



David's score: a more appropriate dominance ranking method than Clutton-Brock et al.'s index

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There are many procedures, of varying complexity, for ranking the members of a social group in a dominance hierarchy (reviewed by de Vries 1998; also Jameson et al. 1999; de Vries & Appleby 2000; Albers & de Vries 2001). Roughly, two types of method can be distinguished, one in which the dominance matrix is reorganized such that some numerical criterion, calculated for the matrix as a whole, is minimized or maximized, and one that aims to provide a suitable measure of individual overall success, from which a rank order can be directly derived. Two relatively simple, and somewhat similar, ranking methods belonging to the latter type are Clutton-Brock et al.'s index (Clutton-Brock et al. 1979, 1982) and David's score (David 1987, 1988). Both methods can be used to calculate dominance ranks for individuals in a group, based on the outcomes of their agonistic interactions with other group members, while taking the relative strengths of their opponents into account. Clutton-Brock et al.'s index (CBI) was originally developed as a measure of fighting success for red deer, *Cervus elaphus*, but is more usually used as a general dominance ranking method in behavioural studies (e.g. Goodwin et al. 1999; Mateos & Carranza 1999; Pélabon & Joly 2000; McElligott et al. 2001), whereas David's score (DS), which was developed as a standard ranking method, has been largely overlooked in the behavioural literature. However, as we show below, DS is a more appropriate ranking method than CBI, because DS deals logically with repeated interactions between group members when calculating a hierarchy. Furthermore, when every dyad has an equal number of interactions, DS reduces to row-sum

scoring (David 1987), whereas CBI lacks this desirable property. We therefore recommend that DS should always be used in preference to CBI when calculating dominance ranks based on interaction success.

The Problem with Clutton-Brock et al.'s Index

CBI for each member, i , of a group, is calculated with the formula:

$$\text{CBI} = (B + \Sigma b + 1) / (L + \Sigma l + 1)$$

where B represents the number of individuals that i defeated in one or more interactions, Σb represents the total number of individuals (excluding i) that those represented in B defeated, L represents the number of individuals by which i was defeated and Σl represents the total number of individuals (excluding i) by which those represented in L were defeated. One is added to the numerator and the denominator in the equation because some group members might not have been observed either winning or losing an interaction (Clutton-Brock et al. 1979).

It is important to recognize that CBI does not take into account the total number, or win/loss asymmetry, of interactions recorded between different dyad members (de Vries 1998). Consequently, individual scores calculated with this method may result in illogical hierarchical rankings of group members. For example, if one individual had interacted 10 times with another, winning on nine occasions and losing on one, the above formula would treat this circumstance as if each individual had beaten the other once (essentially the dyad would be considered tied). Therefore, once an individual has beaten, or been beaten by, another member of the group at least once, his own index is weighted according to how successful (or unsuccessful) his opponent was. Since the win/loss asymmetry within the dyad is irrelevant, a relatively unsuccessful individual may have his index disproportionately raised because of a single win against a

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Table 1. Artificial interaction matrix (in which one dyad contains a deviation from the main dominance direction) and dominance ranks calculated using CBI and DS

	r	s	t	u	v	CBI	DS
r	*	100	100	100	99	2.20	9.95
s	0	*	100	100	100	2.67	5.00
t	0	0	*	100	100	1.0	0.00
u	0	0	0	*	100	0.38	-5.00
v	1	0	0	0	*	0.45	-9.95

Table 2. Artificial interaction matrix with dominance proportions (P_{ij}) in parentheses and dominance ranks calculated using DS (values for w , w_2 , l and l_2 are also presented)

	a	b	c	d	e	w	w_2	DS
a	*	4 (1.0)	4 (0.8)	8 (1.0)	0 (0.0)	2.8	5.08	7.2
b	0 (0.0)	*	3 (1.0)	9 (0.9)	7 (0.7)	2.6	3.24	3.3
c	1 (0.2)	0 (0.0)	*	4 (0.4)	5 (1.0)	1.6	1.84	-2.2
d	0 (0.0)	1 (0.1)	6 (0.6)	*	2 (0.5)	1.2	1.62	-3.5
e	0 (0.0)	3 (0.3)	0 (0.0)	2 (0.5)	*	0.8	1.38	-4.8
l	0.2	1.4	2.4	2.8	2.2			
l_2	0.48	1.14	3.24	3.52	4.78			

highly successful individual. Alternatively, a highly successful individual may have his index disproportionately lowered because of a single loss against a relatively unsuccessful group member that he has beaten on numerous other occasions. This can affect the overall rank order of individuals in the group. For example, in the artificial interaction matrix shown in Table 1 a single deviation from the main dyadic dominance direction for individuals r and v resulted in a respective lowering and raising of their scores, and an illogical rank order of s , r , t , v , u using CBI. It is unlikely that anyone would accept such a rank order in this artificially exaggerated case, but CBI may also calculate illogical rank orders in more realistic cases, where the anomaly is more likely to go unnoticed.

David's Score: Problem Solved

An illogical rank order was not calculated for the individuals in Table 1 using DS. Individual ranks calculated with DS are not disproportionately weighted by minor deviations from the main dominance direction within dyads because win/loss asymmetries are taken into account by the use of dyadic dominance proportions in the calculations. The proportion of wins by individual i in his interactions with another individual j (P_{ij}) is the number of times that i defeats j (a_{ij}) divided by the total number of interactions between i and j (n_{ij}), i.e. $P_{ij}=a_{ij}/n_{ij}$. The proportion of losses by i in interactions with j , $P_{ji}=1-P_{ij}$. If $n_{ij}=0$ then $P_{ij}=0$ and $P_{ji}=0$ (David 1988; de Vries 1998). DS for each member, i , of a group is calculated with the formula:

$$DS=w+w_2-l-l_2$$

where w represents the sum of i 's P_{ij} values, w_2 represents the summed w values (weighted by the appropriate P_{ij}

values, see below) of those individuals with which i interacted, l represents the sum of i 's P_{ji} values and l_2 represents the summed l values (weighted by the appropriate P_{ij} values) of those individuals with which i interacted (David 1988, page 108; de Vries 1998). Table 2 shows a worked example with calculated w , w_2 , l , and l_2 values. Specifically for individual a , w represents the sum of a 's P_{ij} values (i.e. $w=1.0+0.8+1.0+0.0=2.8$), and w_2 represents the summed w values (weighted by the appropriate P_{ij} values) of those individuals with which a interacted (i.e. $w_2=[(1.0 \times 2.6)+(0.8 \times 1.6)+(1.0 \times 1.2)+(0.0 \times 0.8)]=5.08$). a 's l and l_2 values are calculated in a similar manner. Finally, it should be noted that P_{ij} values are not wholly appropriate for ranking group members when the n_{ij} are considerably different, that is, when interaction frequency varies substantially between dyads (David 1988; de Vries 1998). de Vries (1998, Appendix 2) proposed a correction to P_{ij} (which he termed d_{ij}) for use in such circumstances. de Vries' correction takes into account the possibility that the win/loss asymmetry recorded for a particular dyad is no different to that which would be expected if the dyad were actually tied. This correction to the P_{ij} values also takes the differences in the n_{ij} values into account.

If the social group under investigation is relatively large, and if most dyads do not contain any deviations from the main dominance direction, then CBI and DS will calculate similar hierarchies. For example, prerut CBI and DS rank orders calculated for fallow bucks, *Dama dama*, resident in Phoenix Park, Dublin (Moore et al. 1995) in 1995, 1996 and 1997 correlated significantly with each other in all years (1995: $r_s=0.987$, $N=69$, $P<0.001$; 1996: $r_s=0.963$, $N=61$, $P<0.001$; 1997: $r_s=0.974$, $N=63$, $P<0.001$). However, minor deviations and any resultant positional changes of individuals in the hierarchy will have a relatively greater effect on the overall

rank order in smaller social groups (e.g. Table 1). CBI should not therefore be used to calculate dominance hierarchies for small groups.

There is, however, a logical reason for using DS in preference to CBI for groups of all sizes. It is desirable that prior to ranking group members in a dominance hierarchy as many agonistic interactions as possible have been recorded. However, as minor deviations from a generally stable dominance direction sometimes occur, an increase in the number of recorded interactions would also increase the possibility of recording such minor deviations. Recorded deviations within dyads may also result from observer error, which should again increase with an increase in the number of recorded interactions. In the extreme case of at least one recorded deviation within all dyads, all individuals would receive identical scores if CBI were used, an outcome that could only arise with DS if all dyads were actually tied. Therefore, while DS benefits from increased observer effort and the resultant advantage that additional data bring when calculating a hierarchy, an increase in data actually introduces more ambiguity to CBI as more dyads are considered tied because of the deviations that inevitably occur.

Adapting Clutton-Brock et al.'s Index is not the Answer

There are ways in which CBI can be adapted to circumvent the problem that we have highlighted. For instance, dyadic dominance status could be used as the basis for calculating the index; instead of counting the number of individuals that i defeated, one could count the number of dyads in which i was dominant. Thus B could represent the number of dyads in which i was dominant and Σb the total number of dyads that were dominated by the subordinates in B . L could represent the number of dyads in which i was subordinate and Σl the total number of dyads in which dominants from L were subordinate. A dyadic interpretation of CBI would also necessitate the introduction of a term for incorporating ties into the calculation. Using this process, we can essentially convert the original interaction matrix to a relationship matrix, with the entries in the matrix representing dyadic dominance status (assigned on the basis of win/loss asymmetries within dyads) rather than the actual number and outcomes of interactions.

However, assigning dominance status to dyad members based on win/loss asymmetries is not a simple matter. In Table 1, victory by individual v in only one out of 100 interactions with individual r was considered a minor deviation from the main dyadic dominance direction. Individual r could clearly be classified as the dominant dyad member in this case. But interaction matrices compiled from data collected in the field usually contain less information than this (Jameson et al. 1999; Whitehead & Dufault 1999). A more realistic scenario could involve r having won three out of four interactions with v , or r having won the only recorded interaction between these two individuals. The validity of assigning the status of dominant dyad member to r based on such nonsignificant asymmetries may be questioned (de Vries 1998). One

alternative would be to consider the dominance relationship between two individuals as unknown or tied unless the win/loss asymmetry was significant (e.g. 6/0, 8/1, 10/2 using the binomial test). However, owing to the usual scarcity of information in interaction matrices, such a restrictive criterion would probably result in most relationships being classified as unknown or tied, thereby increasing the difficulty of calculating a hierarchy (de Vries 1998; Jameson et al. 1999). Therefore, although adapting CBI by using dominance status as the basis for calculating the index circumvents the problem inherent in the original formula, it requires a somewhat arbitrary assignation of dominance status to dyad members based (probably) on mainly nonsignificant win/loss asymmetries. Furthermore, this adaptation results in a loss of information, as the degree of dyadic win/loss asymmetry is not taken into account. This means, for example, that there would be no differentiation between a 9/1 asymmetry and a 1/0 asymmetry; both would essentially be treated as a 1/0 asymmetry. (Note that under the original formulation of CBI, both members of a dyad with a 9/1 asymmetry are considered equal in status whereas in a dyad with a 1/0 asymmetry, the status of dominant dyad member belongs to the individual that won the single interaction.)

Another possible solution to the problem inherent in CBI is to use the P_{ij} values in the calculation of CBI. The advantage of this strategy is that it removes the necessity to assign dominance status within dyads and means that degrees of dyadic win/loss asymmetries are taken into account in the calculations. We have already highlighted the fact that CBI and DS are somewhat similar ranking methods, and they are actually calculated using similar mathematical formulae. In fact, the formula for calculating CBI using P_{ij} values can be written in the same notation as the formula for DS, as $CBI^P = (w + w_2 + 1) / (l + l_2 + 1)$.

However, adapting CBI by using P_{ij} values in the calculations is not an entirely satisfactory solution, and is certainly deficient when compared with the use of DS, for the following reason. The artificial interaction matrix shown in Table 3 represents a balanced tournament (where each individual has the same number of interactions with all other members of the group). David (1987, page 434, Property 3) has shown that for a balanced tournament, DS gives the same rank order as w . In other words, DS reduces to row-sum scoring and the resultant rank order therefore equates with individual victories within the group (Table 3). A rank order based on w is acceptable for a balanced tournament and has the desirable feature that each individual's rank score is independent of interactions in which he was not involved. In Table 3, individuals n and m (who had identical w scores) received identical dominance ranks using DS, as did individuals p and q (who also had identical w scores), whereas n was ranked above m and p below q using CBI^P . Therefore the rank order calculated with DS was equivalent to row-sum scoring whereas the rank order calculated with CBI^P was not; the reduction to row-sum scoring for balanced tournaments is a property that clearly does not hold for CBI^P . Thus DS is a more

Table 3. Artificial interaction matrix (in which each individual has exactly one interaction with every other individual, i.e. it is a balanced matrix) and dominance ranks calculated using DS and CBI^P

	m	n	o	p	q	w	w ₂	l	l ₂	DS	CBI ^P †
m	*	1	1	1	0	3.0	6.0	1.0	3.0	5.0	2.00
n	0	*	1	1	1	3.0	4.0	1.0	1.0	5.0	2.67
o	0	0	*	1	1	2.0	2.0	2.0	2.0	0.0	1.00
p	0	0	0	*	1	1.0	1.0	3.0	4.0	-5.0	0.38
q	1	0	0	0	*	1.0	3.0	3.0	6.0	-5.0	0.50

†For this matrix CBI values calculated according to the original formula, or by using dyadic dominance status or P_{ij} values, are identical because there are no deviations from the main dominance direction within dyads.

appropriate ranking method than CBI^P for balanced tournaments, and is consequently a more suitable method for unbalanced tournaments (David 1987, 1988). Furthermore, because each individual in Table 3 interacted only once with every other individual, there were no deviations from the main dominance direction within dyads. Therefore, for this matrix, the dominance ranks calculated with CBI^P are identical to those calculated with the original formulation of CBI or by using dyadic dominance status as the basis for the index, and the reason given for the unsuitability of CBI^P when compared with DS is also applicable to both other formats of the index. Thus, while both adaptations to CBI considered above are obvious improvements over the original formulation of the index, DS is a more appropriate ranking method than either, as well as being more appropriate than the original formulation of CBI.

Discussion

It is certainly possible that others have also recognized and attempted to circumvent the problem with CBI by using dyadic dominance status or dyadic dominance proportions as the basis for calculating the index in their studies (although there is little evidence in the literature to suggest that this is the case). However, as we have shown, such adaptations to CBI are not entirely satisfactory and may still result in the calculation of illogical rank orders in certain circumstances. Therefore, neither adaptation presented here is an appropriate solution to the problem inherent in CBI. As we have clearly shown, the use of DS in preference to CBI is an appropriate solution for two reasons. First, DS takes repeated interactions between dyad members into account in a logical manner and is therefore not disproportionately affected by minor deviations from the main dominance direction within dyads. Second, DS reduces to row-sum scoring for balanced tournaments, meaning that in such tournaments an individual's rank is independent of interactions in which he was not involved.

One of the advantages of DS that we have highlighted is that it prevents individuals from obtaining artificially raised or lowered dominance ranks caused by minor deviations from the main dominance direction within dyads. It should be realized, however, that another way by which certain individuals could potentially obtain artificially raised or lowered dominance ranks is if they had been recorded interacting with only a small subset of

the entire group and had always either won or lost these interactions. Individuals that do not interact with many other group members are always going to cause problems (regardless of the ranking method used) since the reliability of the estimated rank of these individuals will be low. Perhaps the best strategy is to exclude such individuals from the ranking process. This could possibly be justified by arguing that, behaviourally, such animals are not really members of the group. For example, in Phoenix Park, male fallow deer with low interaction rates are usually those that spend most of the year in a separate area from the main group, joining the other animals only occasionally (M. P. Gammell, personal observation).

There may of course be reasons apart from separation from the main group that some individuals are rarely recorded interacting with other group members. If the group is large and the number of interactions recorded is a small proportion of the total number of interactions that actually occurred during the observation period, then some individuals could be rarely recorded by chance, even if individual interaction rates were approximately equal. Because of the assumedly random nature of such a variation in sampling rate, statistical properties on which the ranking procedure is based should not be violated, although the accuracy of the rank calculated for rarely recorded individuals is still questionable. We have already stated that increased sampling effort, with a resultant increase in recorded interactions, will bring more accuracy to the ranks calculated by DS (but not necessarily to the ranks calculated by CBI). This is strictly true only if the additional data result from an increase in the number of recorded interactions within the same observation period (as a result of an increase in the number of observers for example) and not from an extension of the observation period. An extension of the observation period may further obscure any temporal variation in dominance rank that might be present, but that is not properly taken into account when dominance hierarchies are calculated from a matrix summarizing interactions recorded over a lengthy period.

A potentially more serious reason for low individual interaction rates could be that some individuals interact preferentially with certain members of the group, or actively avoid others, or both (Freeman et al. 1992; Appleby 1993). As DS is based on the paired comparisons paradigm (David 1988), an underlying assumption of the method is that every dyadic interaction is independent of every other dyadic interaction. This is obviously not

the case if some individuals interact preferentially with others. The assumption of independence is also not satisfied if winning (or losing) an interaction affects an individual's chance of winning (or losing) a future interaction (de Vries 1998). Such winner/loser effects have been identified in various taxa (Chase et al. 1994). The paired comparisons paradigm also requires that the number of records for each dyad be approximately equal. We have already pointed out that DS is not an entirely appropriate ranking method when interaction frequency varies considerably between dyads; de Vries (1998) has suggested a correction to deal with this problem.

Thus DS, like all dominance ranking methods, has its limitations. However, as far as the investigator can be confident that the P_{ij} values are good estimates of the dyadic dominances, DS appears to be a useful and appropriate method. Other ranking methods may deal more appropriately with some of the problems discussed above, or may simply be more suitable for certain studies depending on their particular aims and the available data. The Elo rating method (Albers & de Vries 2001) takes the sequential order of interactions into account when calculating a hierarchy, which may be important if interaction outcome is affected by prior interaction success (or failure). This method also updates the dominance rank of individuals as their interactions occur, which means that a dominance score calculated at any point in time with the Elo rating method is probably a closer approximation of an individual's actual rank at that time than a dominance score calculated with any other method. If the aim of a study is to find a linear hierarchy using a set of observed dominance relationships, then the I&SI method should probably be used as it is the best currently available for ranking group members in the closest possible order to linearity (de Vries 1998; de Vries & Appleby 2000). It is therefore important to consider carefully the most appropriate ranking method for each data set.

The attraction of a ranking method such as CBI is that it takes the relative strengths of encountered opponents into account when calculating dominance ranks, and that it is relatively simple to compute. For this reason, and also because it has been the ranking method of choice in many previous studies, it is likely to remain a popular method. Therefore it is important that the problem inherent in CBI is highlighted so that the calculation of illogical hierarchies can be avoided. Unlike many of the general problems discussed above, this one has a straightforward solution. As we have shown, the problem can be circumvented by using DS, a simple ranking method, like CBI, that also takes the relative strengths of encountered opponents into account, but that does not suffer from the problem that is inherent in CBI. We therefore recommend that David's score should always be used in preference to Clutton-Brock et al.'s index.

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