

## THE $^{30}\text{Si}(p, \gamma)^{31}\text{P}$ REACTION FOR BOMBARDING ENERGIES BETWEEN 1.00 AND 1.53 MeV

by H. VAN RINSVELT\*) and P. B. SMITH†)

Fysisch Laboratorium van de Universiteit te Utrecht

### Synopsis

Eighteen resonances have been identified as belonging to the  $^{30}\text{Si}(p, \gamma)^{31}\text{P}$  reaction for proton bombarding energies between 1.00 and 1.53 MeV. The gamma-ray yield of the corresponding excited states of  $^{31}\text{P}$  has been measured, and the principle features of the decay have been determined. Double and/or triple angular correlations of the decay radiation were measured at seventeen of the resonances, resulting in unique spin assignments in all cases. Conclusions on parity have been drawn in some cases from the multipole mixing ratios of the capture radiation or from the reduced proton width. The quadrupole-dipole mixing ratio in the decay of the first excited state has been determined as  $0.28 \pm 0.02$ . In an appendix it is proven that the mixing ratio determination in this case is ambiguous if only intensity-direction correlation measurements are made. The unique result given here stems partly from other methods of measurement. A corollary to the theorem mentioned, relating to the first gamma ray in a cascade, is also given.

The spin of the fifth excited state of  $^{31}\text{P}$  is found to be  $7/2$ , and, in disagreement with various other authors, it has been established that the branching ratio of a possible ground-state decay of this level is not more than 1%. The unexpectedly large number of close-lying doublets found in this bombarding energy region is discussed. A resonant absorption measurement at the doublet at  $E_p = 1480.7$  and  $1482.2$  keV demonstrated the predicted array of one central absorption dip, and two satellite dips.

I. *Introduction.* The  $^{30}\text{Si}(p, \gamma)^{31}\text{P}$  reaction between 1.0 and 1.5 MeV has recently been the subject of several investigations. The yield has been measured by Ohmura *e.a.*<sup>1)</sup>, and Barnard *e.a.*<sup>2)</sup>. Spectroscopic work leading to decay schemes and information on resonance levels in this region and lower levels in  $^{31}\text{P}$  has been carried out by Harris and Seagon-dollar<sup>3) 4)</sup>. Polarization measurements on capture gamma rays from this reaction have been made by Tutakin<sup>5) 6)</sup>. Work of a nature very similar to the present has also been reported by Valter *e.a.*<sup>7)</sup>. More recently the  $^{30}\text{Si}(p, p)^{30}\text{Si}$  reaction has also been studied by Valter *e.a.*<sup>8)</sup>.

The results of the investigation are, in general, in good agreement with

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\*) Boursier de l'Institut Interuniversitaire des Sciences Nucléaires de Belgique.

†) Permanent address: Natuurkundig Laboratorium van de Universiteit te Groningen.

those of previous investigations. Five resonances not reported by any of the workers above have been found, however.

II. *Experimental.* A. Beam and Stabilization. The proton beam used in these measurements, ranging in intensity from 1 to 8  $\mu A$ , was provided by the University of Utrecht Van de Graaff accelerator\*). The deflection magnet provides the standard for energy measurement and control in this installation. The magnet and slit systems were designed by C. M. Braams. For 90° deflection, as used in the present investigation, the average radius of curvature of the principle ray is 32.8 cm. The field is homogeneous, and the beam enters and leaves the magnet perpendicular to the field boundary. The energy defining slit jaws, machined from solid silver, are placed symmetrically with respect to the magnet, at 66 cm from the center of the circular pole pieces. This leads to first-order focussing, since the pole-piece radius is 31.5 cm and the effective field boundary is taken as one pole-gap distance (1.5 cm) beyond the edge of the pole pieces. The radius of the beam trajectory should therefore be 33 cm, in agreement with the measured 32.8 cm. The slit openings may be adjusted with an accuracy of 0.05 mm, from completely closed to an opening of 5 mm. No focussing is present in the direction perpendicular to the deflection plane. Slit jaws are nonetheless provided in this direction to help localize the beam and also to prevent the beam from striking the magnet pole faces, or beam tubing.

The only unusual feature of the magnet is a rounding of the pole-face edges so as to avoid a variation of the calibration constant with field. The result of the rounding off is that the magnetic induction shows no maximum in the region of the pole-face edges. This removes the tendency existing in magnets with sharp pole-face edges for saturation to set in earlier in this region than in the uniform field region of the air gap. A measurement of the field strength at a point in the uniform field region will, in this latter case, not be a good measure of the average field strength. It is difficult to obtain an objective measure of the success of this design. The magnet has been used at fields as high as 16 kG in recent studies of ( $\alpha$ ,  $\gamma$ ) reactions<sup>9</sup>). Comparison with the results of Rytz *et al.*<sup>10</sup>) shows that there is probably a change of about 0.3% in the calibration constant of the magnet at 16 kG with respect to its value at 4.5 kG. At the high field 7% more current is required to produce the field than linear extrapolation from low field would predict. The authors know of no measurements on other magnets with which this result may be compared.

The magnet stabilizer, designed by one of the authors (P.B.S.) has been mentioned in an earlier publication<sup>11</sup>). The main magnet power is supplied

\*) Type K, 3 MeV, horizontal pressure tank; mfd. by The High Voltage Engineering Corporation, Burlington, Mass.

by forty 12E1 (Ediswan) power tetrodes, parallel connected. An extra feedback loop to further damp fast fluctuations, not mentioned in ref. 11, has been added to this design. This has resulted in long and short term stability of the order of 1 : 75,000. The accompanying inconvenience of a "follow-through" of magnetic field after a change of setting, with a time constant of about one second, is usually not too inconvenient.

The high voltage of the accelerator is stabilized, in the usual fashion, by feedback from the exit slit jaws of the magnet to a corona control tube loading the high voltage terminal. It is interesting to note that a far higher degree of stabilization is achieved than would be predicted by geometric considerations of beam trajectories in the magnet. Referring to ref. 11, we would expect to find, in the case shown in fig. 2, an energy resolution (full width at half maximum) of 4.2 keV for the input and output slit openings of 0.9 mm used. The observed width is 0.75 keV, equal to the expected energy loss at the energy of the bombarding protons in the target used ( $5 \mu\text{g}/\text{cm}^2$ , elemental  $^{30}\text{Si}$ ). The beam energy spread must therefore not be more than a few hundred electron volts, or about one-tenth of that predicted by the magnet geometry.

The explanation of this small energy spread is obviously that the feedback holds the high voltage accurately enough so that the range of fluctuation is far less than that permitted by the slits. One result of this high stabilization factor is that there is a noticeable tendency among users of the machine to rely on it too much, not taking sufficiently into account that the grid-cathode contact potential at the feedback amplifier input, and the balance setting of the amplifier itself, have an important effect on beam energy. The energy of the beam is determined by setting the frequency of a marginal oscillator proton (or lithium) magnetic resonance detector. Two different types of marginal oscillator have been built in this laboratory, one by P. J. M. Smulders, the other by B. Strasters, both giving excellent service. The frequency is read from a Hewlett Packard Model 524-C electronic counter. The magnet is then set to correspond to the chosen frequency. The energy of the beam is given approximately by  $E = k f^2$ , where  $f$  is the frequency and  $k$  is a constant determined at the  $992.0 \pm 0.5 \text{ keV}^{12)}$  resonance in the  $^{27}\text{Al}(p, \gamma)^{28}\text{Si}$  reaction. The energies thus found are then relativistically corrected by  $\Delta E = -0.53E(E - E_c)$ , where  $E$  is the apparent energy and  $E_c$  the calibration energy, both in MeV, and  $\Delta E$  is in keV.

B. Targets. All targets used in this investigation were of elemental silicon, highly enriched\* in  $^{30}\text{Si}$ . These were furnished by the Atomic Energy Research Establishment, Harwell. Thicknesses of 5, 15, and  $25 \mu\text{g}/\text{cm}^2$  were used in various stages of the work reported here, and some initial work was

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\*) The exact enrichment factor is not known.

done with a target of  $95 \mu\text{g}/\text{cm}^2$ . Very clean targets have been obtained by sending special quality tantalum sheet to the A.E.R.E., with the request that it only be glowed in vacuum before the evaporation onto it of the target material. Other cleaning and/or polishing procedures are unnecessary and have been known to produce layers of aluminum contamination (presumably from the  $\text{Al}_2\text{O}_3$  rouge frequently used in polishing). The special quality tantalum, delivered by Drijfhout and Co., Amsterdam, differs from ordinary tantalum in that sodium hydroxide is not used in the production process. Quite strong sodium contamination had previously been found in ordinary tantalum sheet.

C. Experimental procedures. 1. Yield curves. All yield measurements were carried out with a 10 cm diameter by 10 cm long NaI (Tl) crystal and Dumont 6363 photomultiplier tube\*). The crystal was placed so that its axis formed an angle of  $55^\circ$  with the direction of the beam. The front face of the crystal was placed at approximately 2 cm from the target spot. The voltage supplied to the multiplier was kept low (700 volts, total) in order to avoid calibration shift from fatigue caused by high multiplier tube currents at strong resonances. In the Van de Graaff installation at this laboratory the signal from the multipliers is driven through about 25 meters of terminated  $135 \Omega$  cable from the target room to the control room by White cathode follower circuits. The signal level in the cables is in the 50 to 500 mV range. In the control room it is amplified up to 5 to 50 volts by 0.1  $\mu\text{sec}$  rise-time amplifiers. The design is basically that of a Los Alamos Model 220 amplifier which has been modified to give a better rise time and to which attenuators have been added.

Most yield curves were taken with three differential discriminators and scalers so that the yield could be simultaneously measured in adjacent gamma-ray energy channels from 1.5 to 10.5 MeV. This has repeatedly turned out to be an excellent manner of detecting the presence of doublets. Any slight difference in decay pattern between the two components will give a shift between the different channels in the energy at which maximum yield is reached, even if the spacing is less than the energy resolution. The discriminators, designed by one of the present authors (P.B.S.) represent a compromise between a simple biased trigger pair, and a more complex window amplifier and trigger system.

The level strengths,  $(2J + 1)\gamma$ , were determined by summing over each resonance the pulses above background counted for  $E_\gamma > 4.5 \text{ MeV}$ , and comparing this (correcting appropriately for the energy difference) with the accurately known yield of the  $E_p = 622 \text{ keV}$  resonance in this same reaction. This yield has been determined by resonant absorption, and is

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\*) Matched Window Assembly, furnished by the Harshaw Chemical Company.

known independently of instrumental factors<sup>14</sup>). The actual strength of a level differs somewhat from that calculated as described above because of differences in efficiency of detection stemming from different decay schemes. The choice of  $E_\gamma > 4.5$  MeV is a good one however, since not more than one gamma ray in each cascade can be counted, and these are usually well above 4.5 MeV in energy. Since the efficiency for detection varies only slowly with energy, and because the energy of most primaries lies above 7 MeV, no further refinement of the calculated yield was felt to be necessary.

2. Gamma-ray spectra. Spectra of the decay radiation, detected in the same crystal as described above (resolution 9% for the  $^{137}\text{Cs}$  line) were analysed by the familiar "peeling" procedure, after correction for background (usually measured immediately below the resonance under study). Very few lines showed an appreciable  $P_4(\cos \theta)$  variation, so that it may be assumed that the decay patterns obtained by measurements at  $55^\circ$  are quite accurate. In a few cases coincidence spectra have been taken, but this was found not to be necessary in order to get a picture of at least the major features of each decay.

III. *Angular distributions and triple angular correlations.* The experimental set-up for the angular distribution and correlation work done was the same as that described in a recent publication of this laboratory<sup>13</sup>). The case of channel spin  $S = 1/2$ , as found here, makes the analysis of the data much easier than in the usual case where the bombarded nucleus has non-zero spin. Special computer programs, adapted to this case, were used. The final result of these programs is to give the values of the mixing parameters which, for a given trial set of level spins in the cascade being treated, best fit the data, as determined by the chi-square criterium.

The method of solution is trial and error. This is practical and fast in this case since there are only two unknowns to be fitted; i.e., the mixing ratios of the gamma rays in the cascade measured. The quantity chi-square is given by:

$$\chi^2 = \frac{1}{M - R} \sum_{ij} (a_i - a_i^*) (a_j - a_j^*) X_{ij}^{-1}. \quad (1)$$

In this expression  $M$  is the number of measured quantities, thus one value of  $a_2$ , and one value of  $a_4$  from the expression  $W(\theta) = 1 + a_2 P_2 + a_4 P_4$  used to fit the data, for each angular distribution or geometry. The number of degrees of freedom is  $(M - R)$ , where  $R$  is in general equal to two, but may be either one or zero if the assumption is made that either or both of the gamma rays in the cascade is unmixed. The  $a_i$  are the measured coefficients and  $X_{ij}^{-1}$  is the inverted error matrix related to these quantities. The  $a_i^*$  are the theoretical values of the coefficients for an assumed spin

combination and given values of the mixing parameters. The necessary formulas may be found in Ferguson and Rutledge<sup>15)</sup> or Smith<sup>16)</sup>, and the coefficients needed may be taken directly from the tables included in either of these references. The coefficients calculated from the data of the angular distributions have been corrected for the attenuation due to the finite size of the detectors; this is impossible for the triple correlations in which case the correction is applied to the theoretical coefficients. This inconsistency in the handling of the data is partly the result of tradition, but it is also based on a very definite advantage. For an angular distribution an uncorrected theoretical curve can be used to get a rough idea of whether or not a given spin combination is possible or not (the geometric attenuation factor is readily applied), but this is not practical for angular correlations since it is impossible to know exactly how the coefficients vary with the geometric correction, and one must calculate theoretical curves in which the geometric attenuation factors for a given set-up are already worked into the result. The quantities  $a_i$  are not all independent; in a given measurement  $a_2$  and  $a_4$  have correlated errors. For this reason it is essential to use eq. (1) in place of the simpler expression which would apply if all the input quantities were independent.

In the first step of the analysis a program written by A. Heyligers and one of the present authors (H. V.R.) quickly searches out, for all possible spin combinations, the regions in the mixing parameter plane where an acceptable solution may possibly exist. This is done by letting both mixing parameters vary in coarse steps, over the range  $-\infty$  to  $+\infty$ . In the final step a program written by P. Vuister, in which the minimum of chi-square is assumed to be the lowest point of a paraboloid, finds the best value of both mixing parameters and the probable errors in these values.

IV. *Experimental Results.* A. Yield and spectra. In fig. 1 a yield curve is shown of the region of proton bombarding energy covered in this

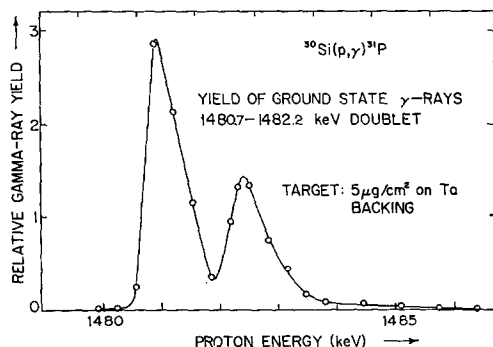


Fig. 2. The 1480.7 - 1482.2 keV doublet. The same as in fig. 1 except that the target thickness is  $5 \mu\text{g}/\text{cm}^2$  and only the groundstate transition gamma rays were detected.

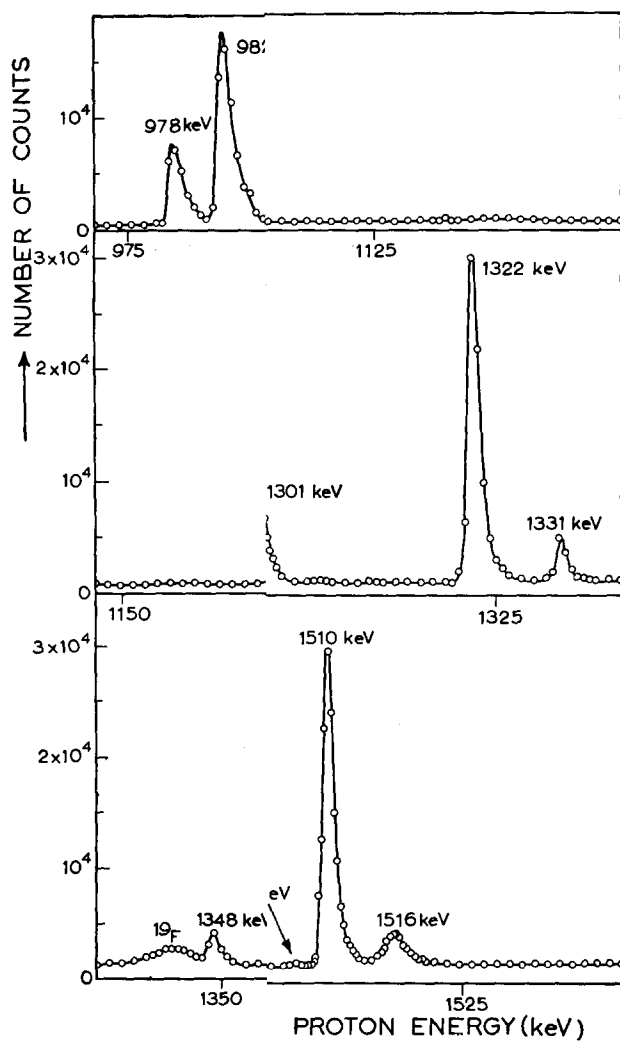


Fig. 1e of the crystal  
is  $2.5 \text{ MeV}$  and  $10.5$

investigation. Each point corresponds to 175  $\mu\text{C}$  charge deposited on the target. The target thickness for this curve was 15  $\mu\text{g}/\text{cm}^2$ . Shown is the sum of the counts registered in all three gamma-ray energy channels (see section IIC1). For comparison the resonances at  $E_p = 978$  and 982 keV are shown, also. These have been studied by Broude, Green, and Willmott<sup>17)</sup> The doublet at  $E_p = 1481$  keV, just visible in fig. 1, is shown to better advantage in fig. 2. Here only the ground-state transition gamma rays were detected, and the target thickness was 5  $\mu\text{g}/\text{cm}^2$ .

In table I can be found, for each of the resonances in this energy region,

TABLE I

The resonances found in this investigation. In the first column the resonance energies are listed, together with the error in the proton energy determination. The level strengths (estimated error, 30%), $(2J + 1)\gamma$ , where $\gamma = \Gamma_\gamma \Gamma_p / \Gamma_t$ , are reported in the third column. From the fourth to the eighth column can be found information on the decay of each resonance. The entries in these columns are branching ratios in percent. The subscripts give the excited state to which the primary transition leads, or, above the third excited state, the energy of the state in MeV. In the last column are given references to other publications wherein the same levels have been reported. In cases where it was not clear which of the members of a doublet had been found by another worker, it was assumed that the resonance found was either that of lowest energy or the strongest of the two.								
$E_p$ (keV)	$E_{ex}$ (MeV)	$(2J + 1)\gamma$ (eV)	Principal decay modes (in percent)					Also re- ported by:
			$\gamma_0$	$\gamma_1$	$\gamma_2$	$\gamma_3$	Others	
1094.7 $\pm$ 0.6	8.346	0.17		65	5		$\gamma_{4.59} = 9$ ; $\gamma_{5.01} = 15$ ; others = 6	1) <sup>7)</sup>
1175.6 $\pm$ 0.7	8.424	0.29			48		$\gamma_{3.29} = 35$ ; $\gamma_{4.19} = 5$ ; others = 12	1) <sup>2)3)7)</sup>
1203.6 $\pm$ 0.7	8.451	1.1		5	27		$\gamma_{3.29} = 33$ ; $\gamma_{3.51} = 10$ ; $\gamma_{4.26} = 25$	1) <sup>2)3)7)</sup>
1213.3 $\pm$ 0.7	8.460	0.08		38	7		$\gamma_{3.29} = 24$ ; $\gamma_{5.01} = 31$	
1289.0 $\pm$ 0.8	8.533	0.23		17		71	$\gamma_{3.51} = 12$ ;	1) <sup>7)</sup>
1297.9 $\pm$ 0.8	8.542	0.9		90			$\gamma_{5.01} = 5$ ; others = 5	1) <sup>8)</sup>
1301.2 $\pm$ 0.8	8.546	0.7	60	7	20	7	$\gamma_{5.01} = 6$	1) <sup>2)7)</sup>
1322.1 $\pm$ 0.8	8.565	1.6	5	32	21		$\gamma_{3.51} = 30$ ; $\gamma_{4.26} = 12$	1) <sup>2)3)7)8)</sup>
1330.9 $\pm$ 0.8	8.574	0.22	10	40	9	5	$\gamma_{3.51} = 26$ ; others = 10	
1348.3 $\pm$ 0.8	8.591	0.08					$\gamma_{3.41} \cong 100$	
1389.9 $\pm$ 0.8	8.631	2.5	4	60	18		$\gamma_{3.29} = 12$ ; $\gamma_{3.51} = 6$	1) <sup>2)3)6)7)</sup>
1398.4 $\pm$ 0.8	8.640	3.1	2	75	4	10	$\gamma_{4.43} = 4$ ; $\gamma_{5.01} = 5$	1) <sup>2)3)7)</sup>
1480.7 $\pm$ 0.9	8.719	3.4	89	5	6			2) <sup>3)7)</sup>
1482.2 $\pm$ 0.9	8.721	2.3	81	9	6		$\gamma_{5.01} = 4$	
1489.9 $\pm$ 0.9	8.728	1.4	5	10	16		$\gamma_{3.29} = 46$ ; $\gamma_{5.01} = 23$	2) <sup>3)7)</sup>
1506.7 $\pm$ 0.9	8.744	0.05						
1509.9 $\pm$ 0.9	8.747	2.5		64	7		$\gamma_{3.41} = 29$	2) <sup>3)7)</sup>
1516.1 $\pm$ 0.9	8.754	0.6	91	3	3	3		8)



TABLE II

Experimental results of the angular distribution and correlation measurements. The resonance energies of table I are rounded off here to the nearest kilovolt, except for the doublet at  $E_p = 1481$  keV. In the table are given  $a_2$  and  $a_4$  of the expansion,  $W(\theta) = 1 + a_2 P_2(\cos \theta) + a_4 P_4(\cos \theta)$  in terms of which the original data were analysed. The spin and, in a few cases, the parity, of the resonance levels as determined by the analysis of the data (see section III) are shown, as well as the mixing ratio of the first gamma ray.

$E_p$ (keV)	Decay studied	Angular distribution (of first gamma ray)				Angular correlations; $\varphi = 180^\circ$				Resonance Spin	Mixing parameter, $\kappa_1$ (of first gamma ray)
		$a_2$		$a_4$		Geometry I; $\theta_2 = 90^\circ$		Geometry II, $\theta_1 = 90^\circ$			
						$a_2$	$a_4$	$a_2$	$a_4$		
1095	$r \rightarrow (1) \rightarrow (0)$	$-0.39 \pm 0.01$	$0.02 \pm 0.01$	$-0.47 \pm 0.04$	$0.03 \pm 0.04$	$-0.74 \pm 0.02$			5/2	$-0.010 \pm 0.004$	
1176	$r \rightarrow (2) \rightarrow (0)$	$-0.33 \pm 0.01$	$0.00 \pm 0.01$	$0.02 \pm 0.15$	$0.02 \pm 0.16$	$0.46 \pm 0.08$	$0.34 \pm 0.08$		7/2 5/2(+)	$-0.021 \pm 0.003$ $-0.100 \pm 0.005$	
1204	$r \rightarrow (1)$ $r \rightarrow (2) \rightarrow (0)$	$-0.20 \pm 0.01$ $0.56 \pm 0.01$	$-0.02 \pm 0.02$ $-0.03 \pm 0.01$	$0.24 \pm 0.05$	$0.03 \pm 0.06$	$-0.02 \pm 0.06$	$0.18 \pm 0.06$			$-0.100 \pm 0.011$	
1213	$r \rightarrow (1) \rightarrow (0)$	$-0.55 \pm 0.08$	$-0.02 \pm 0.04$	$-0.41 \pm 0.18$	$0.01 \pm 0.18$	$-0.76 \pm 0.11$			5/2	$+0.06 \pm 0.03$	
1289	$r \rightarrow (3) \rightarrow (0)$	$0.01 \pm 0.01$	$0.00 \pm 0.01$	$-0.04 \pm 0.05$	$0.00 \pm 0.05$	$-0.02 \pm 0.05$			1/2+		
1298	$r \rightarrow (1) \rightarrow (0)$	$0.02 \pm 0.01$	$-0.01 \pm 0.01$	$0.01 \pm 0.04$	$-0.01 \pm 0.04$	$0.06 \pm 0.04$			1/2+	$\begin{cases} +0.30 \pm 0.02 \\ -4.1 \pm 0.3 \end{cases}$	
1301	$r \rightarrow (0)$	$-0.70 \pm 0.01$	$0.05 \pm 0.01$						3/2+	$\begin{cases} +0.114 \pm 0.007 \\ +1.38 \pm 0.02 \\ +0.09 \pm 0.05 \end{cases}$	
1322	$r \rightarrow (2) \rightarrow (0)$			$0.23 \pm 0.06$	$0.04 \pm 0.06$	$0.58 \pm 0.06$	$-0.22 \pm 0.06$			$-0.004 \pm 0.004$	
1331	$r \rightarrow (1) \rightarrow (0)$ $r \rightarrow (1) \rightarrow (0)$	$-0.37 \pm 0.01$ $-0.38 \pm 0.01$	$-0.04 \pm 0.01$ $0.01 \pm 0.01$	$-0.48 \pm 0.03$ $-0.52 \pm 0.04$	$0.01 \pm 0.03$ $0.03 \pm 0.06$	$-0.72 \pm 0.02$ $-0.65 \pm 0.03$			5/2 5/2	$-0.008 \pm 0.006$	
1390	$r \rightarrow (1) \rightarrow (0)$	$-0.48 \pm 0.01$	$0.01 \pm 0.01$	$-0.53 \pm 0.02$	$-0.02 \pm 0.02$	$-0.71 \pm 0.02$			5/2	$+0.042 \pm 0.003$	
1398	$r \rightarrow (1) \rightarrow (0)$	$0.47 \pm 0.01$	$-0.01 \pm 0.01$	$0.88 \pm 0.05$	$0.18 \pm 0.05$	$0.19 \pm 0.05$			3/2(+)	$-0.043 \pm 0.003$	
1480.7	$r \rightarrow (0)$	$-0.41 \pm 0.01$	$-0.01 \pm 0.01$						3/2(+)	$\begin{cases} -0.050 \pm 0.005 \\ +1.97 \pm 0.03 \\ +0.20 \pm 0.07 \end{cases}$	
1482.2	$r \rightarrow (2) \rightarrow (0)$ $r \rightarrow (0)$	$-0.46 \pm 0.01$	$-0.01 \pm 0.01$	$0.23 \pm 0.11$	$0.10 \pm 0.11$	$0.44 \pm 0.08$	$-0.12 \pm 0.08$		3/2	$\begin{cases} -0.022 \pm 0.005 \\ +1.83 \pm 0.03 \\ +0.01 \pm 0.13 \end{cases}$	
1490	$r \rightarrow (2) \rightarrow (0)$ $r \rightarrow (1) \rightarrow (0)$			$0.23 \pm 0.11$	$0.10 \pm 0.11$	$0.67 \pm 0.13$	$-0.20 \pm 0.13$				
1490	$r \rightarrow (1) \rightarrow (0)$	$0.22 \pm 0.02$	$-0.02 \pm 0.03$	$0.81 \pm 0.17$	$-0.18 \pm 0.17$	$0.01 \pm 0.11$			3/2	$-0.12 \pm 0.05$	
1510	$r \rightarrow (1) \rightarrow (0)$	$0.03 \pm 0.01$	$0.01 \pm 0.01$	$-0.10 \pm 0.03$	$0.00 \pm 0.03$	$-0.76 \pm 0.01$			5/2+	$-0.206 \pm 0.003$	

the bombarding energy\*) (see section IIA), the excitation energy of the corresponding level in  $^{31}\text{P}$  using  $Q_0 = 7.286 \text{ MeV}^{12}$ , the approximate (probable error estimated as 30%) level strength  $(2J + 1)\gamma$  (see section IIC1), and the most important primary lines in the decay spectrum. The only cases where no primary lines were observed to either the first, second, or third excited state of  $^{31}\text{P}$  are the resonances at  $E_p = 1348 \text{ keV}$  which is seen to decay almost entirely to the  $3.41 \text{ MeV}$ , fifth excited state of  $^{31}\text{P}$ , and the  $E_p = 1507 \text{ keV}$  resonance level which is so weak that the decay scheme could not be determined. The identification of this resonance level as belonging to the  $^{30}\text{Si}(p, \gamma)^{31}\text{P}$  reaction rests solely on the detection of a  $1.27 \text{ MeV}$  gamma ray.

**B. Angular distributions and correlations.** The results of most of the angular distribution and triple angular correlation measurements are given in table II. The results for the resonance at  $E_p = 1348 \text{ keV}$  are given separately in the following section. In many cases errors of the angular distribution coefficients are given in table II as 0.01. In general the standard deviation calculated from counting statistics is smaller than 0.01, but we have nonetheless placed this lower limit on the error, because experience has shown that the results are not reproducible to better than this precision. The reason for this is not known with certainty; it may be due to insufficient rigidity of our goniometer table.

As indicated in section III, there is a correlation error relating each  $a_2$  and the corresponding  $a_4$ . These are not listed in table II, however. In cascades where it is certain that the intermediate state is of spin  $3/2$  (all cascades which go through the first excited state of  $^{31}\text{P}$ ) there can be no term in the angular correlation higher than  $P_2(\cos \theta)$  in geometry II (first gamma ray detected at  $90^\circ$  to the beam direction). The analysis of the data in such cases is therefore carried out only to  $P_2(\cos \theta)$ .

Three examples of the data from the angular distributions and correlations are shown in figs. 3, 4 and 5. The first two are unusual in that strong  $P_4(\cos \theta)$  terms appear, whereas the last shows a combination of a practically isotropic angular distribution and one extremely anisotropic angular correlation.

**C. Level widths.** The apparent width of each resonance level was determined from yield measurements with the thinnest available target,  $5 \mu\text{g}/\text{cm}^2$ . The results are shown in fig. 6. It is seen that all but two widths lie on a smooth curve the slope of which is probably the result of two effects which

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\*) Here the proton bombarding energies are given as measured, but in the rest of the text they are rounded off to the nearest kilovolt. For clarity, however, the decimal fractions are retained at the  $E_p = 1480.7$  and  $1482.2 \text{ keV}$  doublet.

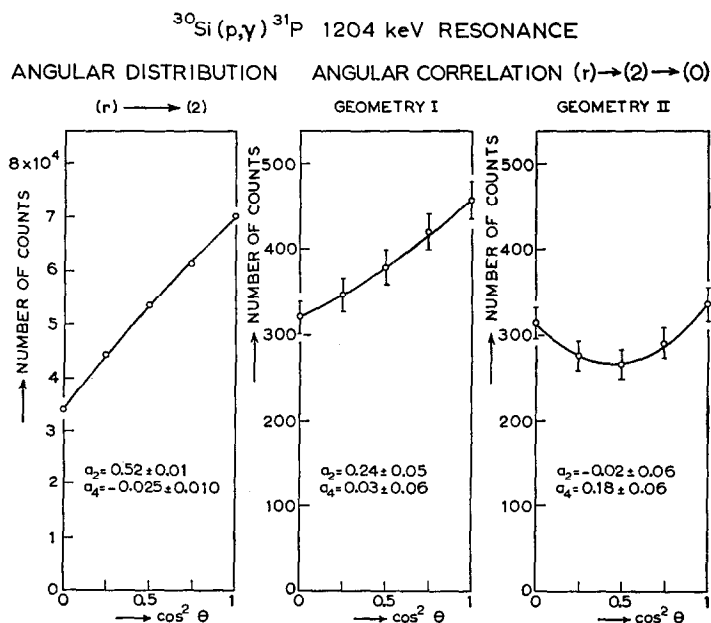


Fig. 3. Measured angular distribution of the  $r \rightarrow (2)$  transition and the angular correlations of  $r \rightarrow (2) \rightarrow (0)$  in geometries I and II at the 1204 keV resonance. The quantities  $a_2$  and  $a_4$  are the coefficients of the Legendre polynomials in the expansion  $W(\theta) = 1 + a_2 P_2(\cos \theta) + a_4 P_4(\cos \theta)$  which best fit the experimental data. The solid lines are the best fits in each case.

almost compensate as the terminal voltage of the accelerator is increased: (1) the diminishing stopping power of the target, and (2) the increasing energy spread of the beam. For the two cases found to exceed this instrumental curve, the natural width was calculated under the assumption that the instrumental and the natural width add quadratically. The results for the total natural widths,  $\Gamma_t$ , of these two levels are:

$$\Gamma_t(1298 \text{ keV}) = 0.7 \pm 0.4 \text{ keV, and}$$

$$\Gamma_t(1516 \text{ keV}) = 1.5 \pm 0.4 \text{ keV.}$$

We note that both levels have  $J = 1/2$  and that both have been identified from elastic proton scattering<sup>8)</sup> as  $J^\pi = 1/2^+$  levels with widths that agree roughly with those given here.

D. Measurements on the  $E_p = 1348$  keV resonance level and on the 3.41 MeV level in  $^{31}\text{P}$ . 1. Angular distribution and correlations. The resonance at  $E_p = 1348$  keV is the only one of the seventeen resonances studied which decays almost exclusively to a level of unknown spin. In fig. 1 it can be seen that this resonance is superimposed on a strong  $^{19}\text{F}(p, \alpha\gamma)^{16}\text{O}$  resonance, so that the analysis of the spectrum was difficult.

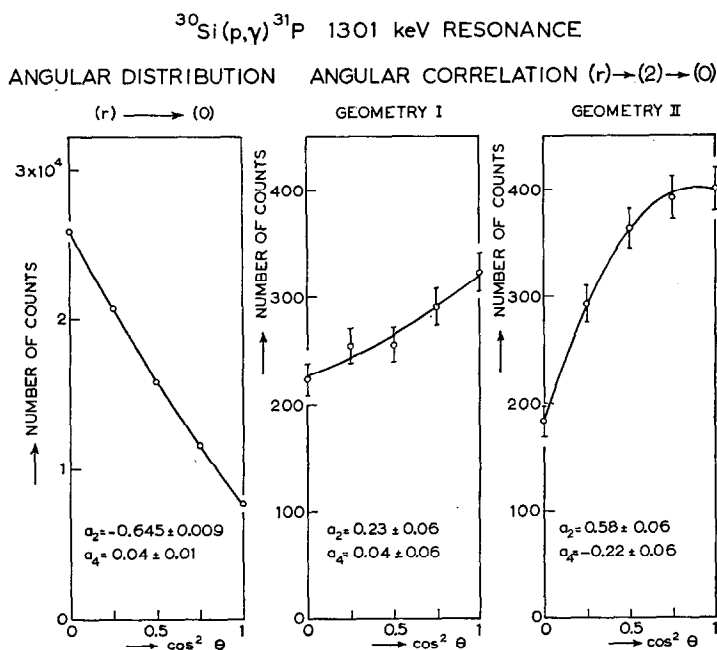


Fig. 4. The same as fig. 3 except that here the resonance is that at  $E_p = 1301$  keV. The angular distribution shown is that of the  $r \rightarrow (0)$  transition.

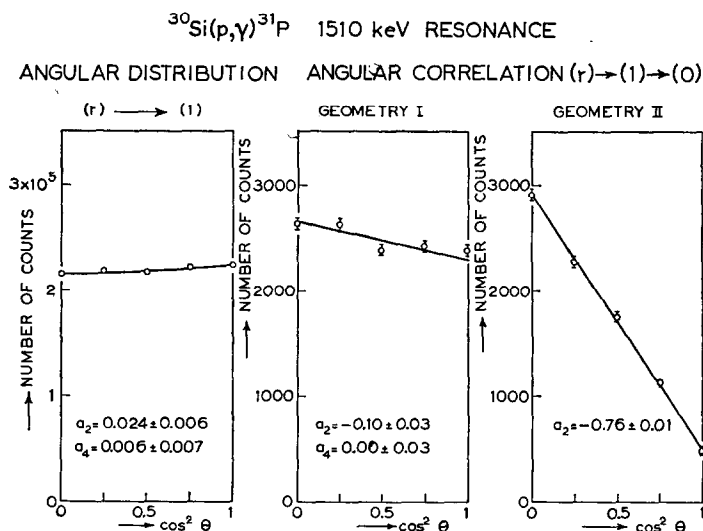


Fig. 5. The same at  $E_p = 1510$  keV as in figs. 3 and 4 except that the transitions are  $r \rightarrow (1)$  and  $r \rightarrow (1) \rightarrow (0)$ . As explained in the text, for geometry II the best fit is given to  $W(\theta) = 1 + a_2 P_2(\cos \theta)$ .

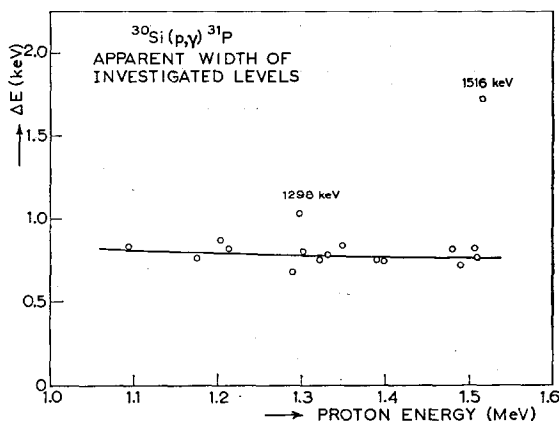


Fig. 6. Measured widths  $\Delta E$  for seventeen resonances. (target thickness:  $5 \mu\text{g}/\text{cm}^2$ ). Two levels, corresponding to  $E_p = 1298$  and  $1516$  keV, show a natural width great enough to be measured.

There may be other weak transitions that we were not able to detect, but the main transition is certainly that given in table I; to the  $3.41$  MeV level, the fifth excited state of  $^{31}\text{P}$ . The angular distribution of this transition was measured, although the error in the measurement was quite large because of the background lines from the fluorine reaction. In order to have enough information to permit a meaningful analysis, more angular correlations were measured than is usually necessary. The only cascade of measurable intensity was the triple cascade  $r \rightarrow (5) \rightarrow (1) \rightarrow (0)$ . The measurements on the first two gamma rays of this cascade are summarized in table III.

TABLE III

Coefficients, $a_2$ and $a_4$ , of the Legendre polynomials found as best fit to the data in the angular distribution and correlations of the cascade $r \rightarrow (5) \rightarrow (1)$ in the decay of the $E_p = 1348$ keV resonance level.					
Geometry	$\theta_1$	$\theta_2$	$\varphi$	$a_2$	$a_4$
I	variable	$90^\circ$	$180^\circ$	$-0.09 \pm 0.07$	$0.03 \pm 0.07$
II	$90^\circ$	variable	$180^\circ$	$0.47 \pm 0.05$	$-0.18 \pm 0.05$
V	variable	$90^\circ$	$90^\circ$	$0.01 \pm 0.07$	$-0.15 \pm 0.07$
VI	$90^\circ$	variable	$90^\circ$	$0.31 \pm 0.05$	$-0.15 \pm 0.05$
Angular distribution of first gamma				$-0.11 \pm 0.02$	$-0.04 \pm 0.02$

Angular correlations in geometries I and II for the cascade  $(5) \rightarrow (1) \rightarrow (0)$  were also measured. Four coincidence circuits were used for each geometry. For geometry I, for instance, one differential discriminator was set on the  $3.41 \rightarrow 1.27$  MeV transition as detected by the rotating counter (channel 1), and another equally wide channel ( $1'$ ) was set just above this transition. For the stationary counter two channels ( $2$  and  $2'$ ) were set in the same way with respect to the  $1.27 \rightarrow 0$  MeV transition. The assumption

was made that channel 1' represented the background for channel 1, and channel 2' for channel 2. All four possible coincidence combinations were recorded. We will call the counting rates of these coincidence circuits  $R(1, 2)$ ,  $R(1, 2')$ ,  $R(1', 2)$ , and  $R(1', 2')$ . The desired effect is clearly  $R(1, 2) + R(1', 2') - R(1, 2') - R(1', 2)$ . Because the corrections were quite large it was necessary to measure continuously for about forty hours in order to reach adequate counting statistics. The data are displayed in fig. 12 (Section VA).

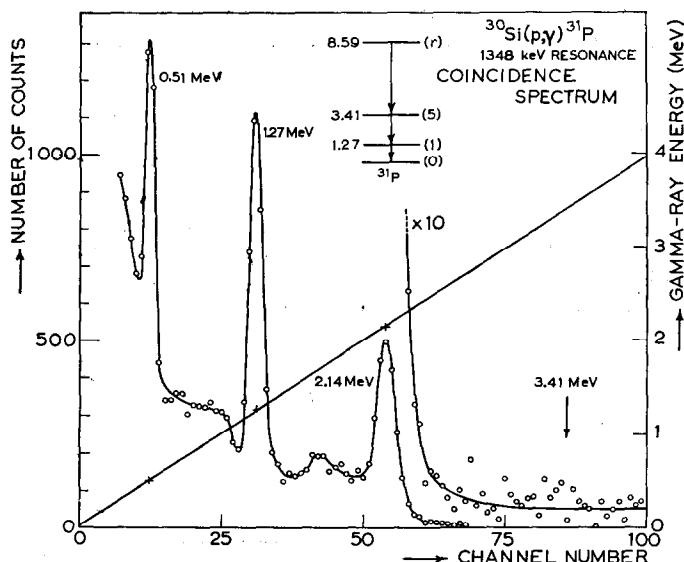


Fig. 7. A gamma-gamma coincidence spectrum gated by the 5.18 MeV line at the  $E_p = 1348$  keV resonance level. The high purity of the 1.27 and 2.14 MeV transitions in the  $(5) \rightarrow (1) \rightarrow (0)$  cascade, and absence of any 3.41 MeV line, are clear.

2. The decay of the 3.41 MeV level in  $^{31}\text{P}$ . During the entire measurement in geometry I of the decay of the 3.41 MeV level an extra differential discriminator was set on the output of the rotating counter with a channel set to select the 5.18 MeV line feeding the 3.41 MeV level. This was used to gate a multi-channel analyzer which recorded the spectrum of the stationary counter. The result is shown in fig. 7. It is clear that the 3.41 MeV level does not decay detectably to the ground state\*). An upper limit of 1% can be set for the branching ratio of such a transition. The fact that other authors have reported this transition<sup>17) 4)</sup> when the 3.41 MeV level is fed at other resonances, cannot be explained at this time. The energy differences come out badly for an accurate determination of the branching ratio of a

\*) The slight indication of a peak at about 3.41 MeV in fig. 7 is due to sum pulses.

possible  $(5) \rightarrow (2) \rightarrow (0)$  cascade. The symmetry of the observed lines of energy 2.14 and 1.27 MeV rules out a branching ratio of more than 2% to the second excited state (the lines in that case would have energies of 2.23 and 1.18 MeV).

E. Resonant absorption. A resonant absorption experiment <sup>14)</sup> was performed on the radiation emitted by the levels excited at  $E_p = 1480.7$  and 1482.2 keV. A relatively thick target was used which gave a saturated intensity for both components. A 30 cm long lead collimator, 10 cm high, was used with an opening of 5 mm. The absorber was red phosphorus powder placed in the collimator opening. A total thickness of

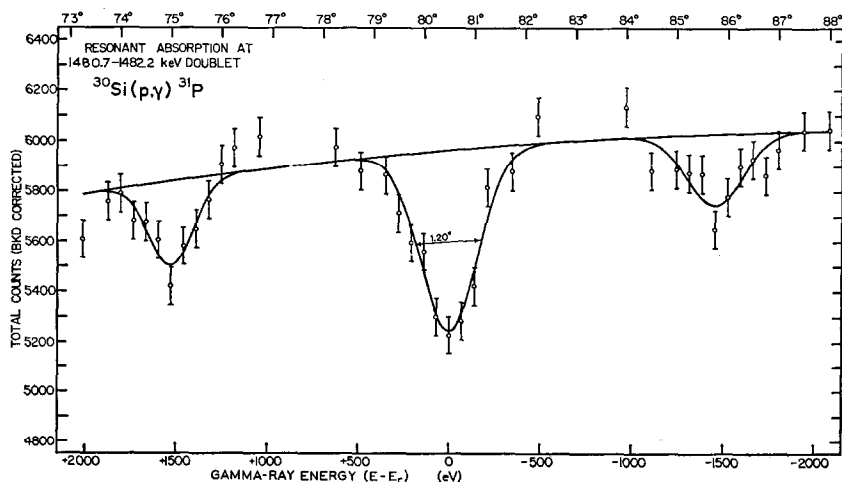


Fig. 8. The results of the resonant absorption measurement at the  $E_p = 1480.7$  and 1482.2 keV doublet. For interpretation, see text.

$7.19 \times 10^{23}$  atoms/cm<sup>2</sup> was used. In fig. 8, the results of the measurement are shown. The central absorption dip is that to be expected for a single resonance, and the two satellite dips are the result of "cross-shooting" of one component into the other, and vice versa. The energy variation of the emitted gamma rays, as well as the angle of measurement with respect to the proton beam, is shown.

The sloping base line in fig. 8 was fitted to the twelve points lying farthest from the center of the absorption dips. The slope is due to the angular distribution which is almost the same (see table II) at the two resonances. The absorption integrals were calculated, leading to the following results:

$$\begin{aligned} A_E \text{ (central dip)} &= 45.3 \pm 2.8\text{eV}, \\ A_E \text{ (left-hand satellite)} &= 16.8 \pm 2.7\text{eV}, \\ A_E \text{ (right-hand satellite)} &= 17.9 \pm 2.8\text{eV}. \end{aligned}$$

The distance between the dips is  $1.5 \pm 0.1$  keV, this being the separation of the members of the doublet. The difference between the increment in excitation energy and in proton bombarding energy is less than the error, and so is not taken into account.

The center solid curve is a gaussian fitted to the points. The same shape and width was used for the solid curves drawn in for the satellite dips.

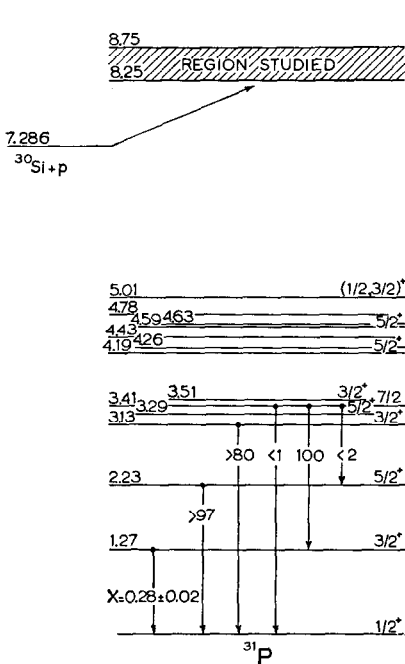


Fig. 9. Level scheme of  $^{31}\text{P}$  showing the resonance region studied in this investigation, and the region of lower levels reached in the decay of the resonance. The new results of this investigation are shown: the mixing ratio of the  $(1) \rightarrow (0)$  transition, the spin of the 3.41 MeV level, and its decay.

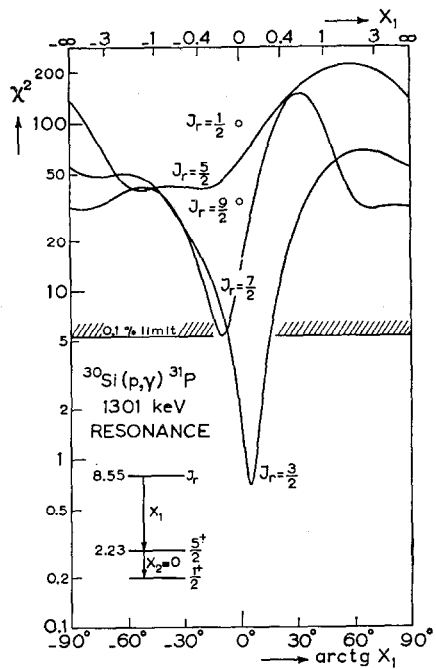


Fig. 10. Chi-square of the fit to the angular correlation measurements at  $E_p = 1301$  keV of the  $r \rightarrow (2) \rightarrow (0)$  cascade in geometries I and II (see fig. 4) plotted as a function of the mixing parameter  $x_1$  of the first gamma ray, with the resonance spin as parameter. The second gamma ray is assumed to be pure quadrupole. For the meaning of the 0.1% limit, see text.

V. Analysis and discussion. A. Spin assignments and mixing parameters. There is included as a guide in reading the discussion, a level scheme of the  $^{31}\text{P}$  nucleus<sup>12</sup>). This is given in fig. 9. The only difference between this scheme and that shown in ref. 12 is that the more precise mixing ratio of the  $(1) \rightarrow (0)$  radiation found in the present investigation is given, and the spin of the 3.41 MeV level as established here is shown, as well as the decay of this level.



The spin assignments resulting from the analysis according to the method described in section III, of the results of the angular distributions and correlations measured at each resonance, are given in table II. The resonance level at  $E_p = 1348$  keV is treated separately, below.

A typical example of the results of the analysis is given in fig. 10. The value of chi-square is plotted against the mixing ratio of the first gamma ray. The second gamma ray is assumed unmixed in this case. In those cases where it could be mixed, the mixing was allowed to vary, and the two-dimensional minimum was found. If, for a given spin combination, the minimum value of chi-square is larger than that indicated by the horizontal line marked "0.1% limit" in fig. 10, then there is less than 0.1% statistical chance that that spin combination is correct<sup>18)</sup>. In this case, the spin of  $3/2$  can be considered to be well established. Most cases studied showed this same degree of uniqueness.

In the last column of table II are given the mixing parameters,  $x_1$ , found from the analysis for the first gamma ray in each cascade. At the  $E_p = 1289$  and  $1516$  keV resonance levels not enough information was available to give a result. In transitions direct to the ground state of  $^{31}\text{P}$  the results are ambiguous, and both possible values are given.

In all of the cases referred to above, the only unknown spin was that of the resonance level itself. The analysis was therefore quite simple. In the case of the  $E_p = 1348$  keV resonance level, however, neither the resonance spin nor the intermediate level spin were known. The final spin is known to be  $3/2$ , of course. All possible spin combinations (with the restriction that the lowest of the interfering multipoles is not greater than quadrupole) were attempted. Only two combinations gave solutions with chi-square under the 0.1% level. These are shown in fig. 11. The minimum value of chi-square reached in both cases is quite large. The probable reason is that to the original data (Table III) were attributed only statistical errors, whereas the instrumental errors, particularly in the case of the angular distribution, while unknown, may be appreciably larger.

Both solutions give a spin of  $7/2$  for the  $3.41$  MeV, fifth excited state of  $^{31}\text{P}$ . The existing tables of  $C_{KM}^N$  coefficients<sup>16)</sup> were extended to octupole transitions in this case. It was then attempted to better the fit already found with a pure  $7/2 \rightarrow 3/2$  transition by including small amounts of octupole radiation. Little improvement was found; the minimum value of chi-square (2.1) for the  $5/2 \rightarrow 7/2 \rightarrow 3/2$  case was reached for an  $x_2$  (the octupole/quadrupole mixing ratio) of less than 0.01.

The above analysis makes it clear that the  $3.41$  MeV level has a spin of  $7/2$ , but the spin of the resonance level may be  $9/2$  or  $5/2$ . In either case the population parameters of the  $3.41$  MeV level may be calculated easily from the vector-coupling coefficients. The results are shown in table IV. As mentioned above, the  $3.41 \rightarrow 1.27 \rightarrow 0$  MeV cascade was also studied.

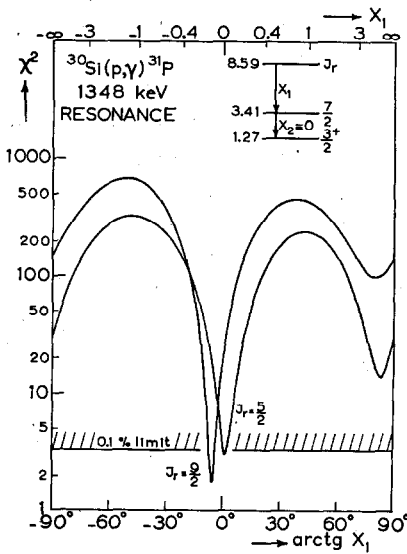


Fig. 11. Chi-square of the fit to the angular distribution and angular correlation measurements shown in table IV ( $E_p = 1348$  keV) plotted as a function of the mixing parameter  $x_1$  for two values of the resonance spin ( $5/2$  and  $9/2$ ). In both cases the spin of the fifth level in  $^{31}\text{P}$  is taken as  $7/2$ , and we suppose that the  $(5) \rightarrow (1)$  transition is pure quadrupole.

$^{30}\text{Si}(p, \gamma)^{31}\text{P}$  1348 keV RESONANCE  
ANGULAR CORRELATION  $(5) \rightarrow (1) \rightarrow (0)$

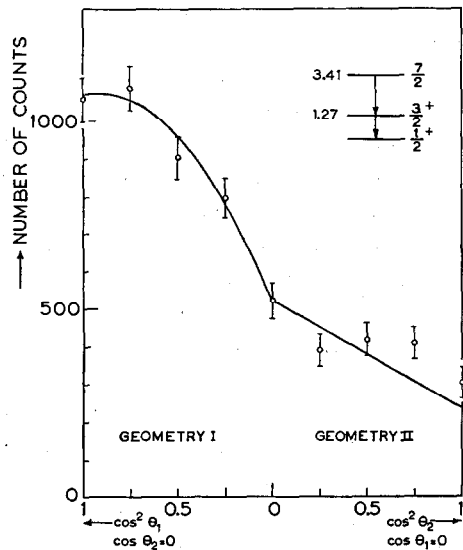


Fig. 12. The best fit to the data measured at  $E_p = 1348$  keV for geometries I and II of the  $(5) \rightarrow (1) \rightarrow (0)$  cascade. The spin sequence is  $7/2 \rightarrow 3/2 \rightarrow 1/2$ .

TABLE IV

Values of the population parameters of the 3.41 MeV, $J = 7/2$ level in $^{31}\text{P}$ as predicted for resonance spins $5/2$ and $9/2$ at $E_p = 1348$ keV, and as measured in the cascade $3.41 \rightarrow 1.27 \rightarrow 0$ MeV excited at that resonance.		
$J_r$	$P(1/2)$	$P(3/2)$
$5/2$	0.64	0.36
$9/2$	0.83	0.17
Measured	$0.66 \pm 0.06$	$0.34 \pm 0.07$

The results of the angular correlations were analysed in a program of the type earlier described<sup>16)</sup> by one of the present authors (P.B.S.). This program finds, by least squares analysis, the best value of the population parameters of the initial state (in this case the  $J = 7/2$ , 3.41 MeV level) from the original data (as opposed to the Legendre polynomial coefficients) for

given values of the radiation mixing parameters. The solutions of both the  $9/2 \rightarrow 7/2 \rightarrow 3/2$  and  $5/2 \rightarrow 7/2 \rightarrow 3/2$  cases show very little mixing of quadrupole in the first gamma ray, so it was possible to restrict the occupied states in the second analysis to  $m = \pm 1/2$  and  $m = \pm 3/2$ . Further, considering the results of the first analysis, there was no need to take appreciable octupole radiation into account in the  $7/2 \rightarrow 3/2$  transition. The mixing ratio of the second gamma ( $3/2 \rightarrow 1/2$ ) could be considered as given (see section VC below). In the light of these very limiting restrictions, the excellent fit to the data shown in fig. 12, and the good agreement of the population parameters found with the values predicted for a resonance spin of  $5/2$  (see table IV), must be considered as rendering this resonance spin as very highly probable, despite the unfavorable impression given by the large value of chi-square in the first analysis (fig. 11).

**B. Parity assignments.** It can be seen from table II that very few parity assignments have been made. Very conservative criteria have been applied. These are:

(1) Positive parity is assigned to a resonance if  $|M|^2$  (the measured radiation width divided by the single-particle width) for magnetic quadrupole radiation would be greater than unity if negative parity were to be assumed. A tentative positive parity assignment is made (+) if  $|M|^2$  is between 0.1 and unity.

(2) A parity assignment is rejected if it leads to a value of the reduced width  $\theta_p^2$ , defined by<sup>19)</sup>

$$\theta_p^2 = \frac{\Gamma_p}{2P_l} \bigg/ \frac{\hbar^2}{\mu a^2}$$

equal to one tenth or more. In this formula,  $\Gamma_p$  is the proton width of a level,  $2P_l$  is the Coulomb penetrability for  $l$ -capture, and  $\hbar^2/\mu a^2$  is the Wigner limit.

Very few resonances are seen to submit to an assignment on this rigorous basis, and all of these assignments depend on the first criterium. The highest  $\theta_p^2$  would be found if the  $E_p = 1176$  keV resonance level were assigned a positive parity ( $g$ -capture). We would find in this case a lower limit of 0.01 for  $\theta_p^2$ . Even this large a value is rendered unlikely by the absence of any shell-model prediction of a  $g_{7/2}$  proton level anywhere near this excitation. An  $f_{7/2}$  proton level would not be at all surprising, on the other hand. A decision can certainly not be made at this stage, however.

**C. Mixing ratio of the transition de-exciting the first excited state of  $^{31}\text{P}$ .** It is shown in the appendix to the present article that it is not possible to remove the ambiguity in the determination of this mixing ratio by intensity-direction correlation measurements alone. By means of

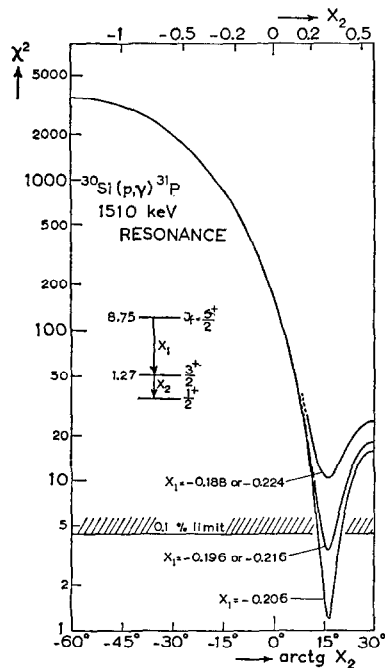


Fig. 13. Chi-square of the fit to the data of fig. 5 ( $E_p = 1510$  keV). The abscissa is the value of  $x_2$ , and in the region of the minimum of  $x_2$  three curves (in fact five) are shown for different values of  $x_1$  in the region of the minimum of that parameter.

TABLE V

The values of the mixing parameter, $x_2$ , of the gamma ray de-exciting the first excited state of $^{31}\text{P}$ , as found at six resonances in the $^{30}\text{Si}(p, \gamma)^{31}\text{P}$ reaction.	
$E_p(\text{keV})$	$x_2$
1095	$0.292 \pm 0.039$
1322	$0.280 \pm 0.038$
1331	$0.197 \pm 0.036$
1390	$0.264 \pm 0.021$
1398	$0.445 \pm 0.079$
1510	$0.295 \pm 0.018$
Weighted average	$0.28 \pm 0.02$

the polarization-direction correlation measurements of Mc Callum<sup>20)</sup> as well as the Coulomb excitation measurements of Andreev *et al.*<sup>21)</sup> combined with the resonant fluorescence measurements of the life-time by Booth<sup>22)</sup>, it is clear that the mixing ratio of this gamma ray is of the order of 0.3. The second value which intensity-direction correlation measurements yield is in the region of  $+1$ , and is thus ruled out by these results. In this in-

vestigation there were six resonances where reliable values of this constant could be extracted from the analysis as described in section III. An example of the sensitivity of the determination is demonstrated in fig. 13 where the analysis of the data at  $E_p = 1510$  keV (shown in fig. 5) is given. Chi-square is plotted as a function of  $x_2$ . In the region of the minimum it is plotted for different values of  $x_1$  as parameter.

In table V the results of the determinations are given and also the weighted average. This result agrees exactly with that of Harris and Seagondollar<sup>4</sup>). The quoted error of the weighted average is the external error, this being larger than the internal error.

D. Resonant absorption. The conclusion which may be drawn from the data of section IVE are more extensive than if there is only one resonant absorption dip. Resonant absorption is frequently non-linear<sup>14</sup>), and especially so when, as here, the observed absorption-dip width is hardly greater than the instrumental width (previously determined). If there is non-linearity it will almost certainly be different for the two levels of the doublet, leading to different absorption integrals for the two satellite dips. It is conceivable, of course, that the two levels have identical total widths so that the non-linearity is the same for both. As will be seen below, however, the excellent agreement of the yield measurements (see table I) and the resonant absorption measurements make this possibility extremely unlikely.

We will analyse the data on the assumption, that there is, practically speaking, linearity. This assumption coincides with the condition that  $\Gamma_p$  is much larger than  $\Gamma_\gamma$  for both members of the doublet. The intensity of the emitted gamma rays is then proportional to the radiation width. In this approximation, calling  $\Gamma_1$  and  $\Gamma_2$  the ground-state transition probabilities of the two levels, the absorption integral (expressed in the same units as the level widths) of the central absorption dip is given by

$$A_c = \frac{ng \lambda^2}{4} \cdot \frac{\Gamma_1^2 + \Gamma_2^2}{\Gamma_1 + \Gamma_2},$$

and each satellite dip has an area:

$$A_s = \frac{ng \lambda^2}{4} \cdot \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2}.$$

Here  $n$  is the thickness of the absorber in atoms/cm<sup>2</sup>,  $\lambda$  the wave-length of the gamma ray, and  $g$  the statistical weight of the excited level. The value of  $ng \lambda^2/4$  here is 72.6. The experimental results for  $A_c$  and  $A_s$  are given in section IVE. The (almost identical) values of the two satellite dips are averaged and the two equations above solved for  $\Gamma$ . There are two solutions for  $\Gamma$ , of course, and we assign these to the appropriate doublet

member on the basis of the measured yield. The values are given in table VI where they are compared with the result of the yield measurements from table I.

TABLE VI

The values of the ground-state width and total width as derived (on the approximation of linearity of absorption) from the resonant absorption measurement, as compared to the results of the yield measurement of table I			
$E_p(\text{keV})$	$\Gamma_{\gamma_0}(\text{eV})$	$(2J + 1)\Gamma\gamma(\text{eV})$ (resonant absorption)	$(2J + 1)\gamma(\text{eV})$ (yield)
1480.7	$0.75 \pm 0.05$	$3.4 \pm 0.2$	$3.4 \pm 1.0$
1482.2	$0.35 \pm 0.06$	$1.8 \pm 0.3$	$2.3 \pm 0.7$

The conclusion is that neither level shows appreciable non-linearity, which leads to a lower limit of about 25 eV for the two total widths. The fact that the central absorption dip is hardly wider than the known instrumental curve, sets an upper limit of about 100 eV for the level widths. It is thus seen that these two close-lying levels of the same spin and very similar decay schemes have closely equal total widths.

E. Level spacings. For the purpose of this discussion we will consider as doublets pairs of levels within 9 keV of each other. An examination of table I shows that there are nine such small spacings out of a total of seventeen adjacent level spacings in a region in which the average level spacing is 30 keV. There are also three triplets, with overall spacings of 12, 9, and 10 keV. We will not discuss this triplet aspect here. Four of the doublets:  $E_p = 1204$  and  $1213$  keV,  $E_p = 1289$  and  $1298$  keV,  $E_p = 1322$  and  $1331$  keV, and either  $E_p = 1480.7$  and  $1482.2$  keV, or  $E_p = 1482.2$  and  $1490$  keV, are composed of levels having the same spin. Considering the large open spaces between levels it seems strange that levels of equal spin do not repel each other and tend to even out the spacings.

It would be premature to attempt to carry these speculations further, since no certainty exists that the doublet levels have the same parity as well as spin (except in the  $E_p = 1289$  and  $1298$  keV case).

VI. *Conclusions.* Of the seventeen resonances to which spin has been assigned here, eight had been assigned on the basis of  $(p, \gamma)$  work earlier<sup>4) 7)</sup> and two on the basis of elastic proton scattering<sup>8)</sup>. Except in the case where these earlier assignments are inconsistent<sup>3) 4)</sup>, they have all been corroborated here. We have hesitated to assign parities unless the assignment could be motivated by a width exceeding a theoretical limit.

In agreement with Harris and Seagondollar<sup>4)</sup>, it has been established here that the 3.41 MeV, fifth excited state of  $^{31}\text{P}$  has spin  $7/2$ . A study of

the coincidence spectrum at the  $E_p = 1348$  keV resonance level which decays almost entirely to the 3.41 MeV level has shown, contrary to findings reported in several other publications<sup>4) 17)</sup>, that this level does not decay to the ground state. The branching ratio of a possible ground-state transition is less than 1%, and that of a possible cascade through the 2.23 MeV second excited state is less than 2%.

During the investigation it was repeatedly found that better resolution led to the discovery that resonances previously thought to be single, were, in fact, double. This in itself is rather surprising; while the average spacing of levels throughout this region is about 30 keV, it was found that half of the spacings are 9 keV or less, twice as many as would be expected on the basis of a simple exponential distribution law. It is remarkable that of the nine spacings of 9 keV or less, at least four are associated with pairs of levels having the same spin and very similar decay patterns. The similarity of the members of one of the latter group of doublets was further underlined by a resonant absorption measurement covering a wide enough angular region so that not only the self-absorption peak, but also the cross-absorption peaks appeared. These two satellite peaks were of the same intensity, giving a strong indication that the two levels are quite similar in total width, as well as decay scheme.

#### APPENDIX

The general question as to whether an angular correlation measurement leads to a unique determination of the spin of the states involved and of the values of the radiation mixing parameters is not amenable to analysis. Perhaps the best that can be done is to search for theorems wherein the impossibility of certain unique determinations is established. A complete set of such theorems would at least save experimenters the trouble of trying to prove the impossible. The theorem to be proved below is a good example of just this, because a great deal of effort has been expended<sup>17) 23)</sup> in attempts to determine uniquely the mixing ratio of the gamma rays de-exciting the first and third excited states of  $^{31}\text{P}$ . The determination of this quantity by means of intensity-direction correlations is subject to an irremovable ambiguity\*).

The statement of the theorem is as follows: it is impossible to determine unambiguously the mixing ratio of a gamma ray de-exciting to the ground state (i.e. a gamma ray which is not followed up in cascade) by means of intensity-direction measurements alone, if the spin of the decaying state is less than two.

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\*) Since the preparation of this paper, ref. 4 has been received in preprint form. A heuristic proof of this theorem is given there, and the publication of the rigorous proof is announced.

The proof is as follows: All angular correlations involving the last two gamma rays of a cascade (numbered 1 and 2) may be described by:

$$W(\theta_1, \theta_2, \varphi) = \sum_{KMN} a_{KM}^N \bar{P}_K^N(\cos \theta_1) \bar{P}_M^N(\cos \theta_2) \cos N\varphi \quad (\text{A } 1)$$

The notation is that of ref. 16. The most that can be done in any series of measurements is to determine the coefficients  $a_{KM}^N$ , and we assume here for simplicity that this has been done with infinite precision. The general form of these coefficients is:

$$a_{KM}^N = \sum_{mpq} P(m) C_{KM}^N(m, p, q, \alpha) x_1^p x_2^q. \quad (\text{A } 2)$$

In this equation  $x_1$  and  $x_2$  are the mixing ratios of the first and second gamma rays in the cascade considered, and  $p$  and  $q$  take on the value zero for pure dipole, one for mixed dipole-quadrupole, and two for pure quadrupole radiation. The coefficients  $C_{KM}^N$  are tabulated by Ferguson and Rutledge<sup>15)</sup> or Smith<sup>16)</sup>. The quantum number  $\alpha$  represents all unmentioned quantum numbers. The sub-states of  $J_1$  are given by  $m$ . Each  $C_{KM}^N$  may be written as a product of two factors, one depending only on  $p$ , and the other on  $q$ . Doing this, we find:

$$a_{KM}^N = \sum_{mp} P(m) A_{KM}^N(m, p, \alpha) x_1^p \sum_q B_M(J_2, J_3, q) x_2^q,$$

or

$$R_M = \frac{\sum_{mp} P(m) A_{KM}^N(m, p, \alpha) x_1^p}{a_{KM}^N} = \sum_q B_M(J_2, J_3, q) x_2^q. \quad (\text{A } 3)$$

There is thus only one distinct expression in  $x_2$  †) for each permissible value of  $M$ . The proof of the theorem depends on the following: (1)  $M$  can only take on even values (this is an inherent limitation in intensity-direction measurements) and (2) the largest value of  $M$  is limited by the triangle condition on  $(J_2 J_2 M)$ . Thus if  $J_2$  is less than two,  $M$  can be equal to, at most, zero and two, giving two quadratic expressions in  $x_2$ , only the ratio of which is significant. We find thus two equally good values of  $x_2$  and the theorem is proved (except in the case of equal roots).

An interesting representation of the ambiguity expressed by eq. (A 3) can be found if we first note that \*)

$$R_2/R_0 = \frac{ax_2^2 + bx_2 + c}{1 + x_2^2},$$

and we make the substitution  $\theta = \text{arctg } x_2$ . We then find,

$$R_2/R_0 = \frac{1}{2} \left\{ a + c + \sqrt{b^2 + (a - c)^2} \cos \left( 2\theta - \text{arctg} \frac{b}{c - a} \right) \right\}. \quad (\text{A } 4)$$

\*) For the  $M = 0$  term the mixing term is always zero (triangle condition on  $L_2, L_2'$  and  $M$ ) and the dipole and quadrupole terms are equal.

†) Note added in proof, The authors thank G. I. Harris for drawing their attention to the obvious inversion in eq. (A3).



Since  $\cos x = \cos(-x)$  we find that the two values of  $\theta(\theta_1$  and  $\theta_2)$  which satisfy eq. (A 4) are related by

$$\theta_2 = \arctg \frac{b}{c-a} - \theta_1,$$

or in other words, that a plot of  $R_2/R_0$  against  $\theta$  has an axis of symmetry at  $\theta_s = \frac{1}{2}\arctg 2b/(a-c)$ . It is found for an intermediate state of spin 3/2 and a final state of spin 1/2, that  $c-a = b/\sqrt{3}$ , or  $\theta_s = 30^\circ$ .

A polarization-direction correlation measurement can be shown to lead to a quadratic expression in  $x_2$  which is essentially different from eq. (A3)<sup>20</sup>. A combination of intensity-direction correlation plus polarization-direction correlation measurements can therefore remove the ambiguity.

A corollary to this theorem can be found by reversing the direction of the cascade and noting that if the first level has no directional properties (spin less than one) no information can be obtained from its formation. If the second level has a spin less than two, then the same conditions (in the reversed direction) apply to the first gamma ray as to the second gamma ray in the case given above. The statement is thus: it is impossible to determine unambiguously the mixing ratio of the first gamma ray in a decay by means of intensity-direction correlation measurements alone if the first state in the decay has a spin less than one and the second state has a spin less than two.

This corollary is intrinsically less valuable than the original theorem, but it may be remarked that it is more difficult to find a way around its limitation. Measurements with polarized beams and/or targets would be required.

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