_\mu\alpha_i^{\mathrm{A}}} = i\bar{q}\gamma_\mu\gamma_5\tfrac{1}{2}\lambda_i q + \mathrm{Tr}\{\tfrac{1}{2}\lambda_i\{P,\partial_\mu\Sigma\} - \tfrac{1}{2}\lambda_i\{\Sigma,\partial_\mu P\}\}\ . \tag{2.6}$$

The brackets [,] and { , } mean commutation and anticommutation.

The equations of motion give the PCAC result

$$\partial_\mu A_{\mu i} = -\frac{\delta\mathcal{L}}{\delta\alpha_i^{\mathrm{A}}} = -(d_{ij0}\,c_0 + d_{ij8}\,c_8)\,P_j \tag{2.7}$$

and for the vector currents

$$\partial_\mu V_{\mu i} = -\frac{\delta \mathcal{L}}{\delta \alpha_{\underset{i}{V}}} = f_{ij8}\, c_8\, \Sigma_j\,. \tag{2.8}$$

Due to the simple structure of the symmetry breaking part of the Lagrangian (2.1) we have some useful Ward-Takahashi identities [9]:

$$\begin{aligned}\frac{\partial}{\partial x_\mu}\langle 0|\mathrm{T}(A_{\mu i}(x)P_j(0))|0\rangle = &-(d_{in0}c_0+d_{in8}c_8)\\ &\times\langle 0|\mathrm{T}(P_n(x)P_j(0))|0\rangle - i(d_{ij0}\langle 0|\Sigma_0|0\rangle\\ &+d_{ij8}\langle 0|\Sigma_8|0\rangle)\,\delta^4(x)\,,\end{aligned}\tag{2.9}$$

$$\begin{aligned}\frac{\partial}{\partial x_\mu}\langle 0|\mathrm{T}(V_{\mu i}(x)\Sigma_j(0))|0\rangle = &\, f_{in8}\,c_8\,\langle 0|\mathrm{T}(\Sigma_n(x)\Sigma_j(0))|0\rangle\\ &+if_{ij8}\langle 0|\Sigma_8|0\rangle\,\delta^4(x)\,,\end{aligned}\tag{2.10}$$

$$\begin{aligned}\frac{\partial}{\partial x_\mu}\langle 0|\mathrm{T}(V_{\mu i}(x)P_j(y)P_k(z))|0\rangle = &\, f_{in8}\,c_8\langle 0|\mathrm{T}(\Sigma_n(x)P_j(y)P_k(z))|0\rangle\\ &+i\delta^4(x-y)f_{ijn}\langle 0|\mathrm{T}(P_n(y)P_k(z))|0\rangle\\ &+if_{ikn}\,\delta^4(x-z)\langle 0|\mathrm{T}(P_j(y)P_n(z))|0\rangle\,,\end{aligned}\tag{2.11}$$

$$\begin{aligned}&\frac{\partial}{\partial x_\mu}\langle 0|\mathrm{T}(V_{\mu i}(x)\,q(y)\,\bar{q}(z))|0\rangle = f_{ij8}\,c_8\langle 0|\mathrm{T}(\Sigma_j(x)\,q(y)\,\bar{q}(z)|0\rangle\\ &-\delta^4(x-y)\,\tfrac{1}{2}\lambda_i\langle 0|\mathrm{T}(q(y)\,\bar{q}(z))|0\rangle+\delta^4(x-z)\,\langle 0|\mathrm{T}(q(y)\,\bar{q}(z))|0\rangle\,\tfrac{1}{2}\lambda_i\,,\end{aligned}\tag{2.12}$$

$$\begin{aligned}\frac{\partial}{\partial x_\mu}\langle 0|\mathrm{T}(A_{\mu i}(x)\,q(y)\,\bar{q}(z))|0\rangle = &-(d_{ij0}\,c_0+d_{ij8}\,c_8)\\ &\times\langle 0|\mathrm{T}(P_j(x)\,q(y)\,\bar{q}(z))|0\rangle-\delta^4(x-y)\,\tfrac{1}{2}\lambda_i\gamma_5\,\langle 0|\mathrm{T}(q(y)\,\bar{q}(z))|0\rangle\\ &-\delta^4(x-z)\,\langle 0|\mathrm{T}(q(y)\,\bar{q}(z))|0\rangle\,\gamma_5\,\tfrac{1}{2}\lambda_i\,.\end{aligned}\tag{2.13}$$

The identities (2.9) and (2.10) give essentially the Goldstone theorem [7] as pointed out by Lee [5]. The identities (2.11) and (2.12) assure the vector coupling constants of the currents with $i = 1, 2, 3, 8$ to be unrenormalized, which is a sufficient condition to derive the Ademollo-Gatto theorem [8].

3. RENORMALIZATION AND REGULARIZATION

Some years ago Lévy calculated the physical quantities of the model

(2.1) in a tree-approximation [4]. However, such a calculation does not show some well-known effects like the renormalization of coupling constants. Because we are interested in SU(3) symmetry breaking of the renormalized current coupling constants we have to take into account the effect of closed loops. For convenience we take the coupling constants of the meson-meson interactions (2.2) equal to zero.

Consider now a perturbation series in g. Following Lee [5] (gF_0) and (gF_8) are taken to be of order zero. This corresponds to an expansion with respect to the number of closed loops [10] (not counting loops contained in the tadpoles). F_0 and F_8 are the vacuum expectation values of the Σ_0 and Σ_8 fields, obtained by summing all diagrams representing the vanishing of a Σ_0 or Σ_8 line into the vacuum. As a result of this approach two consistency relations for F_0 and F_8 are obtained, as will be shown more explicitly in sect. 4.

When calculating diagrams we are concerned with the problem of ultraviolet divergencies. By adding certain counterterms one can make the theory finite. Generalizing the arguments for the sigma model [5, 6] it is clear that the counterterms of the SU(3) × SU(3) symmetric theory are sufficient. For the symmetric theory in the one-loop approximation we need only four counterterms:

$$-\tfrac{1}{2}\delta\mu^2 Z_{\mathrm{B}}\,\mathrm{Tr}\{P^2+\Sigma^2\} - \tfrac{1}{2}(Z_{\mathrm{B}}-1)\,\mathrm{Tr}\{(\partial_\mu P)^2 + (\partial_\mu \Sigma)^2 + \mu^2(P^2+\Sigma^2)\}$$
$$-(Z_{\mathrm{F}}-1)\bar{q}\gamma_\mu\partial_\mu q - g^4\,\delta\Lambda\, W_1(P,\Sigma)\,. \tag{3.1}$$

These counterterms make the boson and fermion propagators finite. The fourth counterterm cancels the divergence of the box diagram with a fermion going around.

So our Lagrangian has the following structure:

$$\begin{aligned}\mathcal{L} = & -\bar{q}\gamma_\mu\partial_\mu q - \tfrac{1}{2}\mathrm{Tr}\{(\partial_\mu P)^2 + (\partial_\mu\Sigma)^2 + \mu^2(P^2+\Sigma^2)\} \\ & - g\bar{q}(i\gamma_5 P+\Sigma)q + c_0\Sigma_0 + c_8\Sigma_8 - \tfrac{1}{2}\delta\mu^2 Z_{\mathrm{B}}\,\mathrm{Tr}\{P^2+\Sigma^2\} \\ & - \tfrac{1}{2}(Z_{\mathrm{B}}-1)\,\mathrm{Tr}\{(\partial_\mu P)^2 + (\partial_\mu\Sigma)^2 + \mu^2(P^2+\Sigma^2)\} \\ & -(Z_{\mathrm{F}}-1)\bar{q}\gamma_\mu\partial_\mu q - g^4\,\delta\Lambda\, W_1(P,\Sigma)\,. \end{aligned} \tag{3.2}$$

To fix the counterterms in every order we insist that the pion propagator has a pole at $p^2 = -\mu^2 = -m_\pi^2$, m_π is the physical mass of the pion, and furthermore we require the pion fields and one of the quark fields to be asymptotically normalized to unit amplitude. The fourth counterterm contains one free parameter which may be fixed by the experimental π - π coupling constant.

In order to carry through this renormalization scheme we add a set of regulator fields to the Lagrangian (3.2):

$$\mathcal{L} \Longrightarrow \mathcal{L} + \mathcal{L}_{\mathrm{REG}} ,$$

where

$$\begin{aligned}\mathcal{L}_{\mathrm{REG}} = \sum_{j=1} [& -\bar{Q}^1(\gamma_\mu \partial_\mu + m_j) Q_j^1 - \bar{Q}_j^2(\gamma_\mu \partial_\mu + m_j) Q_j^2 \\ & - g_j \{ \bar{Q}_j^1 \Sigma Q_j^1 - \bar{Q}_j^2 \Sigma Q_j^2 + \bar{Q}_j^1 P Q_j^2 + \bar{Q}_j^2 P Q_j^1 \}] \\ + \sum_{k=1} [& -\tfrac{1}{2}\eta_k \,\mathrm{Tr}\, \{ (\partial_\mu \Pi_k)^2 + (\partial_\mu \Omega_k)^2 + \mu_k^2 (\Pi_k^2 + \Omega_k^2) \} \\ & - \tilde{g}_k \bar{q}(i\gamma_5 \Pi_k + \Omega_k) q] . \end{aligned} \tag{3.3}$$

Q_j^1 and Q_j^2 are triplets of fermion regulator fields. They have opposite parity. So for every j there are six regulator fields which form an SU(3) triplet and a parity doublet. The various fermion regulators $Q_j^{1,2}$ are quantized according to the anticommutation ($\epsilon_j = +1$) or commutation relations ($\epsilon_j = -1$). Π_k and Ω_k are SU(3) nonets of pseudoscalar and scalar boson regulators with normal ($\eta_k = +1$) or indefinite metric ($\eta_k = -1$).

Because we want to conserve the symmetry of the original Lagrangian, we are obliged to require the following SU(3) × SU(3) transformation properties for the regulator fields:

$$\begin{aligned} \delta Q_j^{1,2} &= i\alpha_i^{\mathrm{V}} \tfrac{1}{2}\lambda_i Q_j^{1,2} , & \delta \bar{Q}_j^{1,2} &= -i\alpha_i^{\mathrm{V}} \bar{Q}_j^{1,2} \tfrac{1}{2}\lambda_i , \\ \delta \Pi_k &= i\alpha_i^{\mathrm{V}} (\tfrac{1}{2}\lambda_i \Pi_k - \Pi_k \tfrac{1}{2}\lambda_i) , & \delta \Omega_k &= i\alpha_i^{\mathrm{V}} (\tfrac{1}{2}\lambda_i \Omega_k - \Omega_k \tfrac{1}{2}\lambda_i) , \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} \delta Q_j^1 &= \alpha_i^{\mathrm{A}} \tfrac{1}{2}\lambda_i Q_j^2 , & \delta \bar{Q}_j^1 &= \alpha_i^{\mathrm{A}} \bar{Q}_j^2 \tfrac{1}{2}\lambda_i , \\ \delta Q_j^2 &= -\alpha_i^{\mathrm{A}} \tfrac{1}{2}\lambda_i Q_j^1 , & \delta \bar{Q}_j^2 &= -\alpha_i^{\mathrm{A}} \bar{Q}_j^1 \tfrac{1}{2}\lambda_i , \\ \delta \Pi_k &= -\alpha_i^{\mathrm{A}} (\tfrac{1}{2}\lambda_i \Omega_k + \Omega_k \tfrac{1}{2}\lambda_i) , & \delta \Omega_k &= \alpha_i^{\mathrm{A}} (\tfrac{1}{2}\lambda_i \Pi_k + \Pi_k \tfrac{1}{2}\lambda_i) . \end{aligned} \tag{3.5}$$

Apart from the terms linear in the Σ fields, $\mathcal{L} + \mathcal{L}_{\mathrm{REG}}$ is invariant under the combined transformations (2.3), (3.4) and (2.4), (3.5). Therefore all the symmetry relations like Ward-Takahashi identities remain unchanged.

Concerning the fermion regulators, this procedure is an SU(3) generalization of the concept of parity doublet regulators as proposed by Gervais and Lee for the sigma model [6].

By means of these regulator fields one can make the theory finite. Because these fields are unphysical, their masses will be taken very large. When doing this, four divergences appear in our calculations, which will be denoted by D_1, D_2, D_3 and D_4. They can be absorbed into the counter-

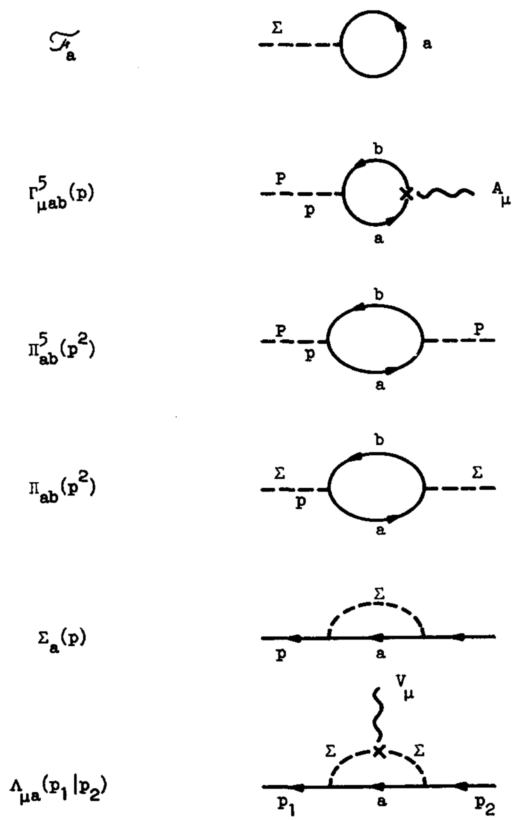


Fig. 1. Diagrams with corresponding functions. Clebsch-Gordan coefficients are not included in the definitions. Tadpoles are attached to internal fermion lines, which give the internal quarks a mass: $m_a = gF_a$. However, such attachments are omitted in all the figures.

terms. In appendix A the regularization procedure is worked out for all diagrams that we need later. Those are listed in fig. 1.

The Feynman rules for the Lagrangian (3.2) are shown in fig. 2. The quark mass in the tree approximation is

$$m_a = gF_a \ , \tag{3.6}$$

where a denotes p, n or λ quark. F_a is defined by

$$F_p = F_n = \frac{1}{\sqrt{3}} (F_0 + \frac{1}{\sqrt{2}} F_8) \ ,$$

$$F_\lambda = \frac{1}{\sqrt{3}} (F_0 - \sqrt{2}\, F_8) \ . \tag{3.7}$$

Because SU(2) symmetry is conserved, we will use only the indices p and λ.

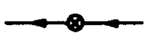
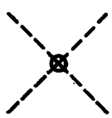
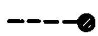
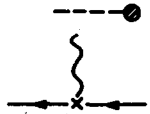
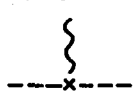
	$i(p^2+\mu^2)^{-1}$	P or Σ line
	$\not{p}\, p^{-2}$	quark line
	$-ig\gamma_5$ or $-g$	quark-P or quark-Σ vertex
	c_0 or c_8	linear Σ_0 or Σ_8 term
	$-\delta\mu^2\, Z_B$	mass renormalization counter-term
	$-(Z_B-1)(p^2+\mu^2)$	wave function renormalization counterterm
	$-(Z_F-1)\,\not{p}$	wave function renormalization counterterm
	$-g^4\delta\Lambda\, W_1(P,\Sigma)$	meson-meson counterterm
	F_0 or F_8	vacuum expectation value of the Σ_0 or Σ_8 field
 and 	V_μ or A_μ	vertices belonging to the vector and axial vector currents

Fig. 2. The Feynman rules for the Lagrangian (3.2).

4. CALCULATION OF THE PROPAGATORS; CONSISTENCY RELATIONS

The counterterm $\delta\Lambda$ cancels the divergence of the fermion loop contributions to the four-point boson vertices. Calculation gives the result:

$$\delta\Lambda = \frac{D_1}{8\pi^2} + \lambda\ . \tag{4.1}$$

λ may be fixed by the experimental $\pi-\pi$ coupling constant.

To determine the counterterms Z_B and $\delta\mu^2$, we impose the pion propagator to exhibit the behaviour

$$D_\pi(s) = \frac{i}{s+\mu^2} \quad \text{as} \quad s \rightarrow -\mu^2 . \tag{4.2}$$

The expression for the pion propagator is given in appendix B, and leads to

$$iD_\pi^{-1}(s) = s+\mu^2 - \frac{g^2}{4\pi^2}\Big\{ s[D_3 - \tfrac{1}{3}] + 6D_2 - \tfrac{11}{3}\ m_p^2 + m_p^2 \log\frac{m_p^2}{\mu^2}$$
$$+\tfrac{1}{2}s\int_0^1 dx \log\frac{sx(1-x)+m_p^2}{\mu^2}\Big\} + 4g^2\lambda m_p^2$$
$$+ Z_B\,\delta\mu^2 + (Z_B - 1)(s+\mu^2) . \tag{4.3}$$

Imposing the condition (4. 2), the contribution of the counterterms is

$$Z_B\,\delta\mu^2 + (Z_B-1)(s+\mu^2) = \frac{g^2}{4\pi^2}\{s[D_3 - \tfrac{1}{3}] + 6D_2 - \tfrac{11}{3}\ m_p^2 + m_p^2\log\frac{m_p^2}{\mu^2}$$
$$+\tfrac{1}{2}sB_{pp}(-\mu^2) - \tfrac{1}{2}(s+\mu^2)\,\mu^2 B'_{pp}(-\mu^2)\} - 4g^2\lambda m_p^2 , \tag{4.4}$$

where we used the notation

$$B_{ab}(s) = \int_0^1 dx \log\frac{sx(1-x)+m_a^2 x + m_b^2(1-x)}{\mu^2} ,$$

$$B'_{ab}(s) = \frac{d}{ds}B_{ab}(s) . \tag{4.5}$$

a and b denote the particular members of the quark triplet.

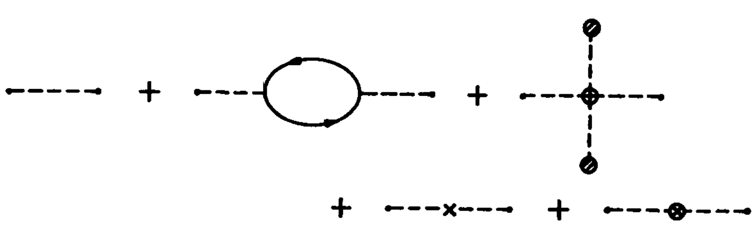


Fig. 3. Diagrams of the meson propagators.

The diagrams that contribute to the meson propagators are shown in fig. 3. Appendix B gives the corresponding expressions. Because a $P_0 - P_8$ and a $\Sigma_0 - \Sigma_8$ mixing arises, the propagators of the $\Sigma_0 - \Sigma_8$ and the $P_0 - P_8$ combinations are represented by two-by-two matrices. A straightforward calculation gives the following result for the pseudoscalar mesons:

$$iD_{\pi}^{-1}(s) = s + \mu^2 - \frac{g^2}{8\pi^2} \{ s[B_{pp}(s) - B_{pp}(-\mu^2)] + (s+\mu^2)\,\mu^2 B'_{pp}(-\mu^2)\} , \tag{4.6}$$

$$\begin{aligned} iD_{K}^{-1}(s) = s + \mu^2 - \frac{g^2}{8\pi^2} \{ & \tfrac{1}{3}(m_p - m_\lambda)(19m_\lambda + 3m_p) + m_\lambda^2 \log\frac{m_\lambda^2}{\mu^2} - m_p^2 \log\frac{m_p^2}{\mu^2} \\ & + s[B_{p\lambda}(s) - B_{pp}(-\mu^2)] + (m_p - m_\lambda)^2 B_{p\lambda}(s) \\ & + (s+\mu^2)\,\mu^2 B'_{pp}(-\mu^2)\} + 4g^2 \lambda m_\lambda (m_\lambda - m_p) , \end{aligned} \tag{4.7}$$

$$\begin{aligned} iD_{08}^{-1}(s) = s + \mu^2 - \frac{g^2}{8\pi^2} \{ & \tfrac{11}{3}(m_p^2 - m_\lambda^2) + m_\lambda^2 \log\frac{m_\lambda^2}{\mu^2} - m_p^2 \log\frac{m_p^2}{\mu^2} \\ & + \tfrac{1}{2} s[B_{\lambda\lambda}(s) + B_{pp}(s) - 2B_{pp}(-\mu^2)] + (s+\mu^2)\,\mu^2 B'_{pp}(-\mu^2)\} \\ & - 2g^2\lambda(m_p^2 - m_\lambda^2) + \tfrac{1}{3}[\frac{g^2}{8\pi^2} \{ \tfrac{11}{3}(m_p^2 - m_\lambda^2) + m_\lambda^2 \log\frac{m_\lambda^2}{\mu^2} - m_p^2 \log\frac{m_p^2}{\mu^2} \\ & + \tfrac{1}{2} s[B_{\lambda\lambda}(s) - B_{pp}(s)]\} + 2g^2 \lambda(m_p^2 - m_\lambda^2)] \begin{pmatrix} 1 & 2\sqrt{2} \\ 2\sqrt{2} & -1 \end{pmatrix} . \end{aligned} \tag{4.8}$$

The propagators of the scalar mesons, denoted by a wiggle, have the following form:

$$\begin{aligned} i\tilde{D}_{\pi}^{-1}(s) = s + \mu^2 - \frac{g^2}{8\pi^2} \{ & -\tfrac{32}{3} m_p^2 + s[B_{pp}(s) - B_{pp}(-\mu^2)] + 4m_p^2 B_{pp}(s) \\ & + (s+\mu^2)\,\mu^2 B'_{pp}(s)\} + 8g^2 \lambda m_p^2 , \end{aligned} \tag{4.9}$$

$$\begin{aligned} i\tilde{D}_{K}^{-1}(s) = s + \mu^2 - \frac{g^2}{8\pi^2} \{ & -\tfrac{1}{3}(m_\lambda + m_p)(19m_\lambda - 3m_p) + m_\lambda^2 \log\frac{m_\lambda^2}{\mu^2} - m_p^2 \log\frac{m_p^2}{\mu^2} \\ & + s[B_{p\lambda}(s) - B_{pp}(-\mu^2)] + (m_p + m_\lambda)^2 B_{p\lambda}(s) \\ & + (s+\mu^2)\,\mu^2 B'_{pp}(-\mu^2)\} + 4g^2 \lambda m_\lambda (m_p + m_\lambda) , \end{aligned} \tag{4.10}$$

$$
\begin{aligned}
i\tilde{D}^{-1}_{08}(s) = {} & s + \mu^2 - \frac{g^2}{8\pi^2}\{-\tfrac{5}{3}m_\mathrm{p}^2 - 9m_\lambda^2 + m_\lambda^2 \log\frac{m_\lambda^2}{\mu^2} - m_\mathrm{p}^2 \log\frac{m_\mathrm{p}^2}{\mu^2} \\
& + \tfrac{1}{2}s[B_{\mathrm{pp}}(s) + B_{\lambda\lambda}(s) - 2B_{\mathrm{pp}}(-\mu^2)] + 2m_\mathrm{p}^2 B_{\mathrm{pp}}(s) \\
& + 2m_\lambda^2 B_{\lambda\lambda}(s) + (s+\mu^2)\mu^2 B'_{\mathrm{pp}}(-\mu^2)\} + 2g^2\lambda(m_\mathrm{p}^2 + 3m_\lambda^2) \\
& + \tfrac{1}{3}[\frac{g^2}{8\pi^2}\{9(m_\mathrm{p}^2 - m_\lambda^2) + m_\lambda^2 \log\frac{m_\lambda^2}{\mu^2} - m_\mathrm{p}^2 \log\frac{m_\mathrm{p}^2}{\mu^2} \\
& + \tfrac{1}{2}s[B_{\lambda\lambda}(s) - B_{\mathrm{pp}}(s)] + 2m_\lambda^2 B_{\lambda\lambda}(s) - 2m_\mathrm{p}^2 B_{\mathrm{pp}}(s)\} \\
& + 6g^2\lambda(m_\mathrm{p}^2 - m_\lambda^2)] \begin{pmatrix} 1 & 2\sqrt{2} \\ 2\sqrt{2} & -1 \end{pmatrix} .
\end{aligned}
\tag{4.11}
$$

By means of the formulae (4.6) - (4.11) we can calculate the masses, the wave function renormalization constants and the mixing angles of the mesons, because these quantities are determined by the poles and the residues of the propagators.

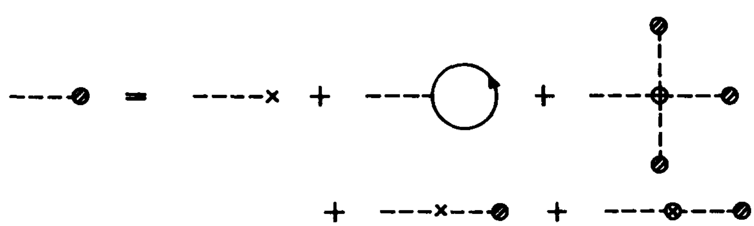


Fig. 4. The pictorial representation of the consistency relations.

For the vacuum expectation values of the Σ_0 and the Σ_8 field two consistency relations hold. They are represented by the diagrams of fig. 4. The corresponding expressions are shown in appendix B. A straightforward computation yields the result:

$$
[\mu^2 - \frac{g^2}{8\pi^2}\mu^4 B'_{\mathrm{pp}}(-\mu^2)]\, F_\mathrm{p} = \frac{1}{\sqrt{3}}[c_0 + \frac{1}{\sqrt{2}}c_8] , \tag{4.12}
$$

$$
\begin{aligned}
& [\mu^2 - \frac{g^2}{8\pi^2}\mu^4 B'_{\mathrm{pp}}(-\mu^2)]\, F_\lambda = \frac{1}{\sqrt{3}}[c_0 - \sqrt{2}c_8] \\
& + 4g^2\lambda(m_\mathrm{p}^2 - m_\lambda^2)F_\lambda + \frac{g^2}{4\pi^2}\{m_\lambda^2[\log\frac{m_\lambda^2}{\mu^2} - \tfrac{11}{3}] - m_\mathrm{p}^2[\log\frac{m_\mathrm{p}^2}{\mu^2} - \tfrac{11}{3}]\}F_\lambda .
\end{aligned}
\tag{4.13}
$$

By means of these consistency relations one can derive certain equations for F_{p}, F_λ and the inverse propagators at the point $s = 0$. These equations, which are a consequence of the Ward-Takahashi identities (2.9) and (2.10), have the following form:

$$iD_\pi^{-1}(0)\, F_{\mathrm{p}} = \frac{1}{\sqrt{3}}\,[c_0 + \frac{1}{\sqrt{2}} c_8]\,, \tag{4.14}$$

$$iD_{\mathrm{K}}^{-1}(0)[F_{\mathrm{p}} + F_\lambda] = \frac{2}{\sqrt{3}}\,[c_0 - \frac{1}{2\sqrt{2}} c_8]\,, \tag{4.15}$$

$$i\tilde{D}_{\mathrm{K}}^{-1}(0)[F_{\mathrm{p}} - F_\lambda] = \sqrt{\tfrac{3}{2}}\,c_8\,, \tag{4.16}$$

$$iD_{08}^{-1}(0)\, \boldsymbol{F} = \boldsymbol{c}\,. \tag{4.17}$$

Where $D_{08}^{-1}(0)$ is a two-by-two matrix and $\boldsymbol{c}$ and $\boldsymbol{F}$ represent the vectors:

$$\boldsymbol{c} = (c_8,\ c_0 - \frac{1}{\sqrt{2}} c_8)\,,$$

$$\boldsymbol{F} = \frac{1}{\sqrt{3}}\,(\sqrt{2}\,(F_{\mathrm{p}} - F_\lambda),\ F_{\mathrm{p}} + 2F_\lambda)\,. \tag{4.18}$$

The equations (4.14) - (4.17) give the well-known result that a spontaneous breakdown of a symmetry entails the existence of massless bosons, which is the Goldstone theorem [7].

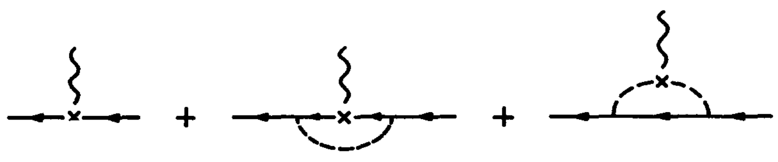


Fig. 5. Diagrams of the quark propagators

The diagrams contributing to the quark propagators are shown in fig. 5. The corresponding expressions are given in appendix B. We fix the counterterm Z_{F} in such a way that the proton quark field is asymptotically normalized to unit amplitude:

$$Z_{\mathrm{F}} = 1 + \frac{g^2}{8\pi^2}\,\{\tfrac{3}{2}[D_4 + \tfrac{1}{2}] + A(-m_{\mathrm{p}}^2) - 2m_{\mathrm{p}}^2 A'(-m_{\mathrm{p}}^2)\}\,, \tag{4.19}$$

where we used the notation

$$A(s) = \int_0^1 \mathrm{d}x\, x \left\{ 2 \log \frac{sx(1-x) + \mu^2 x + m_\mathrm{p}^2(1-x)}{\mu^2} + \log \frac{sx(1-x) + \mu^2 x + m_\lambda^2(1-x)}{\mu^2} \right\},$$

$$A'(s) = \frac{\mathrm{d}}{\mathrm{d}s} A(s) \,. \tag{4.20}$$

Using this result, the quark propagators obtain the following form:

$$S_\mathrm{q}^{-1}(p) = \not{p} - im_\mathrm{q} - \not{p}\, \frac{g^2}{8\pi^2} \{A(p^2) - A(-m_\mathrm{p}^2) + 2m_\mathrm{p}^2 A'(-m_\mathrm{p}^2)\} \,, \tag{4.21}$$

where q denotes a particular member of the quark triplet.

The poles and the residues of the propagators determine the quark masses and wave function renormalization constants:

$$M_\mathrm{p} = M_\mathrm{n} = m_\mathrm{p} \{1 + 2m_\mathrm{p}^2 \frac{g^2}{8\pi^2} A'(-m_\mathrm{p}^2)\} \,,$$

$$M_\lambda = m_\lambda \{1 + \frac{g^2}{8\pi^2} [A(-m_\lambda^2) - A(-m_\mathrm{p}^2) + 2m_\mathrm{p}^2 A'(-m_\mathrm{p}^2)]\} \,,$$

$$Z_\mathrm{p}^{-1} = Z_\mathrm{n}^{-1} = 1 \,,$$

$$Z_\lambda^{-1} = 1 - \frac{g^2}{8\pi^2} \{A(-m_\lambda^2) - A(-m_\mathrm{p}^2) + 2m_\mathrm{p}^2 A'(-m_\mathrm{p}^2) - 2m_\lambda^2 A'(-m_\lambda^2)\} \,. \tag{4.22}$$

5. THE CURRENTS

In this section we calculate the renormalized vector and axial vector coupling constants, which are defined in the matrix elements between quark state of the strangeness changing and the strangeness conserving currents. One has for instance

$$\langle \mathrm{p}(\boldsymbol{p}_1) | V_{\mu 1+i2} | \mathrm{n}(\boldsymbol{p}_2) \rangle = Z_\mathrm{p}^{\frac{1}{2}} Z_\mathrm{n}^{\frac{1}{2}} \bar{u}_\mathrm{p}(\boldsymbol{p}_1) (F_{1+i2}^\mathrm{V}(q^2) \gamma_\mu + G_{1+i2}^\mathrm{V}(q^2) \sigma_{\mu\nu} q_\nu + H_{1+i2}^\mathrm{V}(q^2) q_\mu) u_\mathrm{n}(\boldsymbol{p}_2) \,,$$

and similarly for the other matrix elements.

$q = p_2 - p_1$, where p_2 and p_1 denote the momenta of the in- and outgoing quarks respectively.

The coupling constants g are defined by

$$g^{V}_{1+i2} = Z_p^{\frac{1}{2}} Z_n^{\frac{1}{2}} F^{V}_{1+i2}(0) ,$$
$$g^{V}_{4+i5} = Z_p^{\frac{1}{2}} Z_\lambda^{\frac{1}{2}} F^{V}_{4+i5}(0) ,$$
$$g^{A}_{1+i2} = Z_p^{\frac{1}{2}} Z_n^{\frac{1}{2}} F^{A}_{1+i2}(0) ,$$
$$g^{A}_{4+i5} = Z_p^{\frac{1}{2}} Z_\lambda^{\frac{1}{2}} F^{A}_{4+i5}(0) , \tag{5.1}$$

Z_p, Z_n and Z_λ are the wave function renormalization constants calculated in the previous section.

The diagrams which represent the contributions to the matrix elements of the currents are shown in fig. 6. The diagrams of the third type vanish,

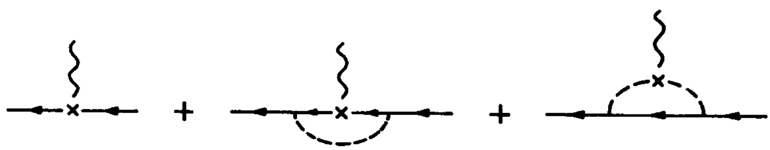


Fig. 6. Diagrams of the current matrix elements.

because the contributions of the various mesons cancel. Diagrams other then shown here will give a contribution only to the scalar and pseudoscalar form factors. For the vector currents $V_{\mu\,1+i2}$ and $V_{\mu\,4+i5}$ the diagrams give the following expression:

$$Z_F \{i\gamma_\mu - i[2\Lambda_{\mu p}(p_1|p_2) + \Lambda_{\mu\lambda}(p_1|p_2)] + i\gamma_5[2\Lambda_{\mu p}(p_1|p_2) + \Lambda_{\mu\lambda}(p_1|p_2)]\gamma_5\} , \tag{5.2}$$

where $\Lambda_{\mu a}(p_1|p_2)$ is calculated in appendix A.

For the axial currents the contribution has the following form:

$$Z_F \{i\gamma_\mu\gamma_5 + i\gamma_5[2\Lambda_{\mu p}(p_1|p_2) + \Lambda_{\mu\lambda}(p_1|p_2)] - i[2\Lambda_{\mu p}(p_1|p_2) + \Lambda_{\mu\lambda}(p_1|p_2)]\gamma_5\} . \tag{5.3}$$

Note that up to order g^2, p_2 gives the only difference between the $(1+i2)$th and the $(4+i5)$th component of the vector and axial vector current. p_2 is the momentum of a neutron quark in the first and of a lambda quark in the second case. When the quarks are on mass-shall this gives an SU(3)-breaking effect. Using eqs. (4.19) and (4.22), a straightforward calculation yields

$$g^{V}_{1+i2} = 1 ,$$

$$g^{V}_{4+i5} = 1 - \frac{g^2}{8\pi^2} \left\{ m_p^2 A'(-m_p^2) + m_\lambda^2 A'(-m_\lambda^2) - \tfrac{1}{2}(m_p + m_\lambda)^2 \frac{A(-m_p^2) - A(-m_\lambda^2)}{m_\lambda^2 - m_p^2} \right\} ,$$

$$g^{A}_{1+i2} = 1 - 2 \frac{g^2}{8\pi^2} m_p^2 A'(-m_p^2) ,$$

$$g^{A}_{4+i5} = 1 - \frac{g^2}{8\pi^2} \left\{ m_p^2 A'(-m_p^2) + m_\lambda^2 A'(-m_\lambda^2) - \tfrac{1}{2}(m_p - m_\lambda)^2 \frac{A(-m_p^2) - A(-m_\lambda^2)}{m_\lambda^2 - m_p^2} \right\} . \tag{5.4}$$

6. SOME RESULTS OF BROKEN SU(3) SYMMETRY

Given the result of our calculations we can verify some theorems concerning SU(3) breaking. We define the SU(3) symmetry breaking parameter ξ by

$$\xi = \frac{F_8}{F_0} . \tag{6.1}$$

6.1. *The Gell-Mann-Okubo mass formula*

From formulae (5.8) and (5.11) one can easily evaluate the pseudoscalar and scalar masses in order g^2. An explicit calculation gives the result.

$$3\overline{m}_\eta^2 + m_\pi^2 - 4m_K^2 = O(\xi^2) , \tag{6.2}$$

for the pseudoscalar as well as for the scalar meson masses. This result is just the Gell-Mann-Okuba mass formula [1, 11]. $\overline{m}_\eta^2$ denotes the pole of the η propagator, which is not equal to the physical η mass because of the singlet-octet mixing. The mixing angles of the scalar and pseudoscalar mesons are given by $\mathrm{tg}\, \varphi = \sqrt{2}$.*

6.2. *The Ademollo-Gatto theorem*

Expanding the coupling constant of the strangeness changing vector current (5.4) in powers of ξ, one obtains

$$g^{V}_{4+i5} = 1 + O(\xi^2) , \tag{6.3}$$

which is the Ademollo-Gatto theorem [8]. This result is not affected by a spontaneous breakdown of the symmetry, which is in contradiction with a recent assertion by Korthals Altes [12]. His result is not correct for the following reason. As shown by Fubini and Furlan [13], the Ademollo-Gatto

* Because the interactions $W_2(P, \Sigma)$ and $W_3(P, \Sigma)$ (eq. (2.2)) were not taken into account, the mixing angles are not proportional to ξ.

theorem can be derived by saturating the equal-time commutation relation (between proton quark states):

$$\langle \mathrm{p}(\boldsymbol{p}) | [F_{4+i5}, F_{4-i5}] | \mathrm{p}(\boldsymbol{p}) \rangle = \langle \mathrm{p}(\boldsymbol{p}) | F_3 + \sqrt{3} F_8 | \mathrm{p}(\boldsymbol{p}) \rangle . \tag{6.4}$$

However, this proof breaks down when the symmetry is spontaneously broken. In that case the commutator is saturated by states with an energy equal to that of the proton quark. Hence the lambda intermediate state gives no contribution, which makes it impossible to derive in this way a theorem concerning the weak vector coupling constant. However, the original proof [8] still holds, which agrees with our result (6.3). Moreover, the theorem may be derived from Ward-Takahashi identities [14], independently of how the symmetry breaking arises. This is shown in appendix C.

6.3. SU(3) *breaking of the axial vector coupling constants*

The axial vector coupling constants, given in the formulae (5.4), are obviously renormalized in first order ξ. Evaluating the difference $g^{\mathrm{A}}_{4+i5} - g^{\mathrm{A}}_{1+i2}$, we obtain the following expression:

$$g^{\mathrm{A}}_{4+i5} - g^{\mathrm{A}}_{1+i2} = \frac{g^2}{8\pi^2} [m_{\mathrm{p}}^2 A'(-m_{\mathrm{p}}^2) - m_{\lambda}^2 A'(-m_{\lambda}^2)] + \mathrm{O}(\xi^2) , \tag{6.5}$$

which is also of first order in ξ.

This contradicts the results of Matsuda and Oneda [3], who prove, under the assumption of asymptotic SU(3) symmetry, that $g^{\mathrm{A}}_{4+i5} - g^{\mathrm{A}}_{1+i2}$ should be of second order in ξ. Asymptotic symmetry means that the charge operator F_{6+i7} behaves in the infinite momentum limit as an exact SU(3) generator, even in a broken world. In this limit the hypothesis enables then to truncate the sum over a complete set of intermediate states, when saturating an equal-time commutator which contains F_{6+i7}. However, this assumption obviously goes wrong in the case of the disconnected diagrams shown in fig. 7. Due to these diagrams the matrix element

$$\langle \alpha(\boldsymbol{p}) \beta(0) | F_{6+i7} | \alpha(\boldsymbol{p}) \rangle$$

does not vanish in the infinite-momentum limit. Therefore several results based on the asymptotic symmetry hypothesis have to be modified.

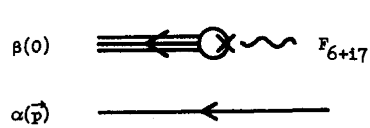


Fig. 7. Diagrams that disturb the asymptotic symmetry hypothesis.

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APPENDIX A

The expressions, represented by the diagrams shown in fig. 1, have the following form:

$$\mathcal{T}_{\mathrm{a}} = \frac{g}{(2\pi)^4\,\mu^2}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - im_{\mathrm{a}}}\Big)$$

$$+\sum_{j=1}\frac{\epsilon_j g_j}{(2\pi)^4\,\mu^2}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - iM_{1j\mathrm{a}}}-\frac{1}{\not k - iM_{2j\mathrm{a}}}\Big)\,, \tag{A.1}$$

$$\Gamma^5_{\mu\,\mathrm{ab}}(p) = \frac{ig}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - im_{\mathrm{b}}}\,i\gamma_\mu\gamma_5\,\frac{1}{\not k+\not p - im_{\mathrm{a}}}\,i\gamma_5\Big)$$

$$+i\sum_{j=1}\frac{\epsilon_j g_j}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - iM_{2j\mathrm{b}}}\gamma_\mu\,\frac{1}{\not k+\not p - iM_{1j\mathrm{a}}}\Big)$$

$$+i\sum_{j=1}\frac{\epsilon_j g_j}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - iM_{1j\mathrm{b}}}\gamma_\mu\,\frac{1}{\not k+\not p - iM_{2j\mathrm{a}}}\Big)\,, \tag{A.2}$$

$$\Pi^5_{\mathrm{ab}}(p^2) = -\frac{g^2}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - im_{\mathrm{b}}}\,i\gamma_5\,\frac{1}{\not k+\not p - im_{\mathrm{a}}}\,i\gamma_5\Big)$$

$$-\sum_{j=1}\frac{\epsilon_j g_j^2}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - iM_{2j\mathrm{b}}}\,\frac{1}{\not k+\not p - iM_{1j\mathrm{a}}}\Big)$$

$$-\sum_{j=1}\frac{\epsilon_j g_j^2}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - iM_{1j\mathrm{b}}}\,\frac{1}{\not k+\not p - iM_{2j\mathrm{a}}}\Big)\,, \tag{A.3}$$

$$\Pi_{\mathrm{ab}}(p^2) = -\frac{g^2}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - im_{\mathrm{b}}}\,\frac{1}{\not k+\not p - im_{\mathrm{a}}}\Big)$$

$$-\sum_{j=1}\frac{\epsilon_j g_j^2}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - iM_{1j\mathrm{b}}}\,\frac{1}{\not k+\not p - iM_{1j\mathrm{a}}}\Big)$$

$$-\sum_{j=1}\frac{\epsilon_j g_j^2}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{Tr}\Big(\frac{1}{\not k - iM_{2j\mathrm{b}}}\,\frac{1}{\not k+\not p - iM_{2j\mathrm{a}}}\Big)\,, \tag{A.4}$$

$$\Sigma_{\mathrm{a}}(p) = -i\,\frac{g^2}{(2\pi)^4}\int \mathrm{d}^4k\,\frac{1}{\not p-\not k-im_{\mathrm{a}}}\,\frac{1}{k^2+\mu^2}$$

$$-i\sum_{k=1}\frac{\eta_k\tilde{g}_k^2}{(2\pi)^4}\int \mathrm{d}^4k\,\frac{1}{\not p-\not k-im_{\mathrm{a}}}\,\frac{1}{k^2+\mu_k^2}\,, \tag{A.5}$$

$$\Lambda_{\mu\mathrm{a}}(p_1|p_2) = -i\,\frac{g^2}{(2\pi)^4}\int \mathrm{d}^4k\,\frac{(p_1+p_2-2k)_\mu}{\not k-im_{\mathrm{a}}}\,\frac{1}{(p_1-k)^2+\mu^2}\,\frac{1}{(p_2-k)^2+\mu^2}$$

$$-i\sum_{k=1}\frac{\eta_k\tilde{g}_k^2}{(2\pi)^4}\int \mathrm{d}^4k\,\frac{(p_1+p_2-2k)_\mu}{\not k-im_{\mathrm{a}}}\,\frac{1}{(p_1-k)^2+\mu_k^2}\,\frac{1}{(p_2-k)^2+\mu_k^2}\,, \tag{A.6}$$

where due to the tadpole attachements to the internal fermion lines (also to the fermion regulators)

$$m_{\mathrm{a}} = gF_{\mathrm{a}}\,,$$

$$M_{1j\mathrm{a}} = m_j+g_jF_{\mathrm{a}}\,,$$

$$M_{2j\mathrm{a}} = m_j-g_jF_{\mathrm{a}}\,. \tag{A.7}$$

a and b denote the members of the fermion triplets. ϵ_j and η_k take the values ± 1, as described in sect. 3. To obtain the formulae (A.1) - (A.6), we used the explicit structure of the currents and the Lagrangian which contain the regulator fields.

To assure the expressions to be finite we impose the following conditions:

$$g^2+2\sum_{j=1}\epsilon_j g_j^2 = 0\,,$$

$$\sum_{j=1}\epsilon_j g_j^2 m_j^2 = 0\,,$$

$$g^4+2\sum_{j=1}\epsilon_j g_j^4 = 0\,,$$

$$g^2+\sum_{k=1}\eta_k\tilde{g}_k^2 = 0\,. \tag{A.8}$$

Because the regulator fields are unphysical we take their masses very large. In that case four divergences arise:

$$D_1 = \sum_{j=1}\frac{\epsilon_j g_j^4}{g^4}\log\frac{m_j^2}{\mu^2}\,,\qquad D_2 = \sum_{j=1}\frac{\epsilon_j g_j^2 m_j^2}{g^2}\log\frac{m_j^2}{\mu^2}\,, \tag{A.9a}$$

$$D_3 = \sum_{j=1} \frac{\epsilon_j g_j^2}{g^2} \log \frac{m_j^2}{\mu^2}\,, \qquad D_4 = \sum_{k=1} \frac{\eta_k \tilde{g}_k^2}{g^2} \log \frac{\mu_k^2}{\mu^2}\,. \tag{A.9b}$$

Computation of expressions (A.1) - (A.6) yields the result:

$$\mathcal{F}_\mathrm{a} = -\frac{g^2}{4\pi^2\mu^2} F_\mathrm{a}\left\{6D_2 + 2D_1 m_\mathrm{a}^2 + m_\mathrm{a}^2\left(\log\frac{m_\mathrm{a}^2}{\mu^2} - \tfrac{11}{3}\right)\right\}\,, \tag{A.10}$$

$$\Gamma^5_{\mu\mathrm{ab}}(p) = ip_\mu \frac{g^2}{4\pi^2}\left\{(F_\mathrm{a}+F_\mathrm{b})(D_3 - \tfrac{1}{3}) + \int_0^1 \mathrm{d}x[F_\mathrm{a}x + F_\mathrm{b}(1-x)]\right.$$
$$\left.\times \log\frac{p^2 x(1-x) + m_\mathrm{a}^2 x + m_\mathrm{b}^2(1-x)}{\mu^2}\right\}\,, \tag{A.11}$$

$$\Pi^5_\mathrm{ab}(p^2) = -i\frac{g^2}{4\pi^2}\left\{p^2(D_3 - \tfrac{1}{3}) + 6D_2 + 2D_1(m_\mathrm{a}^2 + m_\mathrm{b}^2 - m_\mathrm{a} m_\mathrm{b})\right.$$
$$- \tfrac{1}{6}(19m_\mathrm{a}^2 + 19m_\mathrm{b}^2 - 16m_\mathrm{a} m_\mathrm{b}) + \tfrac{1}{2} m_\mathrm{a}^2 \log\frac{m_\mathrm{a}^2}{\mu^2} + \tfrac{1}{2} m_\mathrm{b}^2 \log\frac{m_\mathrm{b}^2}{\mu^2}$$
$$\left. + \tfrac{1}{2}[p^2 + (m_\mathrm{a} - m_\mathrm{b})^2]\int_0^1 \mathrm{d}x \log\frac{p^2 x(1-x) + m_\mathrm{a}^2 x + m_\mathrm{b}^2(1-x)}{\mu^2}\right\}\,, \tag{A.12}$$

$$\Pi_\mathrm{ab}(p^2) = -i\frac{g^2}{4\pi^2}\left\{p^2(D_3 - \tfrac{1}{3}) + 6D_2 + 2D_1(m_\mathrm{a}^2 + m_\mathrm{b}^2 + m_\mathrm{a} m_\mathrm{b})\right.$$
$$- \tfrac{1}{6}(19m_\mathrm{a}^2 + 19m_\mathrm{b}^2 + 16m_\mathrm{a} m_\mathrm{b}) + \tfrac{1}{2} m_\mathrm{a}^2 \log\frac{m_\mathrm{a}^2}{\mu^2} + \tfrac{1}{2} m_\mathrm{b}^2 \log\frac{m_\mathrm{b}^2}{\mu^2}$$
$$\left. + \tfrac{1}{2}[p^2 + (m_\mathrm{a} + m_\mathrm{b})^2]\int_0^1 \mathrm{d}x \log\frac{p^2 x(1-x) + m_\mathrm{a}^2 x + m_\mathrm{b}^2(1-x)}{\mu^2}\right\}\,, \tag{A.13}$$

$$\Sigma_\mathrm{a}(p) = -\frac{1}{4}\frac{g^2}{4\pi^2}\left\{\tfrac{1}{2}\not{p}(D_4 + \tfrac{1}{2}) + im_\mathrm{a}(D_4 + 1)\right.$$
$$\left. + \int_0^1 \mathrm{d}x(\not{p}x + im_\mathrm{a}) \log\frac{p^2 x(1-x) + \mu^2 x + m_\mathrm{a}^2(1-x)}{\mu^2}\right\}\,, \tag{A.14}$$

$$\Lambda_{\mu a}(p_1|p_2) = \frac{1}{4}\frac{g^2}{4\pi^2}\Big\{\tfrac{1}{2}\gamma_\mu(D_4+\tfrac{1}{2}) + \int_0^1 dx \int_0^x dy \Big[\gamma_\mu \log\frac{\chi_a(p_1^2, p_2^2, q^2|x,y)}{\mu^2} + (2p_2(1-x)-q(1-2y))_\mu \frac{\not p_2 x - \not q y + im_a}{\chi_a(p_1^2, p_2^2, q^2|x,y)}\Big]\Big\}\,, \tag{A.15}$$

where $q = p_2 - p_1$, and $\chi_a(p_1^2, p_2^2, q^2|x,y)$ is defined by

$$\chi_a(p_1^2, p_2^2, q^2|x,y) = p_1^2 y(1-x) + p_2^2(x-y)(1-x) + q^2 y(x-y) + \mu^2 x + m_a^2(1-x)\,. \tag{A.16}$$

Two important relations hold for the expressions (A.10)-(A.15). They are essentially a consequence of the Ward-Takahashi identities (2.9) and (2.12).

$$p_\mu \Gamma^5_{\mu ab}(p^2) = i\mu^2[\mathcal{F}_a + \mathcal{F}_b] - [F_a + F_b]\Pi^5_{ab}(p^2)\,, \tag{A.17}$$

$$(p_2 - p_1)_\mu \Lambda_{\mu a}(p_1|p_2) = \Sigma_a(p_1) - \Sigma_a(p_2)\,. \tag{A.18}$$

APPENDIX B

The expressions for the meson propagators, represented by the diagrams shown in fig. 3, have the following form:

$$iD_\pi^{-1}(s) = s + \mu^2 - i\Pi^5_{pp}(s) + \tfrac{1}{3}g^4\delta\Lambda[4F_0^2 + 2F_8^2 + 4\sqrt{2}F_0F_8] + Z_B\delta\mu^2 + (Z_B - 1)(s+\mu^2)\,, \tag{B.1}$$

$$iD_K^{-1}(s) = s + \mu^2 - i\Pi^5_{p\lambda}(s) + \tfrac{1}{3}g^4\delta\Lambda[4F_0^2 + 14F_8^2 - 2\sqrt{2}F_0F_8] + Z_B\delta\mu^2 + (Z_B - 1)(s+\mu^2)\,, \tag{B.2}$$

$$iD_{08}^{-1}(s) = s + \mu^2 - \tfrac{1}{2}i[\Pi^5_{pp}(s) + \Pi^5_{\lambda\lambda}(s)] + \tfrac{1}{3}g^4\delta\Lambda[4F_0^2 + 5F_8^2 - 2\sqrt{2}F_0F_8] + Z_B\delta\mu^2 + (Z_B-1)(s+\mu^2) + \{-\tfrac{1}{6}i[\Pi^5_{pp}(s) - \Pi^5_{\lambda\lambda}(s)] + \tfrac{1}{3}g^4\delta\Lambda[-F_8^2 + 2\sqrt{2}F_0F_8]\}\begin{pmatrix} 1 & 2\sqrt{2} \\ 2\sqrt{2} & -1 \end{pmatrix}\,, \tag{B.3}$$

$$i\tilde{D}_{\pi}^{-1}(s) = s+\mu^2 - i\Pi_{pp}(s) + g^4\,\delta\Lambda[4F_0^2+2F_8^2+4\sqrt{2}F_0F_8]$$
$$+Z_B\,\delta\mu^2+(Z_B-1)(s+\mu^2)\,, \tag{B.4}$$

$$i\tilde{D}_{K}^{-1}(s) = s+\mu^2 - i\Pi_{p\lambda}(s) + g^4\,\delta\Lambda[4F_0^2+2F_8^2-2\sqrt{2}F_0F_8]$$
$$+Z_B\,\delta\mu^2+(Z_B-1)(s+\mu^2)\,, \tag{B.5}$$

$$i\tilde{D}_{08}^{-1}(s) = s+\mu^2 - \tfrac{1}{2}i[\Pi_{pp}(s)+\Pi_{\lambda\lambda}(s)] + g^4\delta\Lambda[4F_0^2+5F_8^2-2\sqrt{2}F_0F_8]$$
$$+Z_B\delta\mu^2+(Z_B-1)(s+\mu^2)+\{-\tfrac{1}{6}i[\Pi_{pp}(s)-\Pi_{\lambda\lambda}(s)]$$
$$+g^4\delta\Lambda[-F_8^2+2\sqrt{2}F_0F_8]\}\begin{pmatrix}1 & 2\sqrt{2}\\ 2\sqrt{2} & -1\end{pmatrix}\,. \tag{B.6}$$

The wiggle denotes the propagators of the scalar mesons. Because a $P_0 - P_8$ and a $\Sigma_0 - \Sigma_8$ mixing arises, the propagators corresponding to those particles are represented by two-by-two matrices. These propagators are denoted by $D_{08}(s)$ and $\tilde{D}_{08}(s)$ respectively. The functions $\Pi^5_{ab}(s)$ and $\Pi_{ab}(s)$ are calculated in appendix A.

The consistency relations, pictorially represented in fig. 4, are given by

$$F_0 = \frac{c_0}{\mu^2} - \frac{1}{3\mu^2}\,g^4\delta\Lambda[4F_0^3+12F_0F_8^2-2\sqrt{2}F_8^3] - \frac{1}{\sqrt{3}}\,[2\mathcal{F}_p+\mathcal{F}_\lambda]$$
$$-\frac{1}{\mu^2}\,[Z_B\delta\mu^2+(Z_B-1)\,\mu^2]\,F_0\,, \tag{B.7}$$

$$F_8 = \frac{c_8}{\mu^2} - \frac{1}{\mu^2}\,g^4\,\delta\Lambda[2F_8^3+4F_0^2F_8-2\sqrt{2}F_0F_8^2] - \frac{2}{\sqrt{6}}\,[\mathcal{F}_p-\mathcal{F}_\lambda]$$
$$-\frac{1}{\mu^2}\,[Z_B\,\delta\mu^2+(Z_B-1)\,\mu^2]F_8\,. \tag{B.8}$$

The function $\mathcal{F}_a$ is calculated in appendix A.

For the quark propagators the following expression holds:

$$S_q^{-1}(p) = \not{p} - igF_q + 2\Sigma_p(p)+\Sigma_\lambda(p) - \gamma_5[2\Sigma_p(p)+\Sigma_\lambda(p)]\gamma_5 + (Z_F-1)\not{p}\,, \tag{B.9}$$

where $\Sigma_a(p)$ is calculated in appendix A.

APPENDIX C

As claimed in sect. 6, one can give a derivation of the Ademollo-Gatto theorem based on Ward-Takahashi identities. This derivation remains unchanged in the case of spontaneously broken SU(3) symmetry. We will give the proof for the current matrix element that is related to $K_{\ell 3}$ decay.

The unrenormalized vertex function for the vector current is given by

$$(p_1+p_2)_\mu f_+(s_1, s_2, t)+(p_2-p_1)_\mu f_-(s_1, s_2, t) \ . \tag{C.1}$$

p_1 is the momentum of the outgoing pion, p_2 of the incoming kaon. $t = (p_2 - p_1)^2$, $s_1 = p_1^2$ and $s_2 = p_2^2$.

We separate the contribution of the kappa pole:

$$f_-(s_1, s_2, t) = \tilde{f}_-(s_1, s_2, t) - iD_\kappa(t) f_\kappa(t) G(s_1, s_2, t) \ ;$$

$G(s_1, s_2, t)$ is the unrenormalized $\pi K \kappa$ vertex function, D_κ the unrenormalized propagator of the kappa meson and f_κ characterizes the weak kappa decay.

Due to the Ward-Takahashi identities (2.10) and (2.11), the following relation holds:

$$(s_2 - s_1) f_+(s_1, s_2, t) + t\tilde{f}_-(s_1, s_2, t) + \sqrt{\tfrac{3}{2}} F_8 G(s_1, s_2, t)$$

$$= iD_K^{-1}(s_2) - iD_\pi^{-1}(s_1) \ . \tag{C.2}$$

D_K and D_π are the unrenormalized kaon and pion propagators. Relation (C.2) exhibits a non-singular behaviour for $t = 0$, even in the case of spontaneously broken symmetry.

If $F_8 = 0$, all observable quantities show SU(3) symmetry and the theory is invariant under a transformation proposed by Sirlin [14]:

$$G_5 = C \exp[i\pi F_5] \ . \tag{C.3}$$

C denotes the charge conjugation operator. This invariance implies:

$$G(s_1, s_2, t) = G(s_2, s_1, t) \ ,$$

$$f_+(s_1, s_2, t) = f_+(s_2, s_1, t) \ .$$

In the case of broken symmetry we expect

$$G(s_1, s_2, t) = G(s_2, s_1, t) + O(\xi) \ ,$$

$$f_+(s_1, s_2, t) = f_+(s_2, s_1, t) + O(\xi) \ , \tag{C.4}$$

where ξ is the symmetry breaking parameter as defined by (6.1).

Differentiating relation (C.2) with respect to s_1 or s_2 we obtain the following results for $t = 0$, $s_1 = -m_\pi^2$ and $s_2 = m_K^2$:

$$f_+(-m_\pi^2, -m_K^2, 0) - (m_\pi^2 - m_K^2)\frac{d}{ds_1} f_+(s_1, -m_K^2, 0)\Big|_{s_1=-m_\pi^2}$$

$$-\sqrt{\tfrac{3}{2}}\, F_8 \frac{d}{ds_1} G(s_1, -m_K^2, 0)\Big|_{s_1=-m_\pi^2} = Z_\pi^{-1}\,, \tag{C.5}$$

$$f_+(-m_\pi^2, -m_K^2, 0) + (m_\pi^2 - m_K^2)\frac{d}{ds_2} f_+(-m_\pi^2, s_2, 0)\Big|_{s_2=-m_K^2}$$

$$+\sqrt{\tfrac{3}{2}}\, F_8 \frac{d}{ds_2} G(-m_\pi^2, s_2, 0)\Big|_{s_2=-m_K^2} = Z_K^{-1}\,. \tag{C.6}$$

Multiplying eq. (C.5) with (C.6), and using relation (C.4) and $m_\pi^2 - m_K^2 = 0(\xi)$, we obtain

$$(f_+(-m_\pi^2, -m_K^2, 0))^2 = Z_\pi^{-1} Z_K^{-1} + 0(\xi^2)\,,$$

which is the Ademollo-Gatto theorem.

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