

BOUND STATES OF THREE MAGNONS IN A TWO DIMENSIONAL SIMPLE CUBIC HEISENBERG FERROMAGNET

J.E. Van HIMBERGEN and J.A. TJON

Institute for Theoretical Physics, University of Utrecht, Utrecht, The Netherlands

Received 26 September 1974

The existence of three magnon bound states in a two dimensional simple cubic lattice of spins $\frac{1}{2}$ with nearest neighbor ferromagnetic exchange is shown, using the Faddeev formalism. Their energies as a function of total momentum are given.

In a recent experiment the existence of bound complexes of excitations has been observed in an one-dimensional magnetic system [1]. In view of this it is interesting to study the possibility of their occurrence in higher dimensional systems. For these systems it is more difficult in general to support bound states as has already been pointed out by Dyson [2]. The existence of two-magnon bound states in the two and three dimensional Heisenberg ferromagnet has extensively been treated by Wortis [3] and Hanus [4]. On the other hand nothing is known about the case of more than two magnons with the exception of Bethe's complete solution in one dimension [5]. In this note we present results showing that in two dimensions bound states of three magnons exist for sufficiently large momentum of the complex.

Our starting point is the Dyson Hamiltonian \mathcal{H}_D , a boson representation of the Heisenberg Hamiltonian

$$\mathcal{H}_D = \frac{1}{2} J \sum_{i,\delta} [(a_i^\dagger - a_{i+\delta}^\dagger)(a_i - a_{i+\delta}) + a_i^\dagger a_{i+\delta}^\dagger (a_i - a_{i+\delta})^2] \quad (1)$$

where i extends over all lattice vectors and $i + \delta$ are the nearest neighbors of i .

Application of the Faddeev formalism [6] to the three magnon bound state problem in two dimensions leads in momentum space to a coupled set of integral equations in two continuous variables [7]. It has the form

$$\begin{aligned} \Phi_d(\vec{K}; E; \alpha) = & \frac{1}{2\pi^2} \int_{-\pi}^{+\pi} dK'_x dK'_y \sum_{j=x,y} c_{dj}(K_x, K_y; E) [\cos(P_j - K'_j - \frac{1}{2}K_j) - \cos \frac{1}{2}K_j] \\ & \times [3 - \frac{1}{2}E - \frac{1}{2}(\cos(P_x - K_x) + \cos(P_y - K_y)) - \cos \frac{1}{2}K_x \cos(P_x - K'_x - \frac{1}{2}K_x) - \cos \frac{1}{2}K_y \cos(P_y - K'_y - \frac{1}{2}K_y)]^{-1} \quad (2) \\ & \times \sum_{d'=x,y} \cos(K_{d'} - P_{d'} + \frac{1}{2}K'_{d'}) \Phi_{d'}(K'_x, K'_y; E; \alpha), \end{aligned}$$

where E represents the energy of the three magnon bound state and $d = x, y$ are the two directions along the lattice sites. The functions $c_{lm}(K_x, K_y; E)$ contain the information of the two magnon T -matrix. If we consider a simple cubic lattice and choose the total momentum \vec{P} of three magnons along the (1,1) direction, i.e. $P_x = P_y = P$, eq. (2) simplifies by symmetry to uncoupled integral equations for Φ_x and Φ_y alone. These equations have been studied numerically by discretizing the integrals. Gaussian quadrature has thereby been employed with up to 16 mesh points in each variable. By varying the number of integration points it was estimated that the error was less than 1%. The results are shown in fig. 1. It should be noted that only the edge of the Brillouin Zone is shown in the figure. As is seen from fig. 1 there exist at most three bound states of three magnons, their energies depending on the value of total momentum \vec{P} . The bottom of the band of three free magnons and that of two bound plus one free magnon (dashed and dashed-dotted curves respectively) are also shown. With respect to the possibility of observing these three-magnon states it is expected that in general this might be more difficult than in the

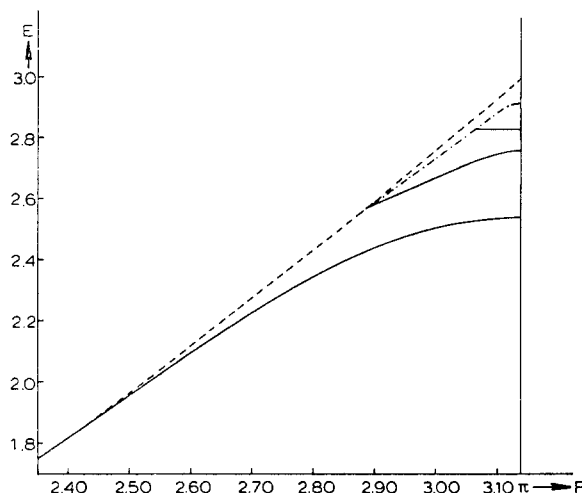


Fig. 1. The energy E of three magnon bound states in the (1,1) direction (drawn lines) as function of the total momentum P . Bottom of the band of three free magnons (dashed line) and of two bound plus one free magnon (dashed-dotted line).

one-dimensional case, since only a small portion of the Brillouin Zone allows for the existence of these states.

As a consequence of the boson character and non-hermiticity of the Dyson Hamiltonian in general there will also be spurious non physical states, which are not present in the original Heisenberg problem [8]. These states have also been determined by studying their existence in a finite lattice as a function of the number of spins and have been found to lie within the continuum. In this way it was verified that the solutions of fig. 1 belong to physical states.

For other directions than the (1,1) direction in the lattice eq. (2) remain in general coupled. Since the matrix size resulting from the kernel is quite large in this case, a different method has been used to solve eq. (2). Including an inhomogeneous term in eq. (2) the Neumann series for the three magnon T -matrix can be determined by numerically iterating these equations. The solution of eq. (2) can in principle be obtained by applying Padé approximants to this series [9]. It was explicitly verified that this method gives identical results in the (1,1) direction. In practice the rate of convergence is reasonable and a [9,9] approximant is always sufficient to find a converging result. Using this method it was shown that also in other directions bound states exist for sufficiently large \vec{P} .

References

- [1] J.B. Torrance, Jr. and M. Tinkham, Phys. Rev. 187 (1969) 595.
- [2] F.J. Dyson, Phys. Rev. 102 (1956) 1217.
- [3] M. Wortis, Phys. Rev. 132 (1963) 85.
- [4] J.G. Hanus, Phys. Rev. Letters 11 (1963) 336.
- [5] H.A. Bethe, Z. Physik 71 (1931) 205.
- [6] L. Faddeev, Sov. Phys. JETP 12 (1961) 1014.
- [7] C.K. Majumdar, Phys. Rev. B1 (1970) 287.
- [8] J.E. van Himbergen and J.A. Tjon, to be published in Physica.
- [9] J.A. Tjon, in Padé approximants and their applications, ed. P.R. Graves-Morris (Academic Press, New York 1973).