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THE REDUCTION IN THE BROWNIAN MOTION OF ELECTROMETERS

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Synopsis

A method is described to reduce the Brownian deflections of electrometer systems. This is achieved by replacing the air damping by a special type of artificial damping. The latter is realised by means of a photoelectric amplifier containing a differentiating circuit, comp. fig. 1. The amount of light entering the photocell is made proportional to the deflection of the electrometer system and the output of the amplifier is fed back to the electrometer.

The motion of the electrometer is studied with a photoelectric relay and a recording galvanometer allowing a magnification up to 14300×. The application of this method of damping has already resulted in a hundred fold reduction of the Brownian energy and an increase in precision of a factor of ten.

In order to reach the same accuracy with normal air-damping it should be necessary to cool the instrument to 3°K.

§ 1. *Introduction.* In a previous paper [1]) ***) the general theory of the Brownian motion of electrometers has been developed. It was demonstrated that by far the greatest part of the Brownian deviations were due to the irregular collisions of the air molecules against the system and that the thermal voltage fluctuations contributed only very little to the ultimate instability of the system.

Although the intensity of the fluctuating mechanical torque appeared to be proportional to the air-damping β_M, reduction of β_M only, does not cause any decrease in the Brownian motion. It is true that evacuation of the instrument brings about a decrease of the irregular collisions, but the damping of the system decreases also

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***) In the following to be referred to as I.

hence, corresponding to the law of aequipartition, the mean square $\overline{\varphi^2}$ of the deflections remains unchanged and subject to $\frac{1}{2}A\overline{\varphi^2} = \frac{1}{2}kT$, where A is the directional force, k the Bolzmann constant and T the absolute temperature.

Therefore, to obtain a reduction of the fluctuations one may resort to one of the following measures,

a. replace air-damping by an artificial damping which supplies much less fluctuations,

b. take the mean of a small number of deflections (cf. I § 5) of the completely undamped instrument,

c. suspend the electrometersystem in such a way, that the restoring torque becomes negligible. Then, the angular acceleration, which may be measured by a photoelectric amplifier containing a double differentiating circuit, becomes proportional to the charge to be measured.

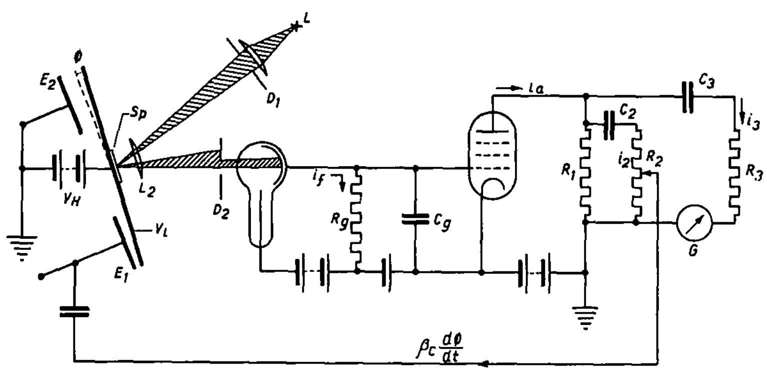

Fig. 1. Schematic arrangement of the electrometer, the damping circuit and the recording galvanometer.

In order to explain the difference between *a.* on one side and *b.* and *c.* on the other, it is necessary to describe first the method of artificial damping we applied.

A lightsource L fig. 1 throws a beam of light through the diafragm D_1 on the electrometermirror S_p. A part of the reflected light enters the photoelectric cell via the diaphragm D_2 which is conjugated to D_1. The diaphragms are placed in such a way that the photoelectric current becomes directly proportional to the angle of deflection φ. This current is supplied to an amplifier, which contains a differentiating circuit R_2C_2. Thus the output is equal to $\beta_C\, d\varphi/dt$

where β_C is a constant: this voltage is supplied to E_1 in order to damp the electrometer.

When using method *a.*, the instrument may be regarded formally to be connected with two reservoirs of different temperatures at the same time.

1. by means of the leakage resistance R and the remainder of the damping air, the instrument is coupled weakly to a body of room-temperature,

2. via the damping circuit, the electrometer is coupled to the amplifier which, in virtue of its low noise level, may be regarded as a body of very low "effective temperature", amounting to 3.10^{-4} °K. This will be discussed below.

By increasing β_C the latter coupling can be made so strong that the instability of the electrometer will reach very closely the noise level determined by the amplifier. In this way we were able to reduce the Brownian energy of the electrometer hundred fold, thus obtaining an increase in precision of a factor of ten.

Now it is evident that there exists a fundamental difference between the methods indicated. In the case of *b.* and *c.* contrary to method *a.* there is only one single temperature in the electrometer and the circuit. Here, using an evacuated instrument it takes a comparatively long time i.e. the RC time of the instrument ($\sim 10^5$ sec.), before the energy fluctuations due to the Brownian motion are able to manifest themselves in the unrest of the system. Thus during the relative short time of observation (~ 20 sec.) the Brownian energy of the instrument may be supposed to be constant.

Our experiments have been restricted to method *a.*

§ 2. *The experimental arrangement.* The apparatus used in our experiments consisted of the electrometer and the photoelectric amplifier. The output of the amplifier was partly fed back to the electrometer after differentiation and partly supplied to a recording galvanometer whose deflections were proportional to those of the electrometer. These galvanometer deflections were recorded on a rotating drum, representing on a greatly magnified scale the Brownian motion of the electrometersystem.

We turn now to a more detailed description of the respective instruments.

§ 3. *The electrometer.* The electrometer was a type designed by Milatz. Although it was of a fairly firm construction, the experiments of H. Vreedenberg[2]) and R. Dorrestein[3]) show that it may be applied to carry out fairly delicate measurements. In fig. 2 and fig. 3 respectively, a transverse and longitudinal section of this instrument are shown. The chamber containing the movable system is surrounded by heavy brass pieces of one cm thickness, to

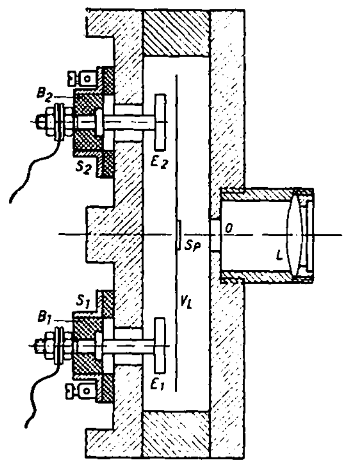

Fig. 2. Transverse section of the electrometer, B_1 and B_2 are amber insulators, S_1 and S_2 are guardrings.

keep air currents, arising from temperature gradients, at a minimum. The air currents due to the lens, heated by radiation, are localized in the chamber *abcd* (comp. fig. 3).

Since the suspension had to be fairly strong in order to support the relatively heavy system, the torsional constant was high and the Brownian motion as well as the sensitivity rather small, when low auxiliary potentials were used. In order to achieve a sufficiently large value of the Brownian deflections, for exemple of 1 mm per m, we could proceed in two different ways:

a. The effective torsional constant could be reduced by applying auxiliary potentials V_H very close to the value where labilisation occurs.

b. The deflections could be magnified by a relay as indicated in fig. 1 and described in § 4.

We have chosen the second method *b*) which permits to adjust V_H

arbitrary. Since the value of the r.m.s. of the Brownian motion of the system amounted to 10^{-6} rad. (when $V_H = 0$), we had to apply factors of magnification up to $14300\times$. At these large values of the magnification sometimes rather troublesome creep occurred, which, for the greater part, should be described to inelastic behaviour of the suspension wire. The influence of this creep could only be eliminated by a special design of the recording relay.

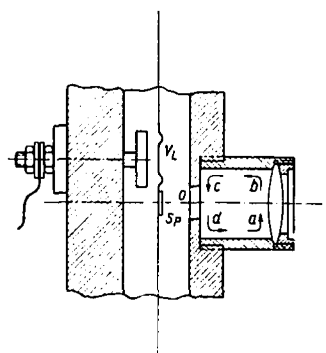

Fig. 3. Longitudinal section of the electrometer.

In order to damp oscillations of the electrometersystem in its own plane *), which appeared after evacuation and which could not be reduced by artificial damping, a comb shaped electrometersystem was applied comp. fig. 4. Now, the air damping for rotational motion became so small that the electrometer was already underdamped at atmospheric pressure when the transversal oscillations were still energetically damped, as is shown in the record 1) of fig. 5. With this electrometersystem there appeared to be a range of pressure where successful experiments could be performed. The constants of the electrometer were: capacity $C_0 = 3$ cm, period of the undamped instrument = 6.3 sec., capacity coefficients a and b resp. 0.825 cm rad^{-1} and 6,55 cm rad^{-2}. Labilisation occurred at 33 V. The other constants are given in table I of I [1]).

§ 4. *The amplifier.* In the amplifier fig. 1 the same valve was used to provide the current i_3 to operate the recording galvanometer G,

*) These oscillations often posessed an energy far greater than $\frac{1}{2}kT$, which, by asymmetry effects in the suspension, were readilly transformed to rotational motion of the system.

as well as to supply the current i_2 to the differentiating circuit.

The main period occurring in the signal supplied to the grid of the tube was the one corresponding to the period of the motion of the electrometer i.e. about 6 sec. In order that proper differentiation may take place, the time constant of the circuit C_2R_2, where the damping voltage originates has to be small compared to the period of the electrometer, actually $R_2C_2 = 0.02$ sec. The damping could be adjusted with potentiometer R_2.

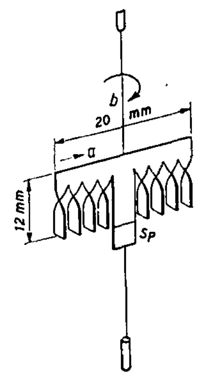

Fig. 4. Comb shaped electrometer system.

In the galvanometer circuit, the condensor C_3 was inserted to prevent the *dc* component of the anode current from flowing through the galvanometer *G*. In order to be able to record signals of long periods with only small loss of magnification, R_3C_3 should be sufficiently large. In our arrangement, harmonic signals with periods of 20 sec. were reproduced with still 96% of the maximum amplification i.e. the amplification for very quickly varying signals, which amounted to 14300×.

However, the deflection of the electrometersystem only leads to a ballistic deflection of the galvanometer. The amplificationfactor depends slightly on the total damping constant β of the electrometer as β determines the frequency spectrum of the signal to be reproduced, compare for instance fig. 5, where on the three records the deflection corresponds to the same test tension of 2.10^{-3} V, the ratio of the total damping in 1), 2) and 3) being as 1 : 10 : 100.

The method to amplify deflections just described, had the advantage that a constant creep, which caused considerable difficulties

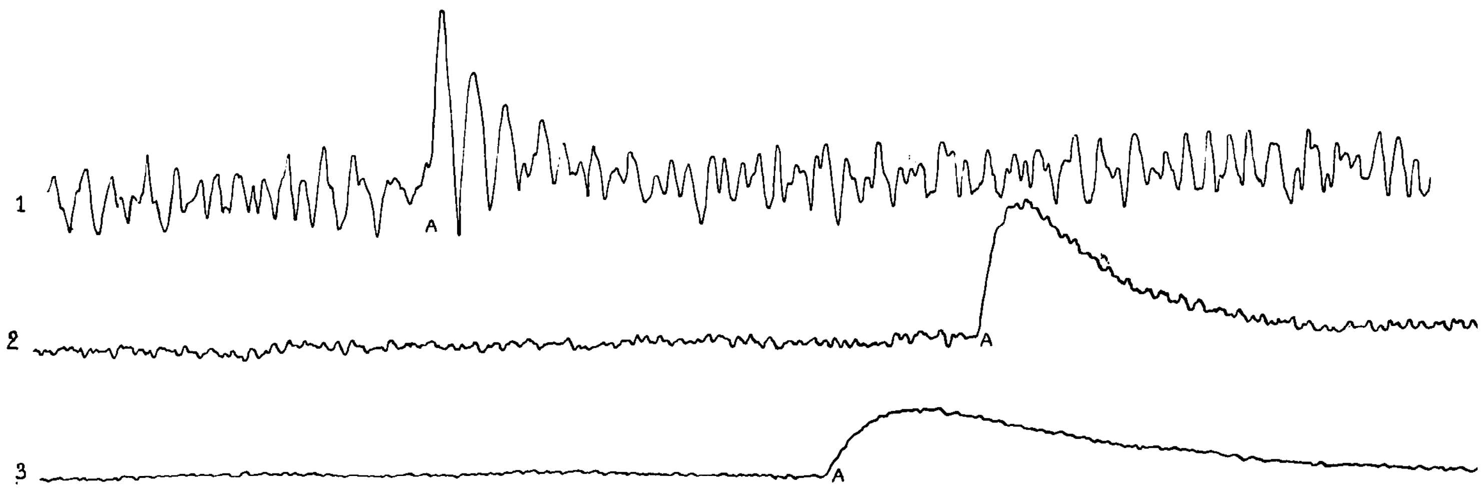

Fig. 5. Records of the Brownian motion at different degrees of artificial damping. In 2) the instrument was critically damped, in the total damping amounted to $\frac{1}{10}\,\beta_{crit}$ and in 3) to $10\,\beta_{crit}$. In all cases the deflection at A corresponds to 2.10^{-3} V.

in previous experiments, now gave rise to a constant displacement of the zero setting of the galvanometer only.

From our provisional experiments it appeared that disturbances arose also from macroscopic fluctuations in the intensity of the light source. We tried to reduce these effects by replacing the single photoelectric cell by two cells inserted in a compensating circuit. Although this increased the stability to some extent, an inconstancy remained which caused fortuitous deflections of the electrometer, amounting to about 1% of the normal Brownian motion. These random deflections surpassed those arising from the noise of the amplifier by a factor of ten.

§ 5. *The noise of the amplifier.* It is a matter of common knowledge now, that the noise produced in an amplifier may be described substantially by the thermal voltage fluctuations in the resistances and by the shot effect of the currents in the circuit elements [5]).

The total noise produced, may be described by a random voltage at the grid of the AF7 tube we used, with a spectral intensity $I_t(\nu)$ given by

$$I_t(\nu) = 2ei_f R_g^2 + 4R_g kT + 2ei_a F_a'^2/s^2 + C_{Fl}^2 i_a^2/s^2\nu \qquad (5.1)$$

where e = charge of the electron; R_g = grid resistance; F' = space charge constant < 1; $C_{Fl}^2 = 2.56 \times 10^{-14}$; s = mutual conductance; k = Boltzmanns constant; T = absolute temperature. The first two terms in the right hand side of eq. (5.1) represents the intensity of the shot effect of the photo current i_f and the thermal voltage fluctuations of R_g respectively. The third term represents the combined effect of the shot effect of the anode current and the noise of partition. The spectral intensity of the flickereffect may be described by the fourth term. C_{Fl}^2 is a constant derived from G r a f f u n d e r s data [6]) for the flicker noise of an AF7 tube. In (5.1) the influence of the grid capacity C_g on I_t has been neglected, since for the frequencies considered, which are determined by the period of the electrometer, $\omega^2 R_g{}^2 C_g{}^2 \ll 1$. The computation of the m.s. $\overline{\varphi_D^2}$ of the deflections of the electrometer system caused by the noise spectrum (5.1) is a matter of straight-forward calculation, since the resonance curve which is given by I (3.6) is known. Therefore, we will refrain ourselves from showing it here and mention the results only.

Experimentally we found that a tenth part of the tension developed over R_2 was sufficient to damp the electrometer critically. In this case one finds for the m.s. of the resulting deflections $\overline{\varphi_D^2}$ by calculation with the constants we used: $i_f = 10^{-7}\,A$, $R_g = 1.1 \times 10^7\,\Omega$, $R_a = 2.5 \times 10^5\,\Omega$, $R_2 = 5 \times 10^5\,\Omega$, $s = 10^{-3}\,A/V$, $C_0 = 3 \times 10^{-8}\,F$, voltage sensitivity $S_v = 2.07 \times 10^{-3}$ rad/V,

$$\overline{\varphi_D^2} = (7.7 \times 10^{-10})^2 + (2 \times 10^{-11})^2 + (6.6 \times 10^{-13})^2 + + (0.6 \times 10^{-10})^2 \text{ rad}^2. \quad (5.2)$$

where the terms are put in the same order of succession as in (5.1).

From eq. (5.2) we observe that the main contribution to the instability of the electrometer is delivered by the shot effect of the photocurrent and the flickernoise. Comparing $\overline{\varphi_D^2}$ with the normal Brownian motion of the system, which for the auxiliary tension we used of 15 V amounts to 7.8×10^{-7} rad, one remarks that by using the "cold" damping just described a gain of a factor of 10^3 in stability may be expected compared to air damping at room temperature. If one would try to reach the same accuracy by applying refregerated air for damping, it should be necessary to decrease the temperature by a factor of 10^6, i.e. one had to cool down to 3×10^{-4} °K.

§ 6. *The measurements.* As it was our primary purpose for the present to demonstrate beyond doubt the possibility of reducing the Brownian motion of the instrument we kept auxiliary potentials as low as 15 V and shunted the electrometer by a resistor R of 0.5 MΩ. Still, by virtue of the high relay magnification, the ultimate voltage sensitivity was moderate.

In fig. 5, some records are shown corresponding to:

1) periodical motion without any artifical damping but with still some airdamping present, $\beta/\beta_{crit} = 0.1$, the fluctuations of the zero-position appears to be about 10% greater than the normal Brownian motion at the ambient roomtemperature of 290°K.

2) critically damped motion $\beta \sim \beta_{crit}$ obtained by supplying the necessary amount of artificial damping to the small airdamping present. The Brownian energy proves now to be nine times smaller than in 1) and thus the r.m.s. of the deflections three times.

3) Strongly overdamped motion $\beta/\beta_{crit} = 10$, attained with the greatest amount of artificial damping which could be supplied. The Brownian energy is here reduced by a factor of one hundred, corresponding to 3°K and consequently the r.m.s. of the deflections by a factor of ten.

In all three records at the points A, the ballistic deflection by a test tension of 2×10^{-3} V is shown.

We have measured from registrograms like those just described the m.s. of the Brownian deflections of the electrometer as a function of the total damping β i.e. artificial damping β_c plus damping of the remaining air β_M.

In I § 3 and I § 4 general formulae have been derived which allow us to calculate the contribution of the respective fluctuations.

The smallest contribution is delivered by the intrinsic electrical damping of the electrometer itself. Starting from the expression I (3.8) for $\overline{\varphi_E^2}$ and substituting the constants concerned (cf. Table I in I), we obtain for the small values of R used,

$$\overline{\varphi_E^2} = [B^2R/(B^2R + \beta_M + \beta_C)]\ (kT/A), \text{ where } B = aV_H \text{ and} \qquad (6.1)$$

$\frac{1}{2}B^2R$ equals the intrinsic electrical damping at low values of R. φ_E appears to be maximally $\frac{1}{2}$ to 1% of the normal Brownian motion at room temperature.

As has been observed in § 4, a small contribution in noise of about the same magnitude is introduced by the damping current from the amplifier.

The main contribution however, which determines the ultimate reduction in the Brownian motion is furnished by the collisions of the remaining damping air molecules. This contribution is described by I (4.1), which expression by virtue of the small value of $B=aV_H$ reduces to

$$\overline{\varphi_M^2} = [\beta_M/(\beta_M + \beta_C)]\ (kT/A) \qquad (6.2)$$

In fig. 6 we have plotted the measured values of the r.m.s. of the deflections against $[\beta_M/(\beta_M + \beta_C)]^{1/2}$. The full line corresponds to (6.2), while the open and black circles correspond to the measurements carried out during two different nights. The error made in the evaluation of the records and the calculations of the mean values amounts to about 10%.

Considering this, one may conclude that there exists fair agreement with the theoretical predictions. The deviations between the measured and the calculated values amount to a factor of two at most. As the instability proves to be smallest at nights without wind, there is reason to believe that the deviations may be ascribed to the influence of stray vibrations of the building. Later experiments carried out by A. H. Wapstra [7]) confirm this belief. In the course of these experiments a further reduction of the Brownian motion was obtained.

§ 7. *Conclusion.* In the experiments described in the previous chapter it has been shown that by means of artificial cold damping it is in deed possible to use an electrometer at room temperature with the very high precision, corresponding to temperatures usually only realisable at Cryogenic Laboratories.

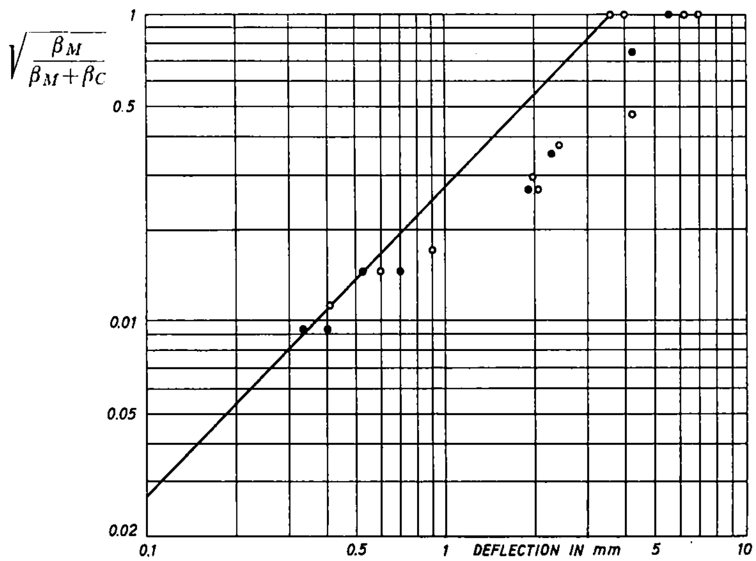

Fig. 6. R.m.s. of the deflections versus $[\beta_M/(\beta_M+\beta_C)]^{\frac{1}{2}}$. The full line represents the values calculated from eq. 6.2, the circles are values actually measured.

Summarizing we may observe that the ultimate reduction to be obtained with the present apparatus appears to be determined by

a) the intrinsic electrical damping of the electrometer used, which amounts to about 1% of the normal Brownian motion, but which may be reduced by suitable design of the electrometersystem;

b) the "macroscopic" fluctuations of the lightsource, presently amounting to about 1% also, but which may certainly be reduced greatly by special design of the filament of the lamp.

By applying a more intense lightsource or by increasing the artificial cold damping way a further reduction may be obtained as is demonstrated in a following paper [7].

It is true that the described experiments do not yet include actual charge measurements but after the corresponding eq. (6.2) has been confirmed experimentally now, there is no reason to doubt the validity of eq. I (5.9) which predicts the precision to be reached in charge measurements. It is quite another question, which will be the most suitable instrument to reach the highest precision the theory allows, in the least difficult manner. It is certainly not apriory sure that an instrument which is very appropriate when used in the ordinary way, as for example a Hoffmann electrometer, will be so as well when used with artificial cold damping. For instance the inhomogeneity of the lightspot, the difficulty of removing the mechanical damping or a rather high intrinsic electrical damping even at low values of V_H, may prove an impedement to the successful application of cold damping to certain types of electrometers.

After closer consideration however, one observes that in the design of an electrometer, to be used with artificial damping, the following points should be regarded.

a. The motion of the electrometer system should be as highly periodic as possible at easily obtainable vacua. This may be reached by using a system with a rather large moment of inertia and a shape, suitable in aerodynamical sense. The increase in the indication time may be compensated by the following measure, viz.

b. The torsion constant of the suspension fibre must be high in order to reach a high value of the lability tension V_H and thus of B. From eq. I (5.10) one observes that this causes a gain in precision. The reduction in sensitivity is not essential and may be overcome easily by applying a magnifying relay.

Finally one should try to design the instrument in such a way that it may be outgassed. This opens the way to seal the instrument.

According to our experiences it may be expected with a high degree of certainty that with such an instrument a r.m.s. of the error in charge of 8 or 9 electrons in a measuring time of 10 sec. may be obtained. This error is of course reduced, if one succeeds in diminishing the indication time of the instrument for example by method *b*).

The application of a photomultiplier cell instead of the combination photocell-amplifier might be shown to bring about not only a considerable simplification in the experimental arrangement, but by the absence — eventually after cooling of the cathode — of low frequent (Flicker-) noise an improvement of the performance of the damping amplifier.

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