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GOLDBERGER-TREIMAN RELATION IN THE RENORMALIZED SIGMA MODEL

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Abstract: The regularization and renormalization of the full sigma model is worked out explicitly in the tree and one-loop approximation. Various renormalized quantities relevant for chiral symmetry breaking are listed. The numerically calculated Goldberger-Treiman relation is also compared with experiment.

1. INTRODUCTION

It is well-known that the Goldberger-Treiman relation (G.T.) implies $d(0)K(0) = g_A m/f_\pi/G_{\pi NN} = 0.9$, where $d(q^2)$ and $K(q^2)$ are respectively the pion propagator form factor and the proper πNN vertex form factor. Such deviation from unity is looked upon as the effect of a small breaking of chiral symmetry. The breaking is supposed to be also responsible for the non-vanishing mass of the pion. Our intention is to study the correlation between $d(0)K(0)$ and the pion mass, μ_π.

The first calculation of $d(0)K(0)$ was done by Pagels [1] with $\mu_\pi = 140$ MeV. He did not use a particular model of chiral symmetry breaking, but merely he made an off-mass-shell extrapolation of the physically dressed pion propagator and πNN vertex. However, this resulted only in a one percent deviation.

In order to be able to consider the pion mass as a variable of the problem, we need a model that gives us the dependence of masses, coupling constants, etc. on μ_π. The linear sigma model [2] was chosen for this purpose because of its simplicity. However, we cannot expect it to be very realistic. For instance, it does not include strange particles. Nevertheless, the type of correction considered here is presumably also part of the corrections to the G.T. relation in the real world.

In this context it does not make sense to use the sigma model in tree approximation, for it always leads to a fixed value of g_A, i.e. $g_A = 1$. The experimental number is 1.24. Fortunately, Lee [3] and Gervais and Lee [4]

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proved that it is possible to renormalize the sigma model without violating its symmetry properties like CVC and PCAC. For the one-loop approximation, the renormalization of the full sigma model was carried out by several authors [5, 6]. Mignaco and Remiddi [5] used the conventional counterterm technique. However, the way they dealt with surface terms of divergent integrals did in fact induce additional breaking of the symmetry. As a consequence, the Goldberger-Treiman relation is not satisfied in their calculation.

In this paper we apply a symmetric regularization, which does not break the symmetry. By this we mean that, even for finite regulator masses, the Ward identities are satisfied. When taking those masses to infinity, the renormalization scheme is easily seen. Moreover, the G.T. relation is explicitly verified to hold in terms of the renormalized quantities.

An alternative renormalization technique was introduced by Symanzik. It was applied to the sigma model by Bessis and Turchetti [6], who calculated their diagrams by means of a dispersion relational method. The subtraction constants are interrelated by Ward identity constraints.

The last section of this paper deals with the numerical calculation of the G.T. relation. The conclusion is, however, that our renormalized model cannot quantitatively describe reality. Whatever pion mass is used, $d(0)K(0)$ will never descend as low as 0.9. Finally, a computation was made in the sense of "Continuous Symmetry Breaking" [7].

2. REGULARIZATION OF THE SIGMA MODEL

Our regularized Lagrangian is the following:

$$
\begin{aligned}
\mathcal{L} = :\ & \bar{\psi}_0(i\partial\!\!\!/ - g_0(\sigma_0 + \boldsymbol{\pi}_0\boldsymbol{\tau}\gamma_5))\,\psi_0 - g_0\,\bar{\psi}_0(S + \boldsymbol{P}\boldsymbol{\tau}\gamma_5)\,\psi_0 \\
& + \sum_{j=1} [\varphi_j^1(i\partial\!\!\!/ - m_j)\varphi_j^1 + \varphi_j^2(i\partial\!\!\!/ - m_j)\varphi_j^2 - g_j\,\sigma_0(\bar{\varphi}_j^1\varphi_j^1 - \bar{\varphi}_j^2\varphi_j^2) \\
& - g_j\boldsymbol{\pi}_0(\bar{\varphi}_j^1\,\boldsymbol{\tau}\varphi_j^2 + \bar{\varphi}_j^2\,\boldsymbol{\tau}\varphi_j^1)] \\
& + \tfrac{1}{2}((\partial_\mu\sigma_0)^2 + (\partial_\mu\boldsymbol{\pi}_0)^2) - \tfrac{1}{2}\mu_0^2(\sigma_0^2 + \boldsymbol{\pi}_0^2) \\
& - \tfrac{1}{2}((\partial_\mu S)^2 + (\partial_\mu\boldsymbol{P})^2) + \tfrac{1}{2}M^2(S^2 + \boldsymbol{P}^2) \\
& - \tfrac{1}{4}\lambda_0^2((\sigma_0 + S)^2 + (\boldsymbol{\pi}_0 + \boldsymbol{P})^2)^2 : + c_0\sigma_0\ .
\end{aligned}
\qquad (2.1)
$$

ψ_0 represents the bare proton and neutron fields. σ_0 is a bare scalar meson field with vacuum expectation value $F_0 = \langle 0|\sigma_0|0\rangle$. $\boldsymbol{\pi}_0$ are the bare fields of the three pseudoscalar pions. S and $\boldsymbol{P}$ are regulator fields with indefinite metric.

φ_j^1 and φ_j^2 are parity doublet regulators. A symbol ϵ_j is defined to be

$-1(+1)$, when $\varphi_j^{1,2}$ are quantized with (anti) commutators. The motivation of this particular way of regularizing is fully explained in ref. [4].

We use the metric $g_{\mu\nu} = (1, -1, -1, -1)$ and Dirac matrices $(\gamma^{1,2,3})^+ = -\gamma^{1,2,3}$; $\gamma^{0+} = \gamma^0$; $\gamma_5^+ = -\gamma_5$. The renormalized quantities are related to the bare ones by means of renormalization constants:

$$\mu_0^2 = \mu^2 + \delta\mu^2 ,$$

$$\psi_0 = Z_F^{\frac{1}{2}} \psi ,$$

$$(\pi_0, \sigma_0, F_0) = Z_B^{\frac{1}{2}}(\pi, \sigma, F) ; \qquad c_0 = Z_B^{-\frac{1}{2}} c ,$$

$$g_0 = g Z_g Z_B^{-\frac{1}{2}} / Z_F ,$$

$$\lambda_0^2 = \lambda^2 Z_\lambda / Z_B^2 . \tag{2.2}$$

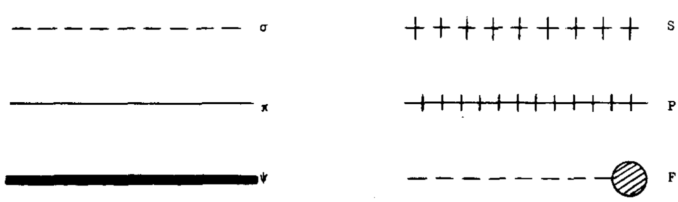

Fig. 1. Graphical representation of the fields.

The graphical symbols, used in our Feynman diagrams are defined in fig. 1. It is important to note that the interaction Lagrangian induces a mixing among the regulator and particle fields. We will only need this effect in tree diagrams, where it arises when coupling tadpoles to the term $((\sigma_0 + S)^2 + (\boldsymbol{\pi}_0 + \boldsymbol{P})^2)^2$. In principle both the σ and S fields can form a tadpole. However, it turns out that it is possible to perform the intended regularization even after requiring $\langle 0|S|0\rangle = 0$. This is a considerable simplification.

Consider now as an example the σ_0, S interaction. The elementary mixing process is given in fig. 2 and leads to a two-by-two propagator $D_\sigma(k)$.

$$D_\sigma^{-1}(k) = \begin{pmatrix} \sigma_0 \to \sigma_0 & \sigma_0 \to S \\ S \to \sigma_0 & S \to S \end{pmatrix}$$

$$= (-i)(2\pi)^4 \begin{pmatrix} k^2 - \mu_0^2 - 3\lambda_0^2 F_0^2 & -3\lambda_0^2 F_0^2 \\ -3\lambda_0^2 F_0^2 & M^2 - k^2 - 3\lambda_0^2 F_0^2 \end{pmatrix} . \tag{2.3}$$

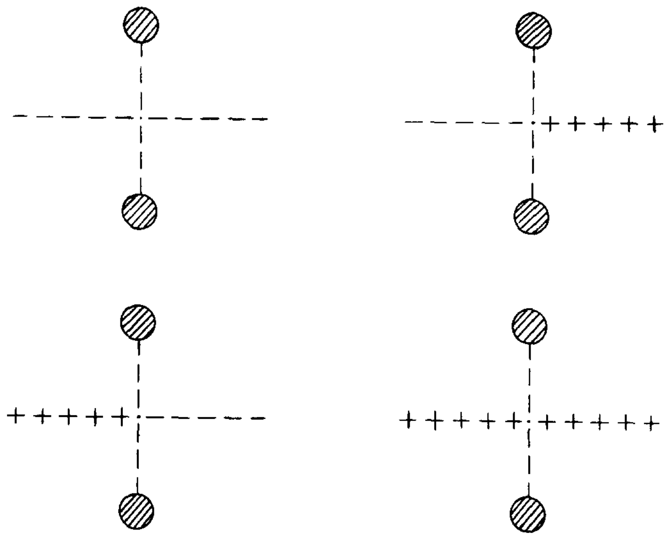

Fig. 2. Elementary processes that cause σ_0 and S to mix.

In most of the cases we will consider later, the σ_0 and S field couple with the same strength to the rest of the Feynman diagram. This makes it useful to define an effective regularized propagator:

$$D_\sigma^R(k) = (D_\sigma(k))_{11} + (D_\sigma(k))_{21} + (D_\sigma(k))_{12} + (D_\sigma(k))_{22}$$

$$= \frac{i}{(2\pi)^4} \frac{\mu_0^2 - M^2}{(k^2 - M^2)(k^2 - \mu_0^2) + 3\lambda_0^2 F_0^2 (M^2 - \mu_0^2)} . \tag{2.4}$$

Similarly, the π_0, P mixing can be considered. The only change in all formulae is the replacing of $3\lambda_0^2 F_0^2$ by $\lambda_0^2 F_0^2$, what yields:

$$D_\pi^R(k) = \frac{i}{(2\pi)^4} \frac{\mu_0^2 - M^2}{(k^2 - M^2)(k^2 - \mu_0^2) + \lambda_0^2 F_0^2 (M^2 - \mu_0^2)} .$$

Finally, we define another useful quantity:

$$D_0^R(k) = \frac{i}{(2\pi)^4} \frac{\mu_0^2 - M^2}{(k^2 - M^2)(k^2 - \mu_0^2)} .$$

3. THE PION PROPAGATOR

Diagrams contributing to the pion propagator $P_\pi(p^2)$ are shown in fig. 3. However, a remark has to be made here. The loop diagrams do in fact stand for a sum of all possible mixing diagrams, which includes also the

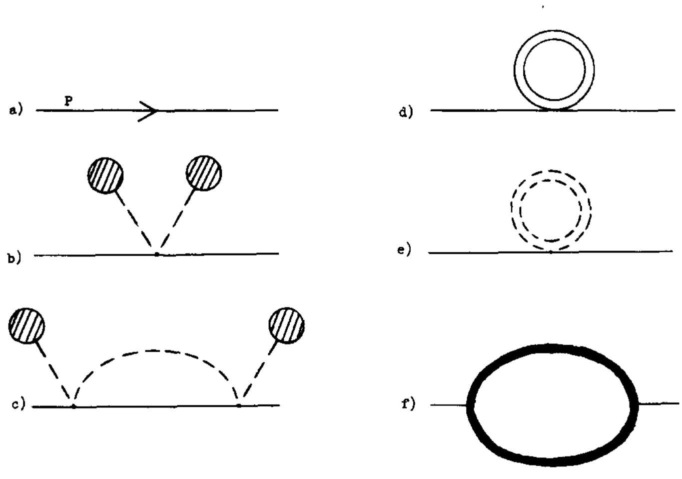

Fig. 3. The proper pion propagator. All additional mixing diagrams are omitted. The double line indicates that the diagrams without mixing are forbidden by normal ordering.

diagrams without any mixing at all. When the latter diagrams are forbidden because of the normal ordering prescription (as is the case in subgraphs d) and e)), a double line is drawn. This will remind us of subtracting out their contributions.

The analytic expressions, corresponding with fig. 3 are listed below:

$$\text{a)} = P_0 = \frac{i}{(2\pi)^4}\frac{1}{p^2-\mu_0^2}\,,$$

$$\text{b)} = (-i)(2\pi)^4\, P_0 \lambda_0^2 F_0^2\, P_0\,,$$

$$\text{c)} = -4(2\pi)^8\, P_0 \lambda_0^4 F_0^2 \int \mathrm{d}^4k\, D_\sigma^{\mathrm{R}}(k) D_\pi^{\mathrm{R}}(k+p)\, P_0\,,$$

$$\text{d)} = -5i(2\pi)^4 P_0 \lambda_0^2 \int \mathrm{d}^4k\, (D_\pi^{\mathrm{R}}(k) - D_0^{\mathrm{R}}(k))\, P_0\,,$$

$$\text{e)} = -i(2\pi)^4 P_0 \lambda_0^2 \int \mathrm{d}^4k\, (D_\sigma^{\mathrm{R}}(k) - D_0^{\mathrm{R}}(k))\, P_0\,,$$

$$\text{f)} = 2P_0\Big[\int \mathrm{d}^4k\, \mathrm{tr}\Big(\gamma_5 \frac{-1}{\not k+\not p-g_0F_0}\gamma_5 \frac{g_0^2}{\not k-g_0F_0}\Big) + \sum_{j=1} \epsilon_j g_j^2 \int \mathrm{d}^4k$$

$$\times\, \mathrm{tr}\Big(\frac{-1}{\not k+\not p-M_{1j}}\frac{1}{\not k-M_{2j}}+\frac{-1}{\not k+\not p-M_{2j}}\frac{1}{\not k-M_{1j}}\Big)\Big] P_0\,. \qquad (3.1)$$

In the last expression we used the notation:

$$M_{1j} = m_j + g_j F_0\,; \qquad M_{2j} = m_j - g_j F_0\,.$$

Finiteness of the integration is obtained by imposing the constraints:

$$\sum_{j=1} \epsilon_j g_j^2 = -\tfrac{1}{2} g_0^2 \,; \qquad \sum_{j=1} \epsilon_j g_j^4 = -\tfrac{1}{2} g_0^4 \,; \qquad \sum_{j=1} \epsilon_j g_j^2 m_j^2 = 0 \,.$$

Working out those integrals leads to two typical functions, S and R, which are defined below, Although integration by parts yields many relations among them, it is convenient not always to reduce the obtained expressions to their most elementary form.

$$S(a_0, a_1, a_2, \ldots; b_0, b_1, b_2, \ldots)$$
$$= \int_0^1 (a_0 + a_1 x + a_2 x^2 + \ldots) \log (b_0 + b_1 x + b_2 x^2 + \ldots) \mathrm{d}x \,,$$

$$R(a_0, a_1, a_2, \ldots; b_0, b_1, b_2, \ldots)$$
$$= \int_0^1 \frac{a_0 + a_1 x + a_2 x^2 \ldots}{b_0 + b_1 x + b_2 x^2 \ldots} \mathrm{d}x \,.$$

It is well-known that renormalization by means of an infinite counterterm is an ambiguous procedure. An additional constraint is required in order to fix δ, the finite part of the counterterm. We choose the following prescription: In the tree approximation $\delta\mu^2 = 0$ and all Z-constants are equal to unity. Moreover, we have

$$\mu_\pi^2 = (\text{mass of pion})^2 = \mu^2 + \lambda^2 F^2 \,,$$

$$\mu_\sigma^2 = (\text{mass of sigma})^2 = \mu^2 + 3\lambda^2 F^2 \,,$$

$$m = (\text{mass of nucleon}) = gF.$$

We now require that, after taking into account the one-loop graphs and after renormalization, the poles of the (three) propagators are not shifted with respect to their tree values. Neither should the normalization of the pion and nucleon propagator be changed. These five constraints fix the finite pieces of all counterterms of the theory. As the σ-particle is assumed to be unstable and never appears in the final states, the normalization of its propagator is irrelevant here.

Finally, in the one-loop diagrams we will replace all bare (= infinite) quantities by their tree values. This prescription does entirely specify our renormalization procedure. From now on, the calculation is straightforward. We first define new scaled variables: $a = \mu_\pi^2/m^2$; $b = \mu_\sigma^2/m^2$; $d = \mu^2/m^2$; $s = p^2/m^2$. Then we work out the integrals, take the regulator masses large, and eliminate λ^2 in favour of g^2 by means of the formula: $\lambda^2 = \frac{1}{2} g^2 (b - a)$. This leads to:

$$c) = -4iP_0^2(\tfrac{1}{2}(b-a)\pi gm)^2\left[2-\log\frac{M^2}{m^2}+S(1;b,a-b-s,s)\right],$$

$$d) = -5iP_0^2(\tfrac{1}{2}(b-a)\pi gm)^2\left[1-\log\frac{M^2}{m^2}-\frac{2d}{b-a}\log d+\frac{2a}{b-a}\log a\right],$$

$$e) = -iP_0^2(\tfrac{1}{2}(b-a)\pi gm)^2\left[3-3\log\frac{M^2}{m^2}-\frac{2d}{b-a}\log d+\frac{2b}{b-a}\log b\right],$$

$$f) = iP_0^2(\pi gm)^2\left[\tfrac{8}{3}s-\tfrac{88}{3}-4sS(1;1,-s,s)\right.$$
$$\left.+\sum_{j=1}\epsilon_j\left(\frac{g_j}{g}\right)^2\left(48\left(\frac{m_j}{m}\right)^2+16\left(\frac{g_j}{g}\right)^2-8s\right)\log\left(\frac{m_j}{m}\right)^2\right]. \quad (3.2)$$

The next step is to define suitable counterterms that make the theory finite, and give the renormalized propagator, $\hat{P}_\pi = P_\pi/Z_B$, the prescribed pole and normalization.

$$\delta\mu^2\left(\frac{4\pi}{gm}\right)^2 = \sum_{j=1}\epsilon_j\left(\frac{g_j}{g}\right)^2\left[48\left(\frac{m_j}{m}\right)^2-8d\right]\log\left(\frac{m_j^2}{\mu^2}\right)-\delta_1,$$

$$Z_B = 1+\left(\frac{g}{4\pi}\right)^2\left[8\sum_{j=1}\epsilon_j\left(\frac{g_j}{g}\right)^2\log\left(\frac{m_j^2}{\mu^2}\right)-\delta_2\right],$$

$$Z_\lambda = 1+\left(\frac{g}{4\pi}\right)^2\left[\frac{32}{b-a}\sum_{j=1}\epsilon_j\left(\frac{g_j}{g}\right)^4\log\left(\frac{m_j^2}{\mu^2}\right)+6(b-a)\log\frac{M^2}{\mu^2}+\delta_5\right]; \quad (3.3)$$

δ_5 is not important for our calculations. δ_1 and δ_2 are given by:

$$\delta_1 = \tfrac{88}{3}-\tfrac{8}{3}a+4aS(1;1,-a,a)+4(2-a)\log d$$
$$+\tfrac{1}{2}(b-a)[\delta_2+\delta_5+5a\log a+b\log b-6d\log d]$$
$$+(b-a)^2[4-3\log d+S(1;b,-b,a)],$$

$$\delta_2 = \tfrac{20}{3}+2(a-2)R(1;1,-a,a)-\frac{(b-a)^2}{a}(1-R(b,a-b;b,-b,a))+4\log d. \quad (3.4)$$

The renormalized propagator is then:

$$\hat{P}_\pi^{-1}(p^2) = (-i)(2\pi)^4m^2(s-a)$$
$$+4i(\pi gm)^2[sS(1;1,-s,s)-aS(1;1,-a,a)+(s-a)$$
$$\times(1+(\tfrac{1}{2}a-1)R(1;1,-a,a))]$$
$$+i(b-a)^2(\pi gm)^2[S(1;b,a-b-s,s)-S(1;b,-b,a)$$
$$-\frac{1}{a}(s-a)(1-R(b,a-b;b,-b,a))]. \quad (3.5)$$

The propagator form factor $d(s)$ is defined by:

$$\hat{P}_\pi(p^2) = \frac{i}{(2\pi)^4}\frac{d(s)}{m^2(s-a)}$$

and has the property $d(s=a) = 1$.

4. THE CONSISTENCY RELATION

The consistency relation is shown in fig. 4. Its analytic expression is given below.

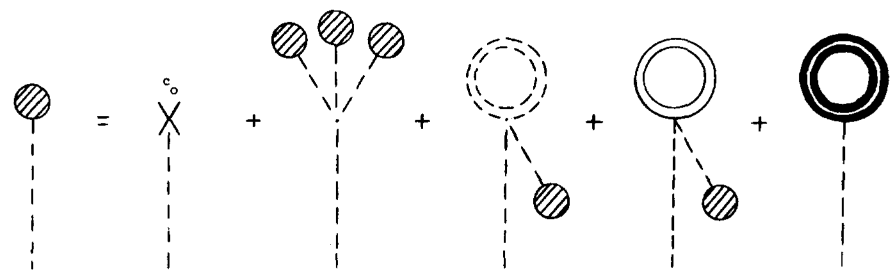

Fig. 4. The consistency relation.

$$F_0 = \frac{c_0}{\mu_0^2} - \frac{\lambda_0^2 F_0^3}{\mu_0^2} - 3\frac{\lambda^2 F}{\mu^2}\left(\int d^4k\,(D_\sigma^R(k) - D_0^R(k))\right.$$
$$\left. + \int d^4k\,(D_\pi^R(k) - D_0^R(k))\right) + \frac{2i}{(2\pi)^4\mu^2}\left[g\int d^4k\,\mathrm{tr}\left(\frac{1}{\not{k}-m}\right)\right.$$
$$\left. + \sum_{j=1} \epsilon_j g_j \int d^4k\,\mathrm{tr}\left(\frac{1}{\not{k}-M_{1j}} - \frac{1}{\not{k}-M_{2j}}\right)\right]. \tag{4.1}$$

Replacing the bare quantities by their actual values (2.2, 3.3 and 3.4), cancels out all infinities and leads to:

$$\frac{c}{Fm^2} = a + \left(\frac{g}{4\pi}\right)^2\left[4a\,(1 - R(1;1,-a,a)) + (b-a)^2\left(2 - b\left(\tfrac{7}{2} - \frac{b}{a}\right)\right.\right.$$
$$\left.\left.\times R(1;b,-b,a)\right) + \left(\tfrac{7}{2}b^2 + \tfrac{1}{2}a^2 - 3ab - \frac{b^3}{a}\right)\ln\frac{b}{a}\right]. \tag{4.2}$$

This can be verified to be consistent with the Ward identity result:

$$\frac{c}{F} = -\frac{i}{(2\pi)^4}\hat{P}_\pi^{-1}(0)\,, \quad \text{or} \quad \frac{c}{F} = \frac{am^2}{d(0)}\,, \tag{4.3}$$

which expresses in fact the Goldstone theorem.

5. THE NUCLEON PROPAGATOR

Diagrams contributing to the nucleon propagator, $S_N(p)$, are shown in fig. 5. Their analytic expressions are given in formula (5.1).

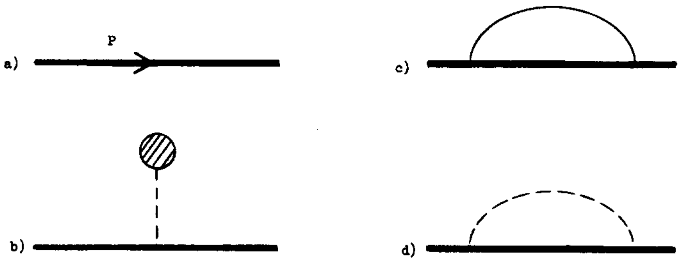

Fig. 5. The proper nucleon propagator.

$$a) = S_o = \frac{i}{(2\pi)^4}\frac{1}{\not p},$$

$$b) = (-i)(2\pi)^4 g_o F_o S_o^2,$$

$$c) = -3i(2\pi)^4 g^2 S_o \int \gamma_5 \frac{d^4k}{\not k + \not p - m} \gamma_5 D_\pi^R(k) S_o$$

$$= 3i\pi^2 g^2 S_o \left[S(m-\not p, \not p; a, 1-a-s, s) + (\tfrac{1}{2}\not p - m)\log\frac{M^2}{m^2} + m - \tfrac{1}{4}\not p\right] S_o,$$

$$d) = (-i)(2\pi)^4 g^2 S_o \int \frac{d^4k}{\not k + \not p - m} D_\sigma^R(k) S_o$$

$$= i\pi^2 g^2 S_o \left[S(-m-\not p, \not p; b, 1-b-s, s)\right.$$

$$\left. + (\tfrac{1}{2}\not p + m)\log\frac{M^2}{m^2} - m - \tfrac{1}{4}\not p\right] S_o . \qquad (5.1)$$

The appropriate counterterms are:

$$Z_F = 1 + \left(\frac{g}{4\pi}\right)^2\left[\delta_3 - 2\log\frac{M^2}{\mu^2}\right],$$

$$Z_g = 1 + \left(\frac{g}{4\pi}\right)^2\left[\delta_4 - 2\log\frac{M^2}{\mu^2}\right],$$

$$\delta_3 = 1 - 2\log d + 3S(1, -1; a, -a, 1) + S(1, -1; b, -b, 1)$$
$$+ 6R(0, 0, 1, -1; a, -a, 1) - 2R(0, 2, -3, 1; b, -b, 1),$$

$$\delta_4 = \delta_3 + 1 + 3S(0, 1; a, -a, 1) - S(2, -1; b, -b, 1), \qquad (5.2)$$

what leads to the finite propagator, $\hat{S}_N = S_N/Z_F$:

$$\hat{S}_N^{-1}(p) = (-i)(2\pi)^4(\not{p}-m) - 3ig^2\pi^2[S(m-\not{p}, \not{p}; a, 1-a-s, s) - S(0, m; a, -a, 1)$$
$$+(\not{p}-m)(S(1,-1;a,-a,1)+2R(0,0,1,-1;a,-a,1))]$$
$$+ig^2\pi^2[S(m+\not{p}, -\not{p}; b, 1-b-s, s) - S(2m, -m; b, -b, 1)$$
$$-(\not{p}-m)(S(1,-1;b,-b,1)-2R(0,2,-3,1;b,-b,1))] . \quad (5.3)$$

6. VECTOR AND AXIAL VECTOR CURRENTS

The infinitesimal transformations:

$$\begin{aligned}
&\psi \to (1+\tfrac{1}{2}i\boldsymbol{\tau}\cdot\boldsymbol{\alpha})\psi , && \psi \to (1-\tfrac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\alpha}\gamma_5)\psi ,\\
&\boldsymbol{\pi} \to \boldsymbol{\pi}+\boldsymbol{\alpha}\times\boldsymbol{\pi} , && \boldsymbol{\pi} \to \boldsymbol{\pi}+\boldsymbol{\alpha}\sigma ,\\
&\sigma \to \sigma , && \sigma \to \sigma-\boldsymbol{\alpha}\cdot\boldsymbol{\pi} ,\\
&\boldsymbol{P} \to \boldsymbol{P}+\boldsymbol{\alpha}\times\boldsymbol{P} , && \boldsymbol{P} \to \boldsymbol{P}+\boldsymbol{\alpha}S ,\\
&S \to S , && S \to S-\boldsymbol{\alpha}\cdot\boldsymbol{P} ,\\
&\varphi_j^1 \to (1+\tfrac{1}{2}i\boldsymbol{\tau}\cdot\boldsymbol{\alpha})\varphi_j^1 , && \varphi_j^1 \to \varphi_j^1-\tfrac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\alpha}\varphi_j^2 ,\\
&\varphi_j^2 \to (1+\tfrac{1}{2}i\boldsymbol{\tau}\cdot\boldsymbol{\alpha})\varphi_j^2 , && \varphi_j^2 \to \varphi_j^2+\tfrac{1}{2}\boldsymbol{\tau}\cdot\boldsymbol{\alpha}\varphi_j^1 ,
\end{aligned} \quad (6.1)$$

generate respectively the vector and axial vector currents:

$$\boldsymbol{V}_\mu = -\overline{\psi}_0\gamma_\mu\tfrac{1}{2}\boldsymbol{\tau}\psi_0 - \boldsymbol{\pi}_0\times\partial_\mu\boldsymbol{\pi}_0 - \sum_{j=1}(\overline{\varphi}_j^1\gamma_\mu\tfrac{1}{2}\boldsymbol{\tau}\varphi_j^1+\overline{\varphi}_j^2\gamma_\mu\tfrac{1}{2}\boldsymbol{\tau}\varphi_j^2)+\boldsymbol{P}\times\partial_\mu\boldsymbol{P} ,$$

$$\boldsymbol{A}_\mu = -i\overline{\psi}_0\gamma_\mu\gamma_5\tfrac{1}{2}\boldsymbol{\tau}\psi_0+(\sigma_0\partial_\mu\boldsymbol{\pi}_0-\boldsymbol{\pi}_0\partial_\mu\sigma_0)+i\sum_{j=1}(\overline{\varphi}_j^2\gamma_\mu\tfrac{1}{2}\boldsymbol{\tau}\varphi_j^1$$
$$-\overline{\varphi}_j^1\gamma_\mu\tfrac{1}{2}\boldsymbol{\tau}\varphi_j^2)-(S\partial_\mu\boldsymbol{P}-\boldsymbol{P}\partial_\mu S) . \quad (6.2)$$

Their charges, $\boldsymbol{Q} = -\int \boldsymbol{V}_0(x)\mathrm{d}^3x$ and $\boldsymbol{Q}^5 = -\int \boldsymbol{A}_0(x)\mathrm{d}^3x$ obey the usual commutation relations of current algebra. Moreover, the regularization does not violate CVC or PCAC, which will be discussed in sect. 8.

We now want to calculate g_A, the axial vector coupling constant. The contributing diagrams are shown in fig. 6. However, subgraph d) shows only a few examples of the full regularization. It has to be pointed out that the regulator part of the axial current contains an additional minus sign. This prevents us from using the effective regularized propagator, defined in formula (2.4). Instead, a careful taking together of the matrix elements of $D_\sigma(k)$ and $D_\pi(k)$ is required. The result is given below.

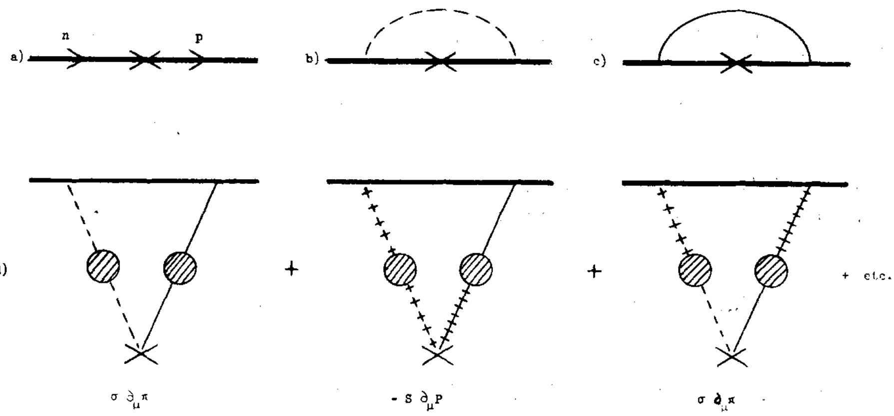

Fig. 6. Coupling of the axial current (represented by a cross) to nucleons. The blob stands for mixing processes.

$$\text{a)} = (-i)(2\pi)^4 Z_{\rm F}\gamma_\mu\gamma_5\,,$$

$$\text{b)} = (-i)(2\pi)^4 g^2 \int \mathrm{d}^4 k D_\sigma^{\rm R}(k)\,\frac{1}{\not k+\not p-m}\,\gamma_\mu\gamma_5\,\frac{1}{\not k+\not n-m}\,,$$

$$\text{c)} = i(2\pi)^4 g^2 \int \mathrm{d}^4 k D_\pi^{\rm R}(k)\gamma_5\,\frac{1}{\not k+\not p-m}\,\gamma_\mu\gamma_5\,\frac{1}{\not k+\not n-m}\,\gamma_5\,,$$

$$\text{d)} = 2(2\pi)^8 g^2 \int \mathrm{d}^4 k\,\frac{M^2+\mu^2-(k-n)^2-(k-p)^2}{M^2-\mu^2}\,(2k-n-p)_\mu$$

$$\times\left[D_\pi^{\rm R}(k-n)D_\sigma^{\rm R}(k-p)\,\frac{1}{\not k-m}\gamma_5 - D_\sigma^{\rm R}(k-n)D_\pi^{\rm R}(k-p)\gamma_5\,\frac{1}{\not k-m}\right]. \qquad (6.3)$$

The contributions of those diagrams to $g_{\rm A}$ are given by

$$\text{a)} = (-i)(2\pi)^4 Z_{\rm F}\,,$$

$$\text{b)} = \tfrac{1}{2}i\pi^2 g^2[\log\frac{M^2}{m^2}+\tfrac{11}{2}-2b-2S(0,\,1;\,b,\,-b,\,1)+2(b-4)R(b,\,1-b;\,b,\,-b,\,1)]\,,$$

$$\text{c)} = -\tfrac{1}{2}i\pi^2 g^2[\log\frac{M^2}{m^2}-\tfrac{5}{2}-2a-2S(0,\,1;\,a,\,-a,\,1)+2aR(a,\,1-a;\,a,\,-a,\,1)]\,,$$

$$\text{d)} = -i\pi^2 g^2[\,2\log\frac{M^2}{m^2}+1+\frac{4}{b-a}\,(S(a,\,-a,\,1;\,a,\,-a,\,1)$$

$$-S(b,\,-b,\,1;\,b,\,-b,\,1))]\,, \qquad (6.4)$$

what leads to the value of $g_{\rm A}$:

$$(g_A - 1)\left(\frac{4\pi}{g}\right)^2 = 2 + \frac{4}{b-a}(S(a, -a, 1; a, -a, 1) - S(b, -b, 1; b, -b, 1))$$
$$+ S(1; b, -b, 1) + S(3, -4; a, -a, 1)$$
$$+ R(0, 0, 6, -7; a, -a, 1) + R(0, 0, 2, -1; b, -b, 1) . \quad (6.5)$$

The calculation of g_V, the vector coupling constant, is very similar and will not be given here. It reproduces the familiar CVC result:

$$g_V = 1 , \quad (6.6)$$

what is crucial for our renormalization (see sect. 8).

7. THE PION-NUCLEON-NUCLEON VERTEX

The next quantity we are interested in, is $G_{\pi NN}$, the pion-nucleon-nucleon coupling constant, which is defined to be the value of the πNN-vertex for all external particles on the mass-shell. This vertex is built up with the diagrams of fig. 7. Here it is no longer possible to analytically carry out the integrations. So we are left with three numerical integrations, represented by G^i, $i = 1, 2, 3$,

$$G^{(1,2,3)}(A, B, C) = -\int_0^1 dx \int_0^x dy\,(1, x, y)[y^2 + q^2x^2 - q^2xy - x(A - B + q^2)$$
$$- y(1 - q^2 + B - C) + A]^{-1} , \quad (7.1)$$

where A, B, C are constants and $q^2 = (n-p)^2/m^2$. One integration is possible by hand; the second, however, has to be done on the computer. The

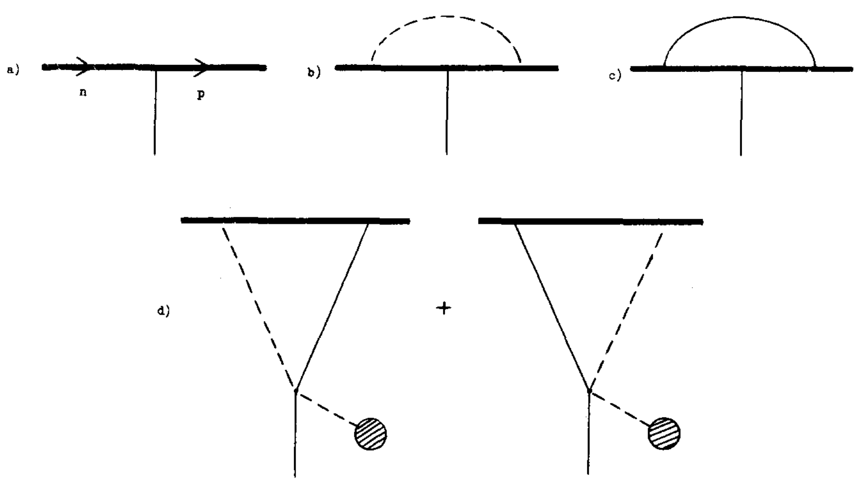

Fig. 7. The pion-nucleon-nucleon vertex.

analytic expressions, corresponding with the diagrams of fig. 7 are:

$$\text{a)} = (2\pi)^4 g Z_g \gamma_5 ,$$

$$\text{b)} = (2\pi)^4 g^3 \int d^4k \frac{1}{\not k + \not p - m} \gamma_5 \frac{1}{\not k + \not n - m} D^R_\sigma(k) ,$$

$$\text{c)} = -(2\pi)^4 g^3 \int d^4k \gamma_5 \frac{1}{\not k + \not p - m} \gamma_5 \frac{1}{\not k + \not n - m} \gamma_5 D^R_\pi(k) ,$$

$$\text{d)} = -2i(2\pi)^8 g^2 \lambda^2 F \int d^4k \Big[D^R_\sigma(k-p) D^R_\pi(k-n) \frac{1}{\not k - m} \gamma_5$$

$$- D^R_\sigma(k-n) D^R_\pi(k-p) \gamma_5 \frac{1}{\not k - m} \Big] . \quad (7.2)$$

For large regulator masses, they can be expressed in terms of the G^i:

$$\text{b)} + \text{c)} = \pi^2 g^3 [2 \log \frac{M^2}{m^2} - 2 - S(1; b, -b, 1) - S(1; a, -a, 1)$$

$$+ 2G^1(1, 1, a) - q^2 G^2(1, 1, a) + (q^2 - 2) G^3(1, 1, a)$$

$$- 2G^1(1, 1, b) - q^2 G^2(1, 1, b) + (q^2 - 2) G^3(1, 1, b)] \gamma_5 ,$$

$$\text{d)} = \pi^2 g^3 (b-a) [-2G^1(b, a, 1) + 2G^2(b, a, 1) - G^3(b, a, 1)$$

$$- 2G^2(a, b, 1) + G^3(a, b, 1)] \gamma_5 . \quad (7.3)$$

Summing up these contributions, putting $q^2 = a$ and dividing by $(2\pi)^4 \gamma_5$ leads to a finite value for $G_{\pi NN}$. When $q^2 \neq a$, the vertex is described by a form factor $K(q^2)$ with $K(a) = 1$.

8. GOLDBERGER-TREIMAN RELATION AND WARD IDENTITIES

The unrenormalized sigma model has three interesting symmetry proper ties: there exists a Goldstone theorem, a conserved vector current (CVC) and a partially conserved axial current (PCAC). We say that the renormalization does not break the symmetry of the model, when those properties are still present after renormalization. That this is so in our calculation can be easily checked.

Formula (4.3), or the consistency relation, is nothing but the Goldstone theorem for the renormalized quantities. The statement of CVC is equivalent with the relation $g_V = 1$, what was already found in formula (6.6). Finally, PCAC is equivalent with the Goldberger-Treiman relation:

$$g_A = f_\pi G_{\pi NN} K(0) d(0)/m , \quad (8.1)$$

where f_π is the pion decay constant, defined by $\langle 0 | A_\mu | \pi(q) \rangle = -i f_\pi q_\mu$. Moreover, $C = f_\pi \mu_\pi^2$. Using this, together with the consistency relation

(4.3), reduces the G.T. relation to:

$$g_A = G_{\pi NN} K(0)/g \ .$$

This formula can be explicitly verified by taking the limit $q^2 \to 0$ in the earlier obtained expression for the πNN vertex (7.3). Intimately related to those three results is the fact that the regularized (axial) vector currents obey the appropriate Ward identities ($q^\mu = n^\mu - p^\mu$):

$$iq^\mu \Gamma^5_\mu(q) = \frac{i}{(2\pi)^4} F \hat{P}^{-1}_\pi(q^2)\, \Gamma^5(q) - i(\hat{S}^{-1}_N(p)\gamma_5 + \gamma_5 S^{-1}_N(n)) \ ,$$

$$iq^\mu \Gamma_\mu(q) = -\hat{S}^{-1}_N(n) + S^{-1}_N(p) \ ,$$

$$-iq^\mu \Lambda_\mu(q) = c + \frac{i}{(2\pi)^4} F \hat{P}^{-1}_\pi(q^2) \ . \qquad (8.2)$$

The Γ^5_μ, Γ_μ, Γ^5 and Λ_μ symbols stand respectively for the proper pA_μn, pV_μn, pπn and $A_\mu \pi$ vertex parts. This last remark is in fact a link between our work and the already mentioned paper by Bessis and Turchetti [6].

9. NUMERICAL CALCULATIONS

The experimental values of the physical quantities related to the Goldberger-Treiman relation are:

$$m = 939.5 \text{ MeV} \ , \qquad \mu_\pi = 139.6 \text{ MeV} \ , \qquad f_\pi = 95.6 \text{ MeV} \ ,$$

$$g_A = 1.24 \ , \qquad G_{\pi NN} = 13.5 \ .$$

These numbers imply a deviation from the exact G.T. relation of 10% (and even 13% when all the errors are taken the other way around). It is possible to fix three parameters of the sigma model in such a way that m, f_π and $G_{\pi NN}$ are reproduced. Varying the fourth parameter leads to a (numerical) relation among g_A and μ_π. In fact, there are always two solutions of the problem for a given μ_π. We only consider here the case with $\mu_\sigma \geqslant \mu_\pi$; the other solution always yields a much smaller sigma mass. Both solutions join together at $\mu_\pi \approx 700$ MeV, beyond which point the parameters of the model become complex.

As a result the obtained value for g_A lies very close to $1.378 = f_\pi G_{\pi NN}/m$, for μ_π ranging from zero to ≈ 200 MeV. For larger masses, g_A keeps growing (fig. 8). The reason is in fact that $d(0)$ is always larger than one, while $K(0) - 1$ is not sufficiently negative to compensate for it. So we are forced to conclude that the here described mechanism alone cannot explain the experimental situation. Finally, in the spirit of the concept of "Continuous Breaking of Chiral Symmetry" [7], we draw in fig. 9 the dependence of various relevant quantities on a symmetry breaking parameter, while keeping μ^2, g, λ^2 fixed. Rather than taking c for that parameter, we chose $F^2/F_G^2 - 1$, where F_G is the vacuum expectation value of the sigma field at the Goldstone point. It has to be mentioned that we kept the

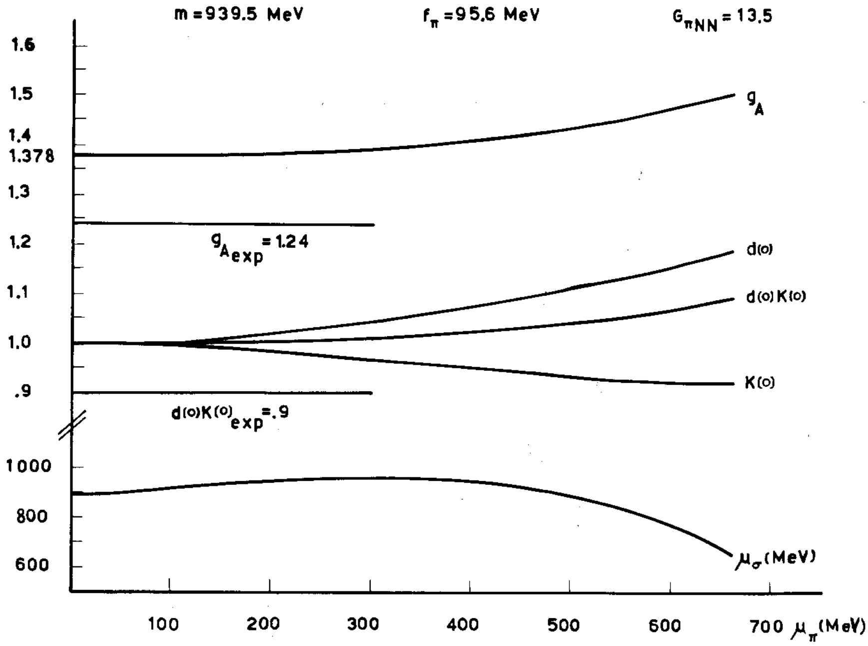

Fig. 8. The dependence of g_A on π_μ when m, f_π, $G_{\pi NN}$ are kept fixed. Only that part of the solution which satisfies $\mu_\sigma \geqslant \mu_\pi$ is drawn.

renormalized parameters constants, and not the unrenormalized ones. In the first case, the normalization of the propagators is fixed, while the value of the counterterms changes; in the latter case, the opposite occurs. The steep dip that g_A and $G_{\pi NN}$ show for small a-values (i.e. $F \approx F_G$) is due to the presence of an "$a \log a$" term. The appearance of such terms in an expansion near the Goldstone point was recently discussed by Li and Pagels [8], who analyzed loop-diagrams with a pion on an internal line. In our calculation, we see that none of the loop-diagrams of fig. 6 or 7 exhibits that behavior. However, graph c) of fig. 5 (the nucleon propagator) goes like "$a \log a$". This graphs contributes to Z_F as well as to Z_g. In this way, both g_A and $G_{\pi NN}$ have an infinite sloop in the symmetry limit and are to be treated with care in perturbation calculations.

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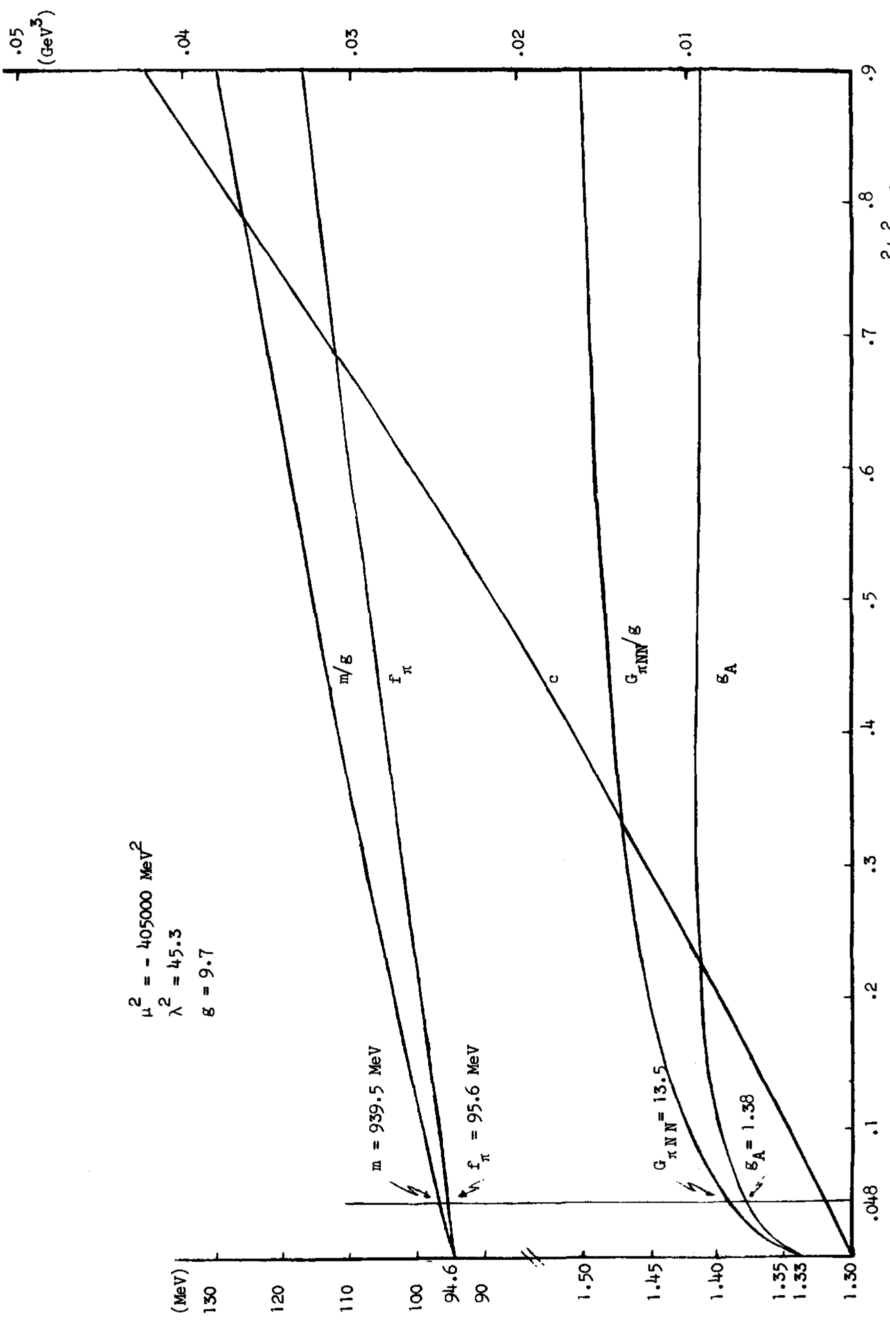

Fig. 9. Continuous symmetry breaking. μ^2, λ^2 and g are constant. The scale on the right corresponds to c.

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