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# GAMMA RADIATION EMITTED BY ORIENTED NUCLEI

## THE INFLUENCE OF PRECEDING RADIATIONS; THE EVALUATION OF EXPERIMENTAL DATA

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**Synopsis**

The angular distribution and polarization of $\gamma$-radiation emitted by oriented nuclei was expressed earlier [1]) with the aid of parameters $f_k$. These parameters characterize the state of orientation of the nuclei from which the radiation is emitted. Here explicit formulae are derived for the change of the parameters $f_k$ if the $\gamma$-radiation under consideration is preceded by a $\beta$ or a $\gamma$-transition.

A discussion is given of the data of physical interest, which may be obtained by the analysis of experimental data on $\gamma$-radiation from oriented nuclei: multipole character of $\gamma$-transitions, nuclear spins and parities, nuclear magnetic moments, data on the Hamiltonian for the $\beta$-interaction and nuclear matrix-elements for $\beta$-decay.

§ 1. *Introduction.* For experiments in which radioactive nuclei are oriented the life time of the nuclei must be rather large. Most half lifes for $\gamma$-transitions are very short, except for the isomeric transitions. However, there are no, or very few, isomeric nuclei which have a suitable half life, a suitable energy (in addition the $\gamma$-radiation must not be entirely converted) and which can be oriented by the present experimental methods. Hence, the situation for observation of the angular distribution and polarization of the $\gamma$-radiation emitted by oriented nuclei will generally be the following:

*a*) the nuclei which are oriented will be $\beta$-radioactive with a sufficiently large life-time.

*b*) the $\beta$-transition is followed by a $\gamma$-transition (fig. 1) or possibly by two or more $\gamma$-transitions in cascade (fig. 2).

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For the calculation of the angular distribution and polarization of the $\gamma$-radiation, the formulae of I, §§ 6–7 [1]) apply. We must only keep in mind that in a $\beta$-transition $j_0 \to j_i$ according to fig. 1 the initial orientation of the nuclei with angular momentum $j_0$ is disturbed. If we consider the distribution of the second $\gamma$-radiation according to fig. 2 we must also account for the change of orientation caused by the emission of the first $\gamma$-radiation.

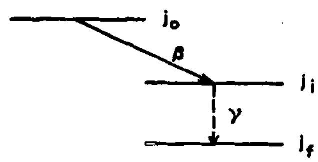


Fig. 1. Decay scheme.

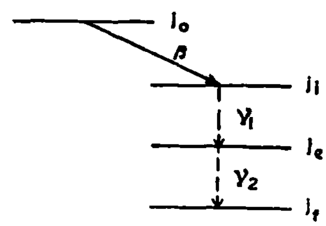


Fig. 2. Decay scheme.

In the next section we shall derive formulae connecting the initial orientation with the orientation after the $\beta$ or $\gamma$-transitions, which precede the observed $\gamma$-radiation. These results together with the formulae of I provide the theoretical formulae, necessary for the discussion of experimental data in this field. Which data of physical interest can be obtained will be discussed in §§ 3 and 4.

§ 2. *Calculation of the change of the orientation parameters $f_k$ by $\beta$ or $\gamma$-transitions.* We assume that there is an axis $\eta$ of rotational symmetry. Then (cf. I, § 2) the orientation of the nuclei with spin $j_0$ is completely characterized by the relative populations $a_{m_0}$ of the sublevels $m_0$ ($\Sigma_{m_0=-j_0}^{j_0} a_{m_0} = 1$). After a $\beta$-transition (or a $\gamma$-transition) $j_0 \to j_i$ the probabilities $a_{m_i}$ are connected with $a_{m_0}$ by (cf. I, § I formulae 3, 4)

$$a_{m_i} = \Sigma_{m_0} a_{m_0} P_{m_0 m_i}, \tag{1}$$

$$\Sigma_{m_i} P_{m_0 m_i} = 1, \tag{2}$$

where $P_{m_0 m_i}$ is the partial transition probability for the transition $(j_0 m_0) \to (j_i m_i)$. For special cases $P_{m_0 m_i}$ are given by the following expressions.

A. $$P^{(0)}_{m_0 m_i} = \delta_{m_0 m_i} \tag{3}$$

This formula is valid for

*a*) allowed $\beta$-transitions with matrix elements

$$|\textstyle\int 1|^2 \text{ or } |\textstyle\int \gamma_5|^2$$

*b*) a forbidden $\beta$-transition with a scalar matrix element.

In these two cases there will often be other matrix elements which play a role in $P_{m_0 m_i}$

B. $$P^{(1)}_{m_0 m_i} = |\langle j_i m_i \, 1M | \, j_i \, 1 j_0 m_0 \rangle|^2 \tag{4}$$

Here $\langle j_i m_i \, 1M | \, j_i \, 1 j_0 m_0 \rangle$ are the transformation coefficients for the addition of angular momenta. Formula (4) applies for

*a*) allowed $\beta$-transitions with matrix element

$$|\int \boldsymbol{\sigma}|^2$$

*b*) first forbidden $\beta$-transitions with matrix elements

$$|\int \boldsymbol{\sigma} \wedge \mathbf{r}|^2, \; |\int \boldsymbol{\alpha}|^2, \; |\int \mathbf{r}|^2 \text{ or } |\int \gamma_5 \mathbf{r}|^2$$

*c*) electric or magnetic $\gamma$-dipole transitions.

C. $$P^{(2)}_{m_0 m_i} = |\langle j_i m_i \, 2 \, M | \, j_i \, | \, 2 j_0 m_0 \rangle|^2 \tag{5}$$

This formula applies for

*a*) first forbidden $\beta$-transitions with matrix element

$$B_{ij}$$

*b*) second forbidden $\beta$-transitions with matrix elements

$$R_{ij}, \; A_{ij}, \; T_{ij}, \text{ or } R^{\gamma}_{ij}$$

*c*) electric or magnetic $\gamma$-quadrupole transitions.

We shall consider a transition for which the formula

$$P^{(L)}_{m_0 m_i} = |\langle j_i \, m_i \, LM \, | \, j_i \, L j_0 \, m_0 \rangle|^2 \tag{6}$$

is valid, of which the above mentioned cases are examples. Since from I, §§ 6–7 it is clear that for the description of the orientation only the parameters $f_k$ are needed, we derive a relation between $f_k(j_0)$ giving the orientation before the transition and $f_k(j_i)$ giving the orientation after the transition. We start with the formula (cf. I, § 2 formulae 25 and 26)

$$f_k(j_i) = w_k(j_i) \, \Sigma_{m_i} \, (-1)^{j_i - m_i} \langle j_i m_i j_i - m_i | \, j_i j_i \, k0 \rangle \, a_{m_i}. \tag{7}$$

According to (1) and (6)

$$a_{m_i} = \Sigma_{m_0} \, a_{m_0} \, |\langle j_i \, m_i \, LM \, | \, j_i L j_0 \, m_0 \rangle|^2. \tag{8}$$

With Racah's definition of the functions $V$ (cf. [2]) formula (16′))

$$\langle j_i \, m_i \, LM | \, j_i L j_0 m_0 \rangle = (-1)^{j_0 + m_0} \, (2j_0 + 1)^{1/2} \, V(j_i L j_0 \, ; \, m_i \, M - m_0), \tag{9}$$

it follows from (7) and (8) that

$$f_k(j_i)=w_k(j_i)\,\Sigma_{m_0m_i}\,a_{m_0}\,(-1)^{j_i-m_i+k}\,(2k+1)^{\frac{1}{2}}\,(2j_0+1)\,V(j_ij_ik;\,m_i-m_i0)\times$$
$$\times\,V(j_iLj_0;\,m_iM-m_0)\,V(j_iLj_0;\,m_iM-m_0). \tag{10}$$

Summation in (10) over $m_i$ and $M$ with formula (41) of Racah's paper [2]) gives the result

$$f_k(j_i)\,=\,w_k(j_i)\,\Sigma_{m_0}\,a_{m_0}\,(2k\,+\,1)^{\frac{1}{2}}\,(2j_0\,+\,1)\,(-\,1)^{L+k-j_i+m_0}\,\times$$
$$\times\,W(j_ij_0j_ij_0;\,Lk)\,V(j_0j_0\,k;\,-\,m_0\,m_0\,0), \tag{11}$$

where the functions $W$ are the Racah coefficients. With the application of (7) for $j_0$ instead of $j_i$ and with the relation (9), formula (11) becomes

$$f_k(j_i)\,=\,w_k(j_i)\,w_k(j_0)^{-1}\,(2j_0\,+\,1)\,W(j_iLkj_0;\,j_0j_i)\,f_k(j_0). \tag{12}$$

From the explicit expressions for $w_k$ (I, § 2 formula 32) the product

$$w_k(j_i)\,w_k(j_0)^{-1}\,=\,(j_0/j_i)^k\left[\frac{(2j_i\,+\,k\,+1)!\,(2j_0-k)!}{(2j_i-k)!\,(2j_0\,+\,k\,+\,1)!}\right]^{\frac{1}{2}} \tag{13}$$

is obtained.

Tables for the Racah coefficient $W(j_iLkj_0;\,j_0j_i)$ are given in several papers [3]) [4]) [5]). With (12) we can directly calculate $f_k(j_i)$ from $f_k(j_0)$ when $j_0$, $j_i$ and $L$ are known.

In some cases the relation (12) becomes very simple. For the transition $j_0 \to j_0 - L$,

$$f_k(j_i)=\frac{j_0^k\,(2j_0-k)!}{(2j_0)!}\,\frac{(2j_i)!}{j_i^k(2j_i-k)!}\,f_k(j_0) \tag{14}$$

or

$$N_k(j_i)\,f_k(j_i)\,=\,N_k(j_0)\,f_k(j_0), \tag{15}$$

where the functions $N_k(j)$ are the same as defined by I, § 6 formula (96a). For $j_0 \to j_0 + L$,

$$f_k(j_i)=\frac{j_0^k\,(2j_0+1)!}{(2j_0+k+1)!}\,\frac{(2j_i+k+1)!}{j_i^k(2j_i+1)!}\,f_k(j_0) \tag{16}$$

or

$$M_k(j_i)\,f_k(j_i)\,=\,M_k(j_0)\,f_k(j_0), \tag{17}$$

with $M_k(j)$ again defined by I, § 6 formula (96*a*).

For $j_0 \rightarrow j_0$ with total angular momentum quantum number of the emitted radiation $L = 1$ we find

$$f_k(j_i) = \left[1 - \frac{k(k+1)}{2j_0(j_0+1)}\right] f_k(j_0). \tag{18}$$

§ 3. *Example of the application of the formulae to the angular distribution of γ-radiation from oriented $^{60}$Co nuclei.* The formulae (I, § 6–7, formulae 90–102) for the angular distribution and polarization of γ-radiation can be applied to actual cases using the results of § 2. We shall illustrate this by dealing with oriented $^{60}$Co nuclei of which the radiation has been investigated experimentally by Daniels et al. [6]), Gorter et al. [7]), Grace [8]) and Poppema et al. [9]).

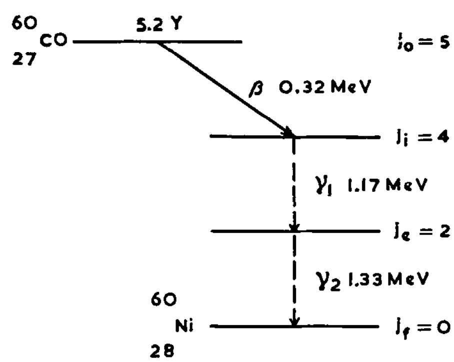


Fig. 3. Decay scheme of $^{60}$Co.

We assume the decay scheme to be as is shown in fig. 3 (cf. Deutsch [10])) where the transitions are electric quadrupole transitions. Then I, § 6, formula 93 for the angular distribution applies i.e.

$$W(\vartheta) = 2(1 - \tfrac{15}{7} N_2(j_i)\, f_2(j_i)\, P_2(\cos\vartheta) - 5N_4(j_i)\, f_4(j_i) P_4(\cos\vartheta)). \tag{19}$$

The formula (19) is written down for the angular distribution of the first γ-radiation. Initially, the orientation parameters $f_k(j_0)$ are given and the $f_k(j_i)$ are calculated for the evaluation of (19). However, for $^{60}$Co the formula (15) may be applied and we need to compute only $N_k(j_0)\, f_k(j_0)$ for $k = 2$ and 4. We have evaluated $f_k(j_0)$ for $j_0 = 5$ as a function of $\beta$ with the assumption

$$a_{m_0} = C \exp. (\beta m_0) \tag{20}$$

which is valid if we have a Boltzmann distribution over equidistant energy levels (with $\beta = \mu B/kTj_0$, $\mu$ nuclear magnetic moment, $B$ magnetic field at the place of the nucleus). The results for $f_1$, $f_2$, $f_3$ and $f_4$ are given in fig. 4 to give an idea of the magnitude of these parameters and their functional dependence on $\beta$.

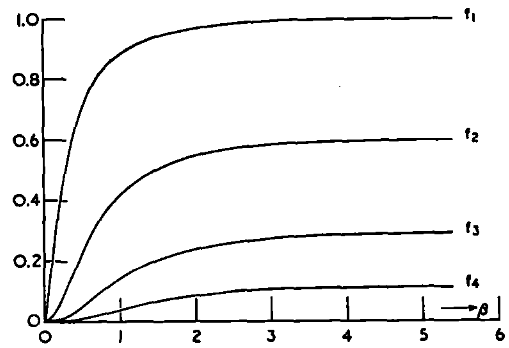


Fig. 4. Diagram of the orientation parameters $f_1$, $f_2$, $f_3$ and $f_4$ as a function of $\beta$.

For orientations of the nuclei, which differ not too much from spherical symmetry, the parameters $f_k$ may be evaluated with the aid of approximate formulae for (20). If e.g. the probabilities $a_{m_0}$ can be approximated by a polynomial of the fourth degree we use the formulae 20, 21 and 22 of. I, § 2.

Using the obtained values for $f_k$ in (19) we obtain numerical results for $W(\vartheta)$. Poppema et al. [9]) compared the results with experimental data and found a rather good agreement. Experimentally, the angular distribution of the first and second $\gamma$-radiation (fig. 3) is measured together since the energy difference of the $\gamma$ quanta cannot be separated easily with scintillation counters. Nevertheless the results obtained from (19) can directly be applied for the following reason. The angular distribution of the second radiation emitted by the nuclei with spin $j_e$ is determined by (19) if $j_i$ is replaced by $j_e$. But again (15) holds and

$$N_k(j_i)\, f_k(j_i) = N_k(j_e)\, f_k(j_e) \tag{21}$$

which gives the result that the first and second $\gamma$-radiation have identical distributions. This will not always be the case for two successive $\gamma$-radiations as follows from the formulae (12) and e.g. (18).

§ 4. *Data of physical interest, which may be obtained from measurements on the $\gamma$-radiation emitted by oriented nuclei.* Depending on the nucleus which is oriented, information about the orientation mechanism or the nuclear process (especially the values of the nuclear spins involved) may be known or still lacking. Generally speaking, knowledge about the unknowns may be obtained if some information is already available, so that the situation is not too complex.

1. Information of *macrophysical or atomic* nature may be obtained if the nuclear physical part is known, especially the nuclear spins involved and the nature of the $\beta$ and $\gamma$ transitions which occur. The measurements of the angular distribution of a $2^L$-pole $\gamma$-radiation provides, in principle, the $f_k$ with $k = 2, 4, \ldots.$ up to the smallest of the numbers $2L$ or $2j_i$. Measurement of linear polarization does not give anything new here, but measurement of the circular polarization would give $f_k$'s with odd $k$. For determination of the $f_k$'s with high $k$, a considerable orientation is required, otherwise they cannot be obtained with any precision. Knowledge of the $f_k$'s obtained from such experiments can be used

*a*) in order to obtain knowledge about the *mechanism of orientation* if this is not known, or

*b*) if this mechanism is known, to obtain the *temperature* from the parameters $f_k$, so that the angular distribution of the $\gamma$-radiation can be used as a *thermometer*.

2. Information concerning *nuclear physical data* can be acquired if the mechanism of orientation is sufficiently known. Data of the following nature may be obtained:

*a*) *The multipole order* of the $\gamma$-radiation may be determined from the angular distribution since $W(\vartheta)$ strongly depends on $L$ (see e.g. I § 6). For the decision as to whether we have *magnetic* or *electric* $2^L$-pole radiation the linear polarization of the $\gamma$-radiation must be measured.

*b*) The values of the *nuclear spins and parities* may follow from the multipole order and the electric or magnetic character of the $\gamma$-transitions. The temperature dependence of the angular distribution will also depend on $j_0$. If $j_i$ and $j_f$ are known, $j_0$ may be determined in this way.

*c*) The *magnetic moment of the initial nucleus* may be obtained if the mechanism of orientation as well as the character of the nuclear process [11]) [12]) is known. If the population of the different $m$-levels is characterized by a Boltzmann factor

$$a_{m_0} = \exp.\ [(\mu B/kTj_0)\ m_0] = \exp.\ (\beta m_0), \tag{22}$$

we can determine from the measured $f_k$ the value of $\beta$ and from $\beta$ the value of $\mu$ if $B/kTj_0$ is known. One obtains only the absolute magnitude of the magnetic moment by measuring the angular distribution (or linear polarization) of the $\gamma$-radiation. The sign could only be obtained by measuring the circular polarization.

This way of measuring magnetic moments of radioactive nuclei is of interest because most radioactive nuclei are not available in sufficient quantities for the application of the usual methods. The magnetic moments of odd-odd nuclei have a special interest because they are mostly radioactive, so that very few data on magnetic moments of odd-odd nuclei exist.

*d*) If the spins and parities involved in the $\gamma$-radiation are known, *information* may be obtained *about the preceding $\beta$-transition*. This will be particularly true if we have an allowed $\beta$ transition for which $j_0 = j_i$. The function $P_{m_0m_i}$ (occurring in (1)) giving the partial transition probability is

$$P_{m_0m_i} = \lambda P^{(0)}_{m_0m_i} + \lambda' P^{(1)}_{m_0m_i} \text{ (with } \lambda + \lambda' = 1) \tag{23}$$

for an allowed $\beta$-transition, where $P^{(0)}_{m_0m_i}$ and $P^{(1)}_{m_0m_i}$ are given by (3) and (5). $\lambda$ and $\lambda'$ are determined by

$$\lambda'/\lambda = [(c_3^2 + c_4^2)/(c_1^2 + c_2^2)]\,[|\textstyle\int \boldsymbol{\sigma}|^2/|\int 1|^2]. \tag{24}$$

We refer for these notions on $\beta$-decay to[13]). $|\int \boldsymbol{\sigma}|^2$ and $|\int 1|^2$ are nuclear matrix elements for $\beta$-decay. If $\lambda$ could be measured and if the ratio of the matrix elements were known one could determine the quantity $(c_3^2 + c_4^2)/(c_1^2 + c_2^2)$, which gives the relative magnitude of Gamow-Teller and Fermi terms in the Hamiltonian for the $\beta$-interaction. If on the other hand, the value of $(c_3^2 + c_4^2)/(c_1^2 + c_2^2)$ were known, one could calculate the ratio of the matrix elements $|\int \boldsymbol{\sigma}|^2$ and $|\int 1|^2$.

We may indicate in somewhat more detail how $\lambda$ could be determined if we measure $f_2(j_i)$ and $f_4(j_i)$ in case of a quadrupole $\gamma$-transition preceded by a $\beta$-transition with $j_0 = j_i$. The connection of $f_2(j_i)$ and $f_4(j_i)$ with $f_2(j_0)$ and $f_4(j_0)$ is given according to (3), (18) and (23) by

$$f_2(j_i) = f_2(j_0) + (1 - \lambda)\,[(j_0^2 + j_0 - 3)/j_0(j_0 + 1)]\,f_2(j_0), \tag{25}$$

$$f_4(j_i) = f_4(j_0) + (1 - \lambda)\,[(j_0^2 + j_0 - 10)/j_0(j_0 + 1)]\,f_4(j_0). \tag{26}$$

We now suppose that the mechanism of orientation is known, then we may assume that $f_4(j_0)$ is a known function of $f_2(j_0)$:

$$f_4(j_0) = F\{f_2(j_0)\}. \tag{27}$$

If $f_2(j_i)$ and $f_4(j_i)$ are measured, the 3 unknowns $f_2(j_0)$, $f_4(j_0)$ and $\lambda$ may be solved from the 3 equations (25), (26), (27). By making the measurements for a number of temperatures, one could reach a higher precision for $\lambda$.

A situation like this is realized for $^{58}$Co for which $j_0 = j_i = 2$; $j_f = 0$. Probably this is an allowed unfavoured transition with $f_{5/2}$—$p_{3/2}$ orbitals for the odd nucleons in $^{58}$Co. (25) and (26) reduce to

$$f_2(j_i) = \tfrac{1}{2}(1 + \lambda)\, f_2(j_0), \tag{28}$$

$$f_4(j_i) = \tfrac{1}{3}(-2 + 5\lambda)\, f_4(j_0). \tag{29}$$

This means that

$$f_4(j_i)/f_2(j_i) = -\left(\tfrac{4}{3}\right) f_4(j_0)/f_2(j_0) \text{ for } \lambda = 0, \tag{30}$$

$$f_4(j_i)/f_2(j_i) = f_4(j_0)/f_2(j_0) \text{ for } \lambda = 1. \tag{31}$$

The strong dependence of $f_4(j_i)/f_2(j_i)$ on $\lambda$ means that $\lambda$ can probably be determined with reasonable accuracy from such measurements.

A determination of $|\int \boldsymbol{\sigma}|^2/|\int 1|^2$ for $^{58}$Co would be of interest to test theories of the nuclear matrix elements for odd-odd nuclei such as proposed by Brysk [14]).

We may add the remark that in all these considerations we assume the nuclei to have no appreciable spin precession after the $\beta$ transition and before the $\gamma$-transition. In the case of $^{58}$Co we have a disintegration by $\beta^+$ transition or by $K$ capture. Since the larger part of the disintegration occurs by $K$ capture it is possible that our assumption does not hold. The disappearance of the $K$ electron may produce a magnetic field strong enough to cause an appreciable spin precession.

In addition to their use for the study of the $\gamma$-radiation itself, sources with oriented nuclei might eventually be suitable sources of linearly or circularly polarized $\gamma$-radiation, which could be used in other experiments.

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## ERRATA

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(22*e*) $\{(2j+1)\,C\}^{-1}$ instead of $C$

(74) $-\alpha_{L\mp 1}$ instead of $\alpha_{L\pm 1}$

(78) $(-1)^{j_i+m_i}$ instead of $(-1)^{j_i-m_i}$

(82) a factor $(-1)^{k}_{-}$ to be added

(84*b*) a factor $-1$ to be added

(84*e*) $\langle LLk2 \mid L1L1\rangle$ instead of $\langle LLk0 \mid L1L1\rangle$.

Reference 12) Physica **18** (1952) 359 ought to be Physica **19** (1953) (to appear shortly).