Anim. Behav., 1995, **50**, 1375–1389

An improved test of linearity in dominance hierarchies containing unknown or tied relationships

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(Received 4 January 1995; initial acceptance 15 February 1995; final acceptance 22 March 1995; MS. number: 4816)

Abstract. Appleby (1983, *Anim. Behav.*, **31,** 600–608) described a statistical test, based on the work of Kendall (1962, *Rank Correlation Methods*), for the significance of linearity in dominance hierarchies. He suggested that unknown relationships should be assigned the value 1/2 and that subsequently the same test procedure can be used. In this paper it is shown that incorrect results are obtained by this method whenever there are unknown relationships. Values of the linearity index are systematically too low. *P*-values can be too high (underestimating the significance) or too low (overestimating), and seem to differ by not much more than a factor two (respectively a half) from the correct *P*-value. An improved method is developed for testing linearity in a set of dominance relationships containing unknown relationships. Furthermore, it is argued that, if one admits the possibility of tied dominance relationships, which should indeed be assigned the value 1/2, Landau's linearity index is to be preferred to Kendall's index. A randomization test is developed for assessing the significance of linearity or non-linearity in a set of dominance relationships containing unknown or tied relationships. The test statistic employed in this testing procedure is based on Landau's linearity index, but takes the unknown and tied relationships into account.

An important topic in social ethology is the analysis of dominance relationships in social groups of individuals. A recent paper by Drews (1993) presents an extensive review of the literature for the purpose of elucidating the concept of dominance. On the basis of the original definition of dominance given by Schjelderup-Ebbe (1922), Drews proposed the following structural definition: dominance is an attribute of the pattern of repeated, agonistic interactions between two individuals, characterized by a consistent outcome in favour of the same dyad member and a default yielding response of its opponent rather than escalation. The status of the consistent winner is dominant and that of the loser subordinate.

In this paper I address the question of how to test for linearity in a set of observed dominance relationships, in particular if this set contains unknown or tied relationships. An unknown dominance relationship (or zero dyad) is the case when the two members of a dyad have not been observed to perform any agonistic interaction towards each other. This observational zero is to be distinguished from a structural zero. If it is structurally impossible for the members of a dyad to have agonistic interactions with each other, this dyad has a structural zero, and a fortiori a dominance relationship between two such individuals is absent. If, on the other hand, the members of a dyad could in principle show agonistic interactions towards each other but were not observed to do so during the observation period, the dyad has an observational zero (also called 'unknown relationship' or 'zero dyad' for short). A dominance relationship is tied if the two individuals in a dyad have performed an equal number of agonistic actions towards each other. I show in this paper that these three different types of dyad (observational zero dyad, structural zero dyad and tied dyad) have not always been clearly distinguished from each other in the literature. A clear distinction, however, is necessary, because each different type requires a different adaptation of the linearity test in a set of dominance relationships.

After the dominance relationships between the individuals in a social group have been

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0003–3472/95/111375+15 $12.00/0

determined, a subsequent question is whether some kind of structure is present in this set of dominance relationships. Mostly, one is interested in how far the dominance relationships can be ordered into a linear hierarchy, although other types of hierarchies could be of interest as well (see e.g. Figure 1 in van Hooff & Wensing 1987). The individuals form a completely linear hierarchy if and only if (1) for every dyad A and B in the group either A dominates B or B dominates A; and (2) every triad is transitive (i.e. not circular), that is, for every three individuals A, B and C the following conditional statement holds: if A dominates B and B dominates C then A dominates C.

Both Landau (1951) and Kendall (1962) have devised an index which can be used to measure the degree of linearity in a set of dominance relationships. Landau derived his index h starting from the concept of 'number of dominated animals', whereas Kendall based his index K (originally denoted by ζ) on the concept of 'circular triad'. Landau's index h is generally used as a descriptive measure in studies analysing dominance hierarchies (e.g. van Hooff & Wensing 1987). Textbooks on ethological methods (Lehner 1979; Martin & Bateson 1993) state that a value of $h>0{\cdot}9$ is generally acknowledged as indicating a strongly linear hierarchy.

As well as a descriptive index which expresses the degree of linearity, a statistical test that gives the significance (the probability) of the observed degree of linearity is also of course very useful in the analysis of hierarchies. Appleby (1983) provided ethologists with just such a test of linearity in a dominance hierarchy, which was originally developed by Kendall (1962). In his paper, Appleby reviewed the literature with regard to this statistical testing method as well as to the problems of how to account for linearity in hierarchies and how to rank the animals if the hierarchy is not completely linear. In the present paper I address the problem of how to test for linearity when there are unknown or tied dominance relationships. Appleby stated that his test procedure can also be applied when information is incomplete (i.e. when there are unknown dominance relationships), although this will bias the test (page 602). He proposed to indicate the unknown relationships by 1/2 in the dominance matrix, and to calculate subsequently the number of circular triads in this matrix using the same formula that is used for a dominance matrix that does not contain zero dyads. I show in this paper that the number of circular triads and the probability value obtained by this method are incorrect. I develop an improved method for testing the significance of linearity in hierarchies containing unknown relationships. Unknown relationships should not be dealt with by putting the value 1/2 into the cells in question of the dominance matrix but here I argue, following but also refining an argument put forward by Nelissen (1986), that tied dominance relationships should be assigned the value 1/2 in the two cells of the matrix.

Landau's linearity index h and Kendall's coefficient K are not completely identical. As Appleby noted, the value of K is slightly smaller than that of h when the number of individuals is even. I show in this paper that this small anomaly is due to the derivation of K being based on the calculation of the number of circular triads, the maximum value of which is constrained when the number of individuals (N) is even. Moreover, I show that circular triads can be sensibly calculated only if there are no tied dyads present, whereas Landau's h is validly defined if tied dyads are present. Therefore, if the possibility of tied dyads is admitted, one should prefer Landau's h to Kendall's K (or, equivalently, the formula of Kendall's K (for N odd) should also be used when N is even).

I also develop a method for testing the significance of linearity in a set of dominance relationships some of which are unknown and some of which may be tied. This statistical test is in the form of a randomization test (see e.g. Edgington 1987) and employs Landau's h as the test statistic, while taking into account the unknown and tied relationships. This method is implemented in the latest version of MatMan, a program for the analysis of sociometric matrices and behavioural transition matrices. The original test procedure of Appleby (1983) has already been implemented in an earlier version of MatMan, which is described by de Vries et al. (1993).

DOMINANCE RELATIONSHIPS WITHOUT TIES

I start this section by recapitulating the test procedure of Appleby/Kendall and then indicate the drawbacks of this method when the dominance matrix contains unknown relationships. Next, I present an improved procedure for testing linearity in a set of dominance relationships some of which are unknown. Finally, I outline a test for linearity in dominance hierarchies containing dyads that are structurally zero (i.e. for which a dominance relationship is impossible). As a general requirement for these tests it is assumed that the relationship between any two individuals predicts nothing about the relationship each has with others. In this section I do not consider sets of dominance relationships that may contain tied dyads. This more general case is treated in the section Dominance Relationships with Ties below.

Appleby/Kendall's Test of Linearity in Hierarchies

The procedure for the test is as follows (from Appleby 1983, page 602).

(1) Construct a matrix of relationships in which the row individual dominant to the column individual is indicated by 1, the column individual dominant to the row individual is indicated by 0 and unknown relationships are indicated by 1/2.

(2) For each individual ($i=1$ to N) obtain the row sum S_i.

(3) Calculate the number of circular triads, d, by

$$d=\frac{N(N-1)(2N-1)}{12}-1/2\Sigma(S_i)^2$$

(4) Obtain the probability of the observed or a lesser value of d by referring to Table I in Appleby's paper (page 603) or (for $N>10$) by calculating a statistic that approximates a chi-squared distribution.

(5) An index K indicating the degree of linearity is defined by:

$$K=1-\frac{d}{maxd}$$

where $maxd$ is the maximum number of circular triads possible in a matrix of size $N\times N$. Kendall (1962, page 156) derived the following formulas for $maxd$:

$$maxd=\frac{1}{24}(N^3-N),\quad N \text{ odd}$$

$$maxd=\frac{1}{24}(N^3-4N),\quad N \text{ even}$$

Although this procedure appears to give satisfactory results, there are some disadvantages connected to it if not all dominance relationships are known. First, the number of circular triads d, thus obtained, is too high in comparison with the unbiased estimate of circular triads. This unbiased estimate of the circular triads is the average number of circular triads in each of the different possible dominance matrices in which the unknown relationships have been changed into a 1–0 or 0–1 dyad. It turns out that there is a simple relation between the unbiased estimate of circular triads (call it d') and the value of d: $d'=d-0{\cdot}25$ (number of unknown dyads). A derivation of this relation is presented in the Appendix. So, the more unknown relationships there are in the matrix the larger is the difference between d and the unbiased estimate of circular triads d'. Thus the test seems to be more or less biased in the direction of not finding linearity, i.e. the test is more or less conservative (however, see below). This conservative bias was also noted by Appleby (page 602). In extreme cases, that is if there are many unknown relationships, d can even become larger than $maxd$, giving rise to a negative degree of linearity K (this was also noted by Appleby, page 602). (See also Example 3 below.)

If this were the only drawback of the procedure then the procedure could easily be corrected, simply by using the unbiased estimate of circular triads instead of the biased one in the assessment of the probability (step 4 above). However, the probability value thus obtained is not the correct probability of the linearity in the set of observed dominance relationships. To obtain the correct P-value of the linearity in the observed dominance matrix one must calculate for each of the different possible dominance matrices in which the unknown dyads have been changed into a 1–0 or 0–1 dyad the P-value of its linearity (i.e. the probability of its value of d). The average of all these P-values is the P-value of the linearity in the observed dominance matrix.

Hierarchies Containing Unknown Relationships

In this section I describe an improved test of linearity that is not troubled by the disadvantages connected to the former procedure when there are unknown relationships. It is assumed in the description of this test that dyads cannot have a tied dominance relationship. That is, for each dyad (A,B) holds: A dominates B or B dominates A or it is unknown who dominates whom. The more general case (unknown and tied relationships possible) is treated in the section Dominance Relationships with Ties below, after the presentation of some examples. The procedure for the test is as follows.

(1) Construct a matrix of relationships in which the row individual dominant to the column individual is indicated by 1, and the column individual dominant to the row individual is indicated by 0. Unknown relationships (zero dyads) have a zero value in both of the cells lying on opposite sides of the diagonal. (These zeros have been indicated by bold printed zeros in the matrix in Fig. 1.)

(2) Let u denote the number of unknown relationships. Construct 2^u matrices each of which contains a unique combination of 1–0 dyads in the place of the unknown relationships. Thus, if individuals A and B have an unknown relationship then cell (A,B) in the new matrix will be assigned a 1 and cell (B,A) a 0 or the other way around. Because there are two possibilities for each unknown relationship to be changed into a dominance relationship (i.e. a 1–0 dyad), the total number of possible matrices thus constructed is 2^u.

(3) For each of the matrices constructed in step 2, calculate the number of circular triads d and determine the associated probability of linearity in this matrix using the method of Appleby/Kendall described above.

(4) Calculate the unbiased estimate of circular triads d' in the original matrix as the average of all the d-values obtained in step 3, and similarly, calculate the P-value of linearity in the original matrix as the average of all the P-values obtained in step 3.

(5) The unbiased estimate of the degree of linearity K' is defined by:

$$K' = 1 - \frac{d'}{maxd}$$

An example will clarify this procedure. Suppose one has obtained a dominance matrix based on observations of a group of seven individuals (Fig. 1). There are four different matrices in which the unknown dyads are changed into 1–0 or 0–1 dyads (Fig. 2). The unbiased estimate of circular triads (d') in the observed matrix is the average of the four d-values: $(0+5+1+5)/4=2{\cdot}75$. Correspondingly, the unbiased estimate of the linearity index ($K'=1-d'/maxd$) is equal to 0·80. The probability (P-value) of the linearity in the observed matrix is obtained by averaging the P-values of the four different possible matrices and thus equals $(0{\cdot}002+0{\cdot}112+0{\cdot}006+0{\cdot}112)/4=0{\cdot}058$. This is to be contrasted with the result that is obtained by means of the Appleby/Kendall procedure. That is, by calculating the number of circular triads for the matrix in which the unknown relationships are indicated by 1/2 (Fig. 3), a value for d of 3·25 is obtained and a corresponding K of 0·77. The probability of obtaining such a value for d or a lesser one is, consulting Table I in Appleby (1983), about equal to 0·04, which is rather different from the value of 0·058 that is obtained by the improved linearity test.

	A	B	C	D	E	F	G
A	*	1	1	1	1	1	***0***
B	0	*	1	1	1	1	1
C	0	0	*	1	1	1	1
D	0	0	0	*	1	1	1
E	0	0	0	0	*	1	***0***
F	0	0	0	0	0	*	1
G	***0***	0	0	0	***0***	0	*

Figure 1. Matrix of dominance relationships in a group of seven individuals; 1: row individual is dominant to column individual; 0: column individual is dominant to row individual; bold printed ***0***: unknown relationship.

Note, by the way, that, for this matrix, the test of Appleby/Kendall does not show a conservative bias, but overestimates the significance of the linearity. In Example 2 below a matrix is presented, the significance of which is underestimated by Appleby's method. Whether the procedure of Appleby/Kendall over- or underestimates the significance of the linearity depends on the number of unknown relationships and on which relationships in the whole set of relationships are unknown.

Two-step randomization test

As an equivalent alternative to the above procedure the following two-step randomization procedure might be used instead. For a general introduction to randomization tests and their usefulness, see Edgington (1987).

(1) Starting from the original matrix, construct a matrix in which all unknown relationships are randomly changed into 1–0 or 0–1 dyads. Next,

(a)

	A	B	C	D	E	F	G
A	*	1	1	1	1	1	1
B	0	*	1	1	1	1	1
C	0	0	*	1	1	1	1
D	0	0	0	*	1	1	1
E	0	0	0	0	*	1	1
F	0	0	0	0	0	*	1
G	0	0	0	0	0	0	*

d=0
K=1
P=0.002

(b)

	A	B	C	D	E	F	G
A	*	1	1	1	1	1	0
B	0	*	1	1	1	1	1
C	0	0	*	1	1	1	1
D	0	0	0	*	1	1	1
E	0	0	0	0	*	1	1
F	0	0	0	0	0	*	1
G	1	0	0	0	0	0	*

d=5
K=0.64
P=0.112

(c)

	A	B	C	D	E	F	G
A	*	1	1	1	1	1	1
B	0	*	1	1	1	1	1
C	0	0	*	1	1	1	1
D	0	0	0	*	1	1	1
E	0	0	0	0	*	1	0
F	0	0	0	0	0	*	1
G	0	0	0	0	1	0	*

d=1
K=0.93
P=0.006

(d)

	A	B	C	D	E	F	G
A	*	1	1	1	1	1	0
B	0	*	1	1	1	1	1
C	0	0	*	1	1	1	1
D	0	0	0	*	1	1	1
E	0	0	0	0	*	1	0
F	0	0	0	0	0	*	1
G	1	0	0	0	1	0	*

d=5
K=0.64
P=0.112

Figure 2. Four different matrices in which the unknown dyads (A,G) and (E,G) have been systematically changed into 1–0 or 0–1 dyads. (a) A>G and E>G; (b) G>A and E>G; (c) A>G and G>E; (d) G>A and G>E. K: Kendall's linearity index; d: number of circular triads; P: probability that this or a smaller number of circular triads is obtained by chance.

calculate the number of circular triads d for this matrix. This value d_0 is one of the possible values of the number of circular triads in the observed matrix if all unknown relationships were known.

(2) Randomize the dominance relationship within each dyad, i.e. for each dyad (A,B) assign randomly the value 1 to cell (A,B) and 0 to cell (B,A) or vice versa.

(3) Calculate the number of circular triads d_r in this randomized matrix.

	A	B	C	D	E	F	G
A	*	1	1	1	1	1	1/2
B	0	*	1	1	1	1	1
C	0	0	*	1	1	1	1
D	0	0	0	*	1	1	1
E	0	0	0	0	*	1	1/2
F	0	0	0	0	0	*	1
G	1/2	0	0	0	1/2	0	*

Figure 3. Matrix of dominance relationships in a group of seven individuals; 1: row individual is dominant to column individual; 0: column individual is dominant to row individual; 1/2: unknown relationship.

(4) If the number of circular triads in this randomized matrix (d_r) is less than or equal to d_0 (the number of circular triads in the observed matrix with unknown relationships randomly changed), increase a counter.

(5) Repeat steps (1)–(4) a number of times (say 10 000), and calculate the left-tailed probability P_l of d as

$$P_l = \frac{\text{Number of times that } d_r \leq d_0}{\text{Number of randomizations}}$$

to obtain an assessment of the significance of the linearity in the set of dominance relationships.

Hierarchies Containing Structural Zero Dyads

If there are some dyads for which there are structural constraints which prevent these individuals from having any agonistic interactions with each other, the set of existing dominance relationships is a subset of the complete set of possible relationships. To test whether there is a significant linearity present in the set of dominance relationships, one has to restrict the data to be used in the test to the existing relationships.

To keep things clear I assume in the description of this test that there are no observational zero dyads present, only structural zero dyads. The procedure outlined here can easily be modified (in a completely analogous way to that described in the previous section) such that it also deals with observational zero dyads.

The procedure for this test is as follows.

(1) Construct a matrix of relationships in which the row individual dominant to the column individual is indicated by 1, the column individual dominant to the row individual is indicated by 0 and non-existing relationships are indicated by −1. (Note that the value of −1 is non-essential and is used only for indicating which relationships are impossible.)

(2) Count the number of circular triads d_0 with the restriction that only those dyads are taken into account that do not have a structural zero.

(3) Determine the probability of the obtained value d_0 or a lesser one by means of a randomization procedure, as follows.

(i) Randomize each existing relationship, i.e. for each dyad (A,B) that does not have a structural zero, assign randomly value 1 to cell (A,B) and 0 to cell (B,A) or vice versa. (ii) Count the number of circular triads d_r in this randomized matrix with the restriction that only those dyads that have a relationship are taken into account. (iii) If the number of circular triads d_r in this randomized matrix is less than or equal to the observed number of circular triads d_0, increase a counter. (iv) Repeat steps (i)–(iii) a number of times (say 10 000), and calculate the left-tailed probability P_l for d_0 by:

$$P_l = \frac{\text{Number of times that } d_r \leq d_0}{\text{Number of randomizations}}$$

to obtain an assessment of the significance of the linearity in the set of existing dominance relationships.

To illustrate the procedure I use the same matrix as that used in the previous test procedure (Fig. 1), but now the zeros of the unknown relationships will be taken to be structural zeros, that is, a dominance relationship does not exist for these dyads. So, the total number of dyads with a dominance relationship is 19 out of the 21 possible relationships. The observed number of circular triads d in this set of 19 dominance relationships is 0. A randomization test in which the dominance relationships are randomized within each dyad excluding the dyads (A,G) and (E,G) results in a left-tailed probability P_l of 0·007. This means that in a 7 by 7 matrix containing 19 known dominance relationships and two structurally impossible relationships the chance probability that there are no circular triads is 0·007. Note that this P-value differs a lot from the P-values obtained above by means of Appleby/Kendall's procedure (P=0·04) and the improved procedure for testing linearity in dominance data containing unknown relationships (P=0·058). This shows that it is important to distinguish structural zero dyads from observational zero dyads.

Iverson & Sade (1990) discussed several statistical methods for the analysis of dominance hierarchies. In the introduction to their paper they used a combinatorial approach to analyse the dominance matrix presented in Appleby's (1983) paper. This combinatorial procedure is in fact equivalent to the test procedure just described. Appleby's 7 by 7 matrix contains eight zero dyads; the remaining 13 non-zero dyads are represented by Iverson & Sade as a directed graph (digraph), thus effectively excluding the eight dyads with a null count. This reduced digraph of Appleby's dominance data is acyclic, i.e. it does not contain circular triads. The total number of possible, different digraphs with 13 edges (dyads) is 2^{13}. By counting the exact number of all possible acyclic dominance digraphs (864), Iverson & Sade obtained the exact probability ($864/2^{13}$) for an acyclic digraph such as Appleby's to occur by chance. In this combinatorial approach zero dyads are excluded, although these null counts are not structural. In Example 2 below, I analyse Appleby's dominance matrix in a different way, that is, by means of the test procedure for a dominance hierarchy containing unknown relationships. So, there, the zero dyads in Appleby's data are taken to be observational zeros and not structural zeros.

EXAMPLES

In this section I first present a very extreme example of a dominance matrix, namely one in which all dominance relationships are unknown. This exceptional matrix may help in elucidating the above proposed linearity test for dominance matrices containing observational zero dyads. Next, I reanalyse the matrix used by Appleby (1983) to illustrate his test procedure, to illustrate the use of the improved method for testing linearity in dominance matrices with unknown relationships.

Example 1

The first example illustrates that the procedure of assigning 1/2 to unknown relationships followed by a calculation of d according to the procedure of Appleby/Kendall leads to incorrect

	A	B	C	D	E	F	G
A	*	0	0	0	0	0	0
B	0	*	0	0	0	0	0
C	0	0	*	0	0	0	0
D	0	0	0	*	0	0	0
E	0	0	0	0	*	0	0
F	0	0	0	0	0	*	0
G	0	0	0	0	0	0	*

Figure 4. Dominance matrix that merely contains unknown relationships (observational zero dyads).

results. Consider a dominance matrix containing merely zero dyads (see Fig. 4). Using the procedure of Appleby/Kendall, the number of circular triads d equals 14, which is the maximum possible number of circular triads in a 7 by 7 matrix. This corresponds to a K-value of zero. The probability that d will attain or exceed the value 14 is 0·001 (according to Appendix Table 9 in Kendall 1962), which would lead to the conclusion that this set of unknown dominance relationships is extremely non-linear. This is evidently a false conclusion.

In contrast, the unbiased estimate d' for this matrix is $d-0{\cdot}25$(number of zero dyads)$=14-0{\cdot}25\times21=8{\cdot}75$, which leads to the correct conclusion that there is neither significant linearity nor significant non-linearity present in these data. For this extreme matrix the value of d' can also be obtained in a different way. Because none of the dominance relationships is known, the best estimate one can have of the number of circular triads is evidently the expected number of circular triads, $E(d)$, in a 7 by 7 matrix. This value can be calculated by means of the following formula (Kendall 1962): $E(d)=N(N-1)(N-2)/24=7\times6\times5/24=8{\cdot}75$. Correspondingly, the correct index of linearity for this matrix is: $K'=1-d'/maxd=1-8{\cdot}75/14=0{\cdot}375$.

Example 2

The next example involves the matrix used by Appleby (1983) to illustrate his test procedure. He presented a matrix (see Fig. 5) of dominance relationships in a group of seven young red deer, *Cervus elaphus*, stags containing eight unknown relationships (here indicated by bold printed zeros, whereas Appleby used the value 1/2 in his Fig. 2). The number of circular triads of this matrix calculated according to the formula

	BST4	COC4	CR14	GRE4	RG14	TA24	TR34
BST4	*	1	1	1	1	1	1
COC4	0	*	1	***0***	***0***	0	0
CR14	0	0	*	***0***	***0***	0	***0***
GRE4	0	***0***	***0***	*	0	***0***	***0***
RG14	0	***0***	***0***	1	*	0	***0***
TA24	0	1	1	***0***	1	*	1
TR34	0	1	0	***0***	***0***	***0***	*

Figure 5. Matrix of dominance relationships between young red deer stags (from Appleby 1983); 1: row individual is dominant to column individual; 0: column individual is dominant to row individual; bold printed ***0***: unknown relationship.

presented in step 3 of the Appleby/Kendall procedure is 6. The probability of a d less than or equal to 6 is 0·198 (Appendix Table 9 in Kendall 1962). The corresponding linearity index equals 0·57. With the improved method an unbiased estimate of circular triads d' of 4 is obtained with a corresponding K' of 0·71. The P-value obtained by the two-step randomization test described above is 0·099, so is much smaller than that obtained by the old method.

If the unknown relationship of the dyad (GRE4, TA24) were known, say for instance TA24 dominated GRE4, then the Appleby/Kendall procedure would result in a d-value of 4·5 (with a corresponding K of 0·68) and the probability of such a d-value or a lesser one would be about 0·08 (Table I in Appleby 1983). In contrast, the improved method yields a d'-value of 2·75 (with a corresponding K' of 0·80) and the probability of the linearity in this dominance matrix is 0·038 (obtained by means of the two-step randomization procedure).

LANDAU'S *h* VERSUS KENDALL'S *K*

Before I treat the general case of sets of dominance relationships which may contain unknown relationships as well as tied dyads, it is necessary first to elucidate and solve the anomaly between Landau's linearity index h and Kendall's coefficient K. The linearity index h is defined by Landau (1951) as:

$$h=\frac{12}{N^3-N}\sum_{i=1}^{N}[V_i-(N-1)/2]^2$$

where V_i is the number of individuals dominated by the individual i; that is, V_i is the sum of row i.

Nelissen (1986) proposed an adaptation of Landau's index h for dominance hierarchies containing individuals with tied rank numbers (which is not the same as tied dominance relationships). He proposed that for an individual i with the same rank number as others, not only the subordinates of individual i must be counted in V_i but also the group members with the same dominance status as i, each with value 1/2. If two individuals i and j are equally dominant over each other (that is, if they have a tied dominance relationship) it is reasonable to increase both V_i and V_j with the value 1/2. So, I concur with Nelissen's proposal, insofar as the equality of the dominance status of the two individuals relative to each other is solely based on the agonistic interactions exchanged between these two individuals. The rank numbers of the two individuals, however, should not be used to decide whether individuals i and j are equally dominant over each other, lest the dominance relationship (which is on the level of the dyad) is confused with the dominance rank number (which is an individual attribute derived from the assumed linear group structure). Two individuals that have a tied dominance relationship (showing from their equal exchange of agonistic interactions) could very well have different rank numbers depending on the number of individuals they dominate or are dominated by.

So, if two individuals i and j are equally dominant over each other, the number of individuals dominated by i (V_i) should be increased by 1/2 and equally the number of individuals dominated by j (V_j) should also be increased by 1/2. In other words, the cells (i,j) and (j,i) should both be assigned the value 1/2, before h is calculated according to the above formula.

	A	B	C	D	E	F
A	*	1/2	1/2	1/2	1/2	1/2
B	1/2	*	1/2	1/2	1/2	1/2
C	1/2	1/2	*	1/2	1/2	1/2
D	1/2	1/2	1/2	*	1/2	1/2
E	1/2	1/2	1/2	1/2	*	1/2
F	1/2	1/2	1/2	1/2	1/2	*

Figure 6. Dominance matrix that merely contains tied relationships.

The following example pertains to a matrix that merely contains tied dominance relationships. This exceptional matrix will be helpful in solving the anomaly between Landau's *h* and Kendall's *K*.

Example 3

Figure 6 shows the dominance matrix for a set of six individuals each of which has tied relationships with every other individual. The value of *h* for this matrix is 0, indicating that the set of tied dominance relationships is completely non-linear, which is indeed a correct conclusion.

When this same matrix is submitted to the procedure proposed by Appleby/Kendall, the following results are obtained. The number of circular triads d equals 8·75, which is larger than the maximum possible number of circular triads, 8, in a 6 by 6 matrix. This corresponds to a negative K-value of $-0{\cdot}09$ which is evidently incorrect. This peculiar result finds its cause in the fact that Kendall (1962) derived his formulas under the assumption that there are no tied relationships and that for each dyad (A,B) holds: A dominates B or B dominates A. Under this strict assumption Kendall showed that if N (the number of individuals) is odd the maximum number of circular triads is $\frac{1}{24}(N^3 - N)$, and if N is even the maximum number of circular triads is $\frac{1}{24}(N^3 - 4N)$. Based on these findings, he defined his coefficient of consistence (measuring the degree of linearity) by the equations:

$$K = 1 - \frac{24d}{N^3 - N}, \quad N \text{ odd}$$

$$= 1 - \frac{24d}{N^3 - 4N}, \quad N \text{ even}$$

where d is the observed number of circular triads.

If, however, tied dominance relationships are present, one runs into the difficulty that the concept of circular triad is not applicable to a triad of individuals that contains one or more tied dyads. So, if one admits the possibility of tied dominance relationships one should base the degree of linearity on the concept of 'number of dominated individuals' rather than on the concept of 'circular triad'. That is, if ties are admitted one should prefer Landau's *h* to Kendall's *K*. However, evidently, Kendall's *K* (for *N* odd) is identical to Landau's *h*; therefore one could just as well say that, if ties are admitted the formula of Kendall's *K* (for *N* odd) should be used in all cases, irrespective of the number of individuals.

DOMINANCE RELATIONSHIPS WITH TIES

In this section I describe a linearity test for the general case that the set of dominance relationships contains unknown relationships and that tied relationships are admitted (although not necessarily present in the observed set of dyads). Finally, I indicate how a randomization test might be developed for the most general case possible where tied dyads are admitted and both observational and structural dyads are present in the data.

An Improved Test of Linearity

The procedure for the test is as follows.

(1) Construct a matrix of relationships in which the row individual dominant to the column individual is indicated by 1, the column individual dominant to the row individual is indicated by 0. For two individuals with a tied dominance relationship the cells in question get the value 1/2. Unknown relationships (zero dyads) have a zero value in both of the cells lying on opposite sides of the diagonal. (Note that the value zero is only used here for indicating which relationships are unknown. These zero values as such do not play a role in the calculations.)

(2) Next, perform the following two-step randomization procedure to obtain an unbiased estimate h' of Landau's index h as well as the probability that this value of h' will be attained or exceeded by chance.

(i) Starting from the dominance matrix, construct a matrix in which all unknown relationships are randomly changed into 1–0 or 0–1 dyads. Next, calculate the value of Landau's h for this matrix:

$$h = \frac{12}{N^3 - N} \sum_{i=1}^{N} [V_i - (N-1)/2]^2$$

where V_i is the sum of row i and N is the number of individuals. This value is one of the possible values of Landau's h for the observed matrix if all unknown relationships would have been known. This value of h will be called h_0. (ii) Randomize the dominance relationship within each dyad, i.e. for each dyad (A,B) assign randomly the value 1 to cell (A,B) and 0 to cell (B,A) or vice versa. (iii) Calculate the value of h for this randomized matrix. This value of h will be called h_r. (iv) If h_r is larger than or equal to h_0 (the h-value for the observed matrix with unknown relationships randomly changed), increase a counter. (v) Repeat steps (i)–(iv) a number of times (say 10 000), and calculate the right-tailed probability P_r by:

$$P_r = \frac{\text{Number of times that } h_r \geq h_0}{\text{Number of randomizations}}$$

An unbiased estimate of Landau's h (h') for the original matrix can be obtained by averaging all the h_0-values. The value of P_r gives the probability that the degree of linearity in the original dominance matrix as expressed by the value of h' results from random processes. One can also calculate the left-tailed probability

$$P_l = \frac{\text{Number of times that } h_r \leq h_0}{\text{Number of randomizations}}$$

which gives the probability that a random dominance matrix has a degree of linearity less than or equal to the linearity in the observed dominance matrix. In this way one can test whether a dominance matrix is significantly non-linear.

Dominance Data with Observational, Structural Zero and Tied Dyads

Finally, one has to consider the most general case possible. Can we test for linearity in a set of dominance relationships for which tied dyads are admitted and which contains both observational and structural zero dyads? I have shown above (in Hierarchies Containing Structural Zero Dyads) that a test of linearity for an incomplete set of relationships can be based on a direct counting of the number of circular triads d, while skipping the structural zero dyads in the counting procedure. If, however, there are tied dyads present it is not possible anymore to count d, because a circular triad is only defined for a triad of individuals that does not contain a tied dyad. Therefore, a linearity test appears not to be possible as soon as the set of dominance relationships contains both tied dyads and structural zero dyads. It might be possible, however, to devise a linearity test by taking the following unnormalized statistic h^* as test statistic.

$$h^* = \sum_{i=1}^{N} [V_i - E(V_i)]^2$$

where

$$V_i = \frac{\text{The number of animals dominated by } i}{\text{The total number of animals that can possibly be dominated by } i}$$

and $E(V_i)$=the total number of animals that can possibly be dominated by i divided by two. Using this test statistic a completely general two-step randomization test can be developed in exactly the same way as described above.

FURTHER EXAMPLES

Example 4

In the paper about MatMan (Table 1 in de Vries et al. 1993) another example of a dominance matrix is presented containing the frequencies of avoiding behaviour among nine geldings in a herd of 31 Icelandic and other horses, *Equus caballus* (see van Dierendonck et al., in press for a complete analysis of dominance in this herd of Icelandic horses). Figure 7 presents the matrix of dominance relationships derived from this interaction frequency matrix: a 1 means that the column individual presented less avoidance behaviour towards the row individual than the other way around. This matrix contains five unknown relationships (i.e. no exchange of avoidance behaviour) and one tied dyad. The number of circular triads d in this matrix is 10·75, with a corresponding K of 0·64. By consulting Table I in Appleby (1983) one obtains a P-value of 0·019 for this or a lesser value of d. By contrast, using the randomization test of linearity in hierarchies containing unknown or tied relationships, a P-value is

	B	C	D	F	H	M	O	S	T
B	*	0	1	1	1	1	1	1	**0**
C	1	*	0	0	1	1/2	0	0	**0**
D	0	1	*	1	1	1	1	1	1
F	0	1	0	*	1	1	1	0	1
H	0	0	0	0	*	**0**	0	0	**0**
M	0	1/2	0	0	**0**	*	0	0	1
O	0	1	0	0	1	1	*	0	**0**
S	0	1	0	1	1	1	1	*	1
T	**0**	**0**	0	0	**0**	0	**0**	0	*

Figure 7. Matrix of dominance relationships between nine geldings (from de Vries et al. 1993); 1: row individual is dominant to column individual; 0: column individual is dominant to row individual; bold printed **0**: unknown relationship; 1/2: tied dominance relationship.

	Yet	Pluis	Ans	Sina	Binjei	Eliza	Getti
Yet	*	3	10	4	2	6	10
Pluis	0	*	**0**	1	2	3	1
Ans	0	**0**	*	**0**	**0**	1	13
Sina	0	0	**0**	*	**0**	**0**	8
Binjei	0	0	**0**	**0**	*	**0**	**0**
Eliza	0	0	1	**0**	**0**	*	**0**
Getti	0	0	0	0	**0**	**0**	*

Figure 8. Matrix of dyadic displacement frequencies of seven female wild orang-utans (S. Utami & S. Wich, unpublished data).

obtained of 0·014. The unbiased estimate h' of Landau's h is 0·68. So, in this example there is only a slight difference between the P-values obtained by the old and the improved method.

Example 5

In 1993/1994 Suci Utami and Serge Wich recorded displacement behaviour in seven female wild orang-utans, *Pongo pygmaeus*, living in the forests of Sumatra. Figure 8 presents the matrix of dyadic displacement frequencies for these animals (unpublished data). The matrix contains eight zero dyads and one tied dyad. Using the test procedure of Appleby (1983), the number of circular triads is found to be 5·25, which corresponds to a Landau's h index of 0·63. The associated probability is 0·12 (according to Table I in Appleby's paper). By contrast, if we use the improved test of linearity a much lower P-value of 0·045 is obtained and a higher h'-value of 0·77 is found (based on the two-step randomization procedure using 10 000 randomizations).

This real example already gives some indication of the possible size of the difference in P- and h-values. In the next section I analyse a series of artificially constructed matrices by means of Appleby's test and the improved linearity test to obtain a more extended view of the size of these differences.

53 Example Matrices

To get some idea of the size of the differences between the P-values obtained by the test procedure of Appleby (1983) and the corresponding values obtained by the randomization test procedure, I subjected 53 dominance matrices with different numbers of unknown relationships to both methods. Table I presents the P-values (P_{old}) and h-values (values of Landau's h index) obtained by Appleby's method alongside the P-values (P_{new}) and h'-values that resulted from application of the randomization test, using 10 000 randomizations in each case. These example matrices were constructed subject to the

Table I. *P*-values and *h*-values of 53 matrices with selected numbers of unknown relationships

Number of unknown relationships	N = 6				N = 7				N = 8				N = 9			
u	P_{old}	h	P_{new}	h'	P_{old}	h	P_{new}	h'	P_{old}	h	P_{new}	h'	P_{old}	h	P_{new}	h'
1	0·03	0·97	0·02	1·0	0·029	0·81	0·038	0·82	0·020	0·73	0·034	0·74	0·014	0·66	0·027	0·67
	0·10	0·80	0·11	0·83	0·060	0·73	0·105	0·75	0·034	0·68	0·054	0·69	0·035	0·59	0·056	0·60
2	0·044	0·91	0·032	0·97	0·017	0·86	0·014	0·89	0·020	0·73	0·052*	0·75	0·009	0·70	0·024*	0·72
	0·10	0·80	0·072	0·86	0·069	0·71	0·108	0·75	0·043	0·66	0·074	0·68	0·019	0·63	0·043*	0·65
3	0·18	0·69	0·14	0·77	0·017	0·86	0·012	0·91	0·023	0·71	0·042	0·75	0·011	0·68	0·036*	0·70
					0·040	0·77	0·034	0·82	0·033	0·68	0·062	0·71	0·019	0·63	0·041*	0·66
4					0·033	0·79	0·017	0·86	0·037	0·67	0·048	0·72	0·030	0·60	0·065*	0·63
					0·080	0·70	0·049	0·77	0·094	0·57	0·144	0·62	0·095	0·50	0·138	0·53
6					0·033	0·79	0·016†	0·89	0·033	0·68	0·039	0·75	0·067	0·53	0·084	0·58
					0·092	0·68	0·041†	0·79	0·050	0·64	0·029	0·71	0·067	0·53	0·050	0·58
8					0·069	0·71	0·029†	0·86	0·063	0·62	0·024	0·71	0·060	0·55	0·035	0·62
					0·155	0·61	0·062†	0·75	0·094	0·57	0·055	0·67	0·067	0·53	0·050	0·60
10					0·175	0·59	0·074†	0·77	0·037	0·67	0·017†	0·79	0·045	0·58	0·033	0·66
									0·094	0·57	0·057	0·69	0·060	0·51	0·055	0·60
12									0·063	0·62	0·029†	0·76	0·067	0·53	0·032†	0·63
									0·094	0·57	0·051	0·71	0·095	0·50	0·040†	0·60
14									0·094	0·57	0·038†	0·74	0·038	0·58	0·015†	0·70
													0·138	0·47	0·052†	0·58

P_{old} and h: P-value and value of Landau's h obtained by means of the test procedure of Appleby (1983). P_{new} and h': P-value and value of Landau's h' obtained by means of the randomization test (10 000 randomizations).

*Old P-value is less than half of the new P-value: P_{old} overestimates the significance level.

†Old P-value is more than two times the new P-value: P_{old} underestimates the significance level.

restriction that the P_{old}-values lie between about 0·01 and 0·15. Also, some care has been taken to construct matrices that show rather large differences between the P_{old} and P_{new} values, with the express purpose to obtain some idea of the maximum possible difference.

The first thing to note is that the h-value is systematically too low compared to the h'-value; and that the difference between h and h' becomes larger with increasing number of unknown relationships u. This is due to the fact that h' and h are directly related to each other as follows:

$$h' = h + \frac{6}{N^3 - N} \times u$$

The derivation of this relation between h' and h is quite similar to the derivation of the relation $d' = d - 0{\cdot}25u$ which holds between d' and d (see Appendix).

The next thing to note is that P_{old} overestimates the significance level if the matrix contains only few unknown relationships. The new P-values appear to differ by not much more than a factor two from the old P-values (an exception is the fifth matrix in the N=9 column, for which P_{old}=0·011 and P_{new}=0·036). In contrast to this overestimation error, P_{old} underestimates the significance level if the matrix contains relatively many unknown relationships. Here also, it appears that P_{old} and P_{new} differ by not much more than a factor two from each other. The largest differences are found for matrices of size 8 and 9 that contain many unknown relationships. For instance, for one 9 by 9 matrix with 12 zero dyads it was found that P_{old}=0·095 whereas P_{new}=0·040. Limited experience with the improved linearity test applied to larger matrices indicates that more or less the same differences in P- and h-values occur for these matrices, although the P-values from the approximate chi-squared tests (for N>10) seem to differ less from the correct P-values than was found above for the smaller matrices.

We may conclude from these findings that, if Appleby's test produces a P-value below 0·01 one may, in all probability, conclude for significant linearity (i.e. the correct P-value is below 0·05). Similarly, if Appleby's test produces a P-value above 0·15 one may conclude there is no significant linearity present (i.e. the correct P-value is above 0·05). However, if a P-value between 0·01 and 0·15 is obtained or if one is interested to know the value of the linearity index h', it is worthwhile to apply the randomization test to obtain the correct P- and h'-values.

CONCLUSION

In this paper I distinguished between four different types of dyads: (1) dyads with a known non-tied dominance relationship (1–0 or 0–1 dyads); (2) dyads with a tied dominance relationship (1/2–1/2 dyads); (3) dyads with an unknown dominance relationship, also called observational zero dyads (0–0 dyads); (4) dyads for which it is structurally impossible to perform any agonistic interactions towards each other, also called structural zero dyads. The last three types of dyads have not been clearly distinguished from each other in the literature. The presence of dyads of these last three types has different consequences for the way the linearity in a set of dominance relationships should be statistically tested. In this paper several results have been derived that pertain to this statistical testing problem. These results can be summarized in the following series of conclusions.

(1) If tied dominance relationships cannot occur in the set of dominance relationships and if all relationships are known (i.e. for each dyad (A,B) holds: A dominates B or B dominates A) then the degree of linearity is given by Kendall's K (according to the two formulas presented on page 602 of Appleby 1983) and the significance of the linearity can be determined by means of the procedure presented in Appleby (1983).

(2) If tied dominance relationships cannot occur in the set of dominance relationships and if some relationships are unknown (i.e. some dyads are zero dyads) then the degree of linearity is given by the index K'. K' is the average of all the Kendall K values calculated for the set of dominance matrices in which the zero dyads are systematically changed into 1–0 or 0–1 dyads. The significance of the linearity can be obtained by calculating the average of the P-values of this set of modified dominance matrices. Alternatively, an equivalent two-step randomization procedure can be used to assess the probability of the observed degree of linearity.

(3) If tied dominance relationships cannot occur and if the set of dyads contains dyads for which a dominance relationship is structurally impossible, the significance of the linearity can be assessed by

means of a randomization test that uses the number of circular triads as the test statistic. The number of circular triads is counted directly from the set of existing dominance relationships thereby skipping the structurally impossible relationships.

(4) If tied dominance relationships are admitted in the set of dominance relationships (although the actual set of relationships observed may not contain any tied dyads at all) and if all relationships are known (i.e. for each dyad (A,B) holds: A dominates B or B dominates A or A and B are tied) then the degree of linearity is given by Landau's h. Equivalently, the formula of Kendall's K for odd number of individuals can be used irrespective of the actual number (odd or even) of individuals.

(5) The general case is when tied dominance relationships are admitted and when not all relationships are known. For this case the degree of linearity is given by the index h'. h' is the average of all the Landau h values calculated for the set of dominance matrices in which the zero dyads are systematically changed into 1–0 or 0–1 dyads (while the tied relationships are assigned the value 1/2 throughout). The following relation holds between h' and h:

$$h' = h + \frac{6}{N^3 - N} \text{ (number of unknown dyads)}$$

where h is the value of Landau's h calculated for the matrix in which the zero dyads have been assigned the value 1/2 (according to the procedure of Appleby 1983). The significance (P-value) of the linearity or non-linearity can be obtained by means of a two-step randomization procedure.

(6) If tied dyads are admitted and the set of dominance relationships contains both observational and structural zero dyads, an unnormalized statistic h^*, which takes the impossible relationships into account, might be used instead of h in the test procedure of conclusion 5 to obtain the P-value of the linearity.

Structural zero dyads are relatively rare in socio-ethological research. I therefore refrained from fully developing the statistical test procedure suggested in conclusion 6. Observational zero dyads and tied dyads, however, are not rare. The procedure for testing linearity in dominance hierarchies containing unknown or tied relationships (conclusion 5) has been implemented in the newest version of the program MatMan, an older version of which is described by de Vries et al. (1993).

APPENDIX

The Relation between d' and d, and between h' and h

In this appendix I show that the following relation holds between the unbiased estimate of circular triads d' and the value of circular triads d calculated according to the procedure outlined in Appleby (1983): $d' = d - 0{\cdot}25$ (number of unknown dyads).

Consider first a dominance relationship matrix which contains only one unknown relationship, say between the individuals k and m. According to Appleby's procedure the values of the cells (k,m) and (m,k) are set to 1/2. The number of circular triads d is then calculated by:

$$d = \frac{N(N-1)(2N-1)}{12} - 1/2\,\Sigma(S_i)^2$$

where S_i is the sum of row $i (i=1$ to $N)$.

The unbiased estimate of circular triads d' is the average of the circular triads of two matrices, one matrix in which cell (k,m) is set to 1 and cell (m,k) is set to 0, and another matrix in which cell (k,m) is set to 0 and cell (m,k) is set to 1. The row sums of the first of these two matrices will be denoted by T_i and the row sums of the second matrix will be denoted by U_i. It can easily be seen that $T_k = S_k + 1/2$, $T_m = S_m - 1/2$, $U_k = S_k - 1/2$, and $U_m = S_m + 1/2$.

Now, by definition,

$$d' = \frac{N(N-1)(2N-1)}{12} - 1/2\,\frac{\Sigma(T_i)^2 + \Sigma(U_i)^2}{2}$$

Using the above equalities, the numerator of the rightmost term, $\Sigma(T_i)^2 + \Sigma(U_i)^2$, can be rewritten as

$$(S_k + 1/2)^2 + (S_m - 1/2)^2 + \sum_{i \neq k: i \neq m} (S_i)^2 + (S_k - 1/2)^2 + (S_m + 1/2)^2 + \sum_{i \neq k: i \neq m} (S_i)^2$$

which is identical to

$$2[(S_k)^2 + (S_m)^2 + 1/2 + \sum_{i \neq k: i \neq m} (S_i)^2]$$

It follows that

$$d' = \frac{N(N-1)(2N-1)}{12} - 1/2\{\Sigma(S_i)^2 + 1/2\} = d - 0{\cdot}25$$

For dominance matrices that contain more than one unknown relationship a similar derivation can be given for each of these unknown relationships. It follows directly that $d' = d - 0{\cdot}25$(number of unknown dyads).

A quite similar derivation can be given to show that between h' and h the following relation holds:

$$h' = h + \frac{6}{N^3 - N} \text{ (number of unknown dyads)}$$

where h is the value of Landau's h calculated for the matrix in which the zero dyads have been assigned the value 1/2 (according to the procedure of Appleby 1983). Note that this relation also holds if the matrix contains tied dyads (which are to be assigned the fixed value of 1/2, as argued in the section Landau's h versus Kendall's K).

ACKNOWLEDGMENTS

I thank Suci Utami and Serge Wich for allowing me to use their unpublished data on orang-utan displacement behaviour to illustrate the linearity test. I thank Linton Freeman for his comments on the hierarchical analysis procedures of MatMan, thus providing the incentive for me to start the investigations that led up to this paper. I also thank two referees, Michael Appleby and C. L. Alados, for their comments on the paper.

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