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THE INFLUENCE OF MICROSEISMIC PERTURBATIONS ON A COLD DAMPED ELECTROMETER

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Synopsis

It is proved that the excess fluctuation of an electrometer above the electrical and mechanical Brownian movements found in a former investigation was due to microseismic movements of the soil. A theory is developed of the influence of mechanical perturbations on a cold damped [1]) electrometer. This theory predicts a characteristic connection between the course of the excess fluctuation with the damping and the Fourier spectrum of the microseismic movement; this connection was checked experimentally. A further proof is the fact, that the Fourier spectrum of the microseismic movement deduced from the spectrum of a recording of the fluctuation of our electrometer is in agreement with a Fourier analysis of a seismogram.

1. *Introduction.* In the course of further experiments with the electrometer with decreased Brownian movement [1]), in which the air damping was decreased, the fluctuations of the electrometer-system appeared much larger than the Brownian movement. On varying the distance between the mirror of the electrometer and the photocell the excess of the fluctuation varied proportional to the distance. This proved that these fluctuations were caused in the electrometer and not by the light source. The fluctuations were probably not due to causes originating in the building: most measurements were made at night. Therefore we concluded, that the microseismic movements of the soil were the cause of the extra movements.

With reference to these facts we developed and tested a theory on the influence of mechanical perturbations on a cold damped electrometer, used as a high resistance voltmeter.

2. *Theory of the electrometer.* The experimental arrangement was the same as that used by Van Zolingen with some improve-

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ments. Because Van Zolingen has shown, that for this arrangement the electrical Brownian motion is neglectable, the equation of the electrometer becomes

$$I\ddot{\varphi} + \beta_L \dot{\varphi} + D\varphi = D_g V + F_B + F_M \tag{2.1}$$

where φ denotes the angle of deflection, I the moment of inertia, β_L the air-damping, D the torsional rigidity, g the sensitivity, V the potential difference and F_B the variable couple for the Brownian movement. The only new term is the couple of the mechanical perturbation F_M. As shown by Van Zolingen the perturbations, caused by the damping amplifier, can be neglected.

In order to find an expression for F_M we only need to consider the horizontal movements perpendicular to the plane through the centre of gravity and the axis of rotation of the electrometersystem since the coupling between the degrees of freedom of the electrometer-system is small. Let the distance between this plane and a fixed plane be $X(t)$. Due to the inertia a force $\varrho \cdot \ddot{X} \cdot \mathrm{d}V$ acts on every mass element $\varrho \cdot \mathrm{d}V$, ϱ being the density of mass. Integrating over the system we find a force of inertia $m\ddot{X}(t)$ (m being the mass of the system), having its point of application at the centre of gravity.

Denoting the distance between the centre of gravity and the axis of rotation by δ, the mechanical disturbance couple becomes

$$F_M = m\,\delta\,\ddot{X}(t).$$

There is no influence due to friction with the air, because the air is moving with the electrometer for low frequencies.

We follow the method introduced by Schottky[2]), developed by Milatz[3]) in the modification of Keller[4]) to calculate the influence of this couple on the moving system.

The disturbing couple $F(t)$ is thought to be limited to a satisfactory long interval τ; outside this interval we take $F(t) = 0$.

Then $F(t)$ can be expanded in a Fourier-series:

$$F(t) = \int_{-\infty}^{+\infty} F(\omega) \exp{(i\omega t)}\, \mathrm{d}\omega$$

The resulting angle of deflection is also expanded in a Fourier-series

$$\varphi(t) = \int_{-\infty}^{+\infty} \varphi(\omega) \exp{(i\omega t)}\, \mathrm{d}\omega$$

If a harmonic disturbance exp $(i\omega t)$ gives an oscillating angle of deflection with amplitude $g(\omega)$, then

$$\varphi(\omega) = g(\omega)\, F(\omega) \tag{2.2}$$

From this result we find the quadratic fluctuation by using Parcevals rule

$$\overline{\varphi^2(t)} = \tau^{-1} \int_0^\infty |\, g^2(\omega)\,| \,.\, |\, F^2(\omega)\,|\, \mathrm{d}\omega$$

If we now take the limit for $\tau \to \infty$ and we define the intensity of the disturbance spectrum

$$I(\omega) = \lim_{\tau\to\infty} \tau^{-1} \,|\, F^2(\omega)\,| \tag{2.3}$$

it follows that $\overline{\varphi^2(t)} = \int_0^\infty |\, g^2(\omega)\,| \,.\, I(\omega)\, \mathrm{d}\omega.$

For the Brownian movement Ornstein found

$$I_B = (2/\pi)\, \beta_L\, kT \tag{2.4}$$

For the mechanical disturbance no explicit formula can be given. But if the Fourier spectrum of a sufficient long interval τ of $X(t)$ is $X(\omega)$, than that of $\ddot{X}(t)$ is $\omega^2\, X(\omega)$, and:

$$I_M \approx m^2 \delta^2 \omega^4\, X^2(\omega)/\tau. \tag{2.5}$$

3. *Fluctuations with cold damped electrometer.* Cold damping is achieved by feeding back to the electrometer a voltage, proportional to $\dot{\varphi}$. Then equation 2.1 becomes

$$I\ddot{\varphi} + (\beta_L + \beta_E)\, \dot{\varphi} + D\varphi = F_B + F_M \tag{3.1}$$

The proportionality-factor β_E is called the electrical damping. Introducing the circular frequency without damping $\omega_0 = (D/I)^{\frac{1}{2}}$ and the critical damping $\beta_0 = 2(DI)^{\frac{1}{2}}$; the factor $g(\omega)$ becomes

$$g(\omega) = \frac{1}{D}\, \frac{1}{1 - (\omega/\omega_0)^2 + 2i(\beta_L + \beta_E)/\beta_0 \cdot (\omega/\omega_0)} \tag{3.2}$$

The fluctuation due to the Brownian movement becomes

$$\overline{\varphi_B^2} = (\beta_L + \beta_E)^{-1} \,.\, \beta_L kT/D \qquad (3.3$$

which describes the decrease in the Brownian fluctuation by the cold damping. The fluctuation due to the mechanical disturbance is found to be

$$\overline{\varphi_M^2} = (\beta_L + \beta_E)^{-1} S(B) \tag{3.4}$$

where

$$S(B) = \frac{\beta_0}{D^2} \int_0^\infty \frac{BI_M(\omega)\, \mathrm{d}\omega}{\{1 - (\omega/\omega_0)^2\}^2 + 4B^2(\omega/\omega_0)^2} \tag{3.5}$$

and

$$B = (\beta_L + \beta_E)/\beta_0 \tag{3.6}$$

$S(B)$ depends, except the Fourier spectrum and the constants of the electrometer, only on the relative damping B. Combined with 3.3 we find for the total quadratic fluctuation

$$\overline{\varphi^2} = (\beta_L + \beta_E)^{-1} \{\beta_L kT/D + S(B)\} \tag{3.7}$$

The most favorable case for measurements with a cold damped electrometer is the aperiodical case $\beta_L + \beta_E = \beta_0$. For larger damping the increase in the time needed for the measurement more than cancels the decrease in $\overline{\varphi^2}$. In this case the fluctuations may be decreased by decreasing the air-damping, but a limit is set to this decrease by the constant fluctuations due to the mechanical motions.

In order to test our formulas we have measured the fluctuations with different air and total damping (section 4). Moreover we have tried to measure the intensity of the mechanical disturbance spectrum (section 5). The function $S(B)$ computed from this spectrum, proved to be identical with that computed from the fluctuations and formula 3.7.

4. *Fluctuation measurements.* After slightly improving Milatz and van Zolingen's experimental arrangement many recordings were taken of the fluctuations of the electrometer. Most of them were made at night because of the fast varying irregular disturbances at daytime. Some of the best results are shown in fig. 1. Mostly the recordings with the highest damping and therefore with the smallest fluctuations showed unexpected characteristic saggings. We attributed these to disturbances in the neighbourhood of the laboratory. Therefore we only measured the parts of the recordings without them, as that shown in fig. 1*c*. In most cases these parts were only a

few (5 to 7)cm long, so the statistical precision of the measurement of the smallest fluctuations is only about 15%.

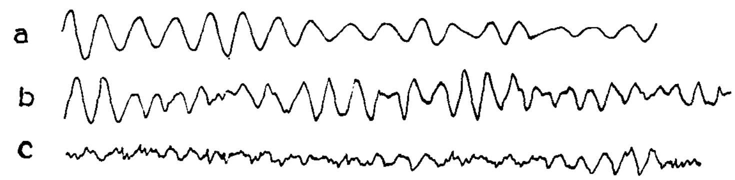

Fig. 1. Recording of the fluctuations of the electrometer on 12-2-1947:
a) only air damping $\beta_L = 0.0007$ units (1 mm = $2.57 \cdot 10^{-6}$ radian)
b) aperiodically damped (1 mm = $2.57 \cdot 10^{-7}$ radian)
c) most strong electrical damping (about 15 β_0) (1 mm = $5.13 \cdot 10^{-8}$ radian).

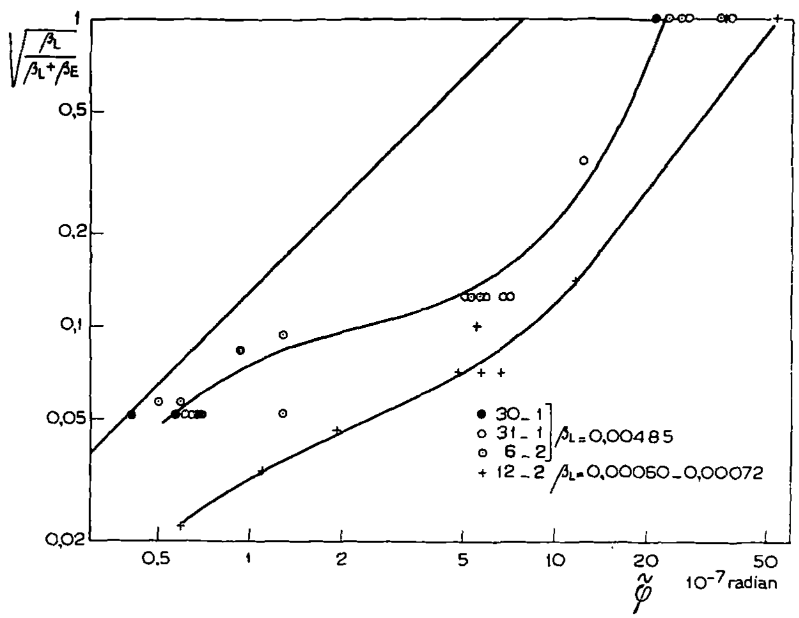

Fig. 2. The fluctuations of the moving system of the electrometer with different air- and electrical dampings. Smooth curves are drawn passing through points with (nearly) the same air damping.

The results of three nights (30–1, 2–2 and 12–2–1947) and one afternoon (31–1) with different total damping are shown in fig. 2. Horizontally the fluctuation $\tilde{\varphi} = (\overline{\varphi^2})^{\frac{1}{2}}$ is set, and vertically the factor $(\beta_L/(\beta_L + \beta_E))^{\frac{1}{2}}$. Without mechanical disturbance the fluctuations would lie on the straight line, according to formula 3.3. The intersection of this line with the upper bording-line of the diagram is the Brownian movement without cold damping. The electrometer was evacuated two months before the first readings were taken. Hence the pressure in the electrometer and the air-damping were stationary ($\beta_L = 0.0048$ dyne cm radian^{-1} sec). About five hours before the last measurements the electrometer was re-evacuated. Therefore the pressure and the damping increased during the measurements ($\beta_L = 0.00060 \ldots. 0.00072$ dyne cm radian^{-1} sec). The smallest fluctuation that we measured is only about 5% of the Brownian movement without cold damping. The temperature should be decreased till 0.8°K to get the same result by cooling.

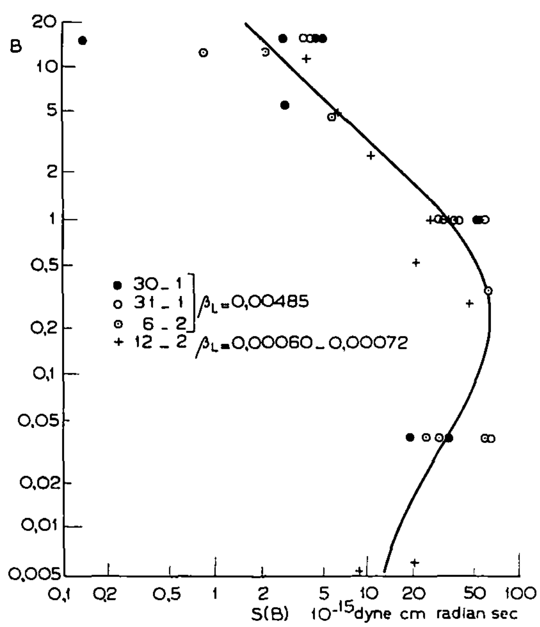

Fig. 3. The function $S(B)$ (explanation in text).

Using formula 3.7 the function $S(B)$ was derived from these measurements. This function is shown in fig. 3. Points taken on various days are nearly on the same curve. This was probably caused by the constancy of the microseismic movements during the period of our measurements. We have checked this constancy by analysing the seismograms which were friendly put at our disposal by the Kon. Ned. Meteorologisch Instituut at de Bilt. Only in not further discussed measurements at 2-2-1947 the fluctuations, and the microseismic movements too, were much larger.

5. *Conclusions from the Fourier analysis.* A Fourier analysis was made of the recording with aperiodical damping shown in fig. 1*b*. The technique of this analysis is described elsewhere [5]). The result is shown in fig. 4. For this analysis a part of the curve was repeated a number of times. Therefore the analysis gives a discrete Fourier spectrum. Because we do not need absolute values we only determined the relative amplitudes. From this result we can get a rather good approximation of the continuous spectrum $\varphi(\omega)$ for an

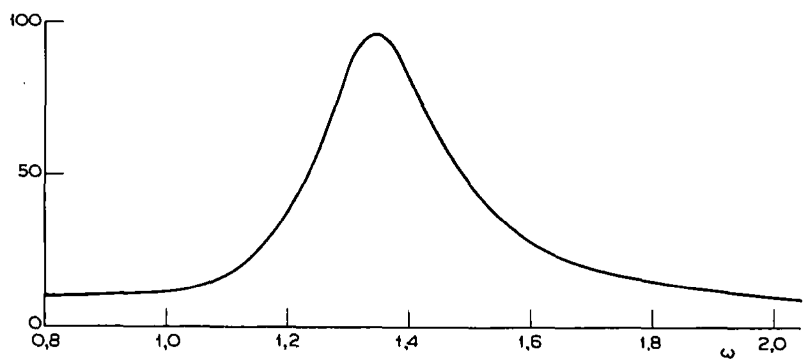

Fig. 4. Fourier analysis of the curve of fig. 1*b*.

infinitely long recording by drawing a smooth curve through the measured points (see fig. 4). From this curve we get, according to formula 2.3 and 2.2, an approximation

$$I_M \approx \varphi^2(\omega)/|\, g^2(\omega)\,|$$

where $g(\omega)$ (formula 3.2) should be taken for the aperiodical case $\beta_L + \beta_E = \beta_0$. Strictly speaking this yields $I_M(\omega) + I_B(\omega)$, but as the total fluctuation is 10.4 times the Brownian fluctuation, I_B is about $(10.4)^2 = 108$ times smaller, hence we can neglect I_B. This intensity $I_M(\omega)$ is shown in fig. 5. It has a rather sharp maximum for a period of 4.6 sec. As a check we have also analysed a seismogram of de Bilt. The agreement was as good as could be expected. Using formula 3.5 we now compute the function $S(B)$ by graphical integration. Taking the experimental value for $S(1)$ from fig. 3 we were able to compute the undetermined factor in $S(B)$ and therefore in the Fourier analysis.

It is easily shown that $S(B)$ tends to a limit for low values of B

$$S(0) = \tfrac{1}{2}\pi\, I_M(\omega)/D.$$

Using this formula we find $S(0) = 6.3 \cdot 10^{-15}$ dyne cm radian sec. For the lowest values B in the fig. 3 this value is already almost reached. This is in agreement with the fact that in the curve for the graphical integration of $S(0.01)$ the maximum due to the resonance of the electrometer is already higher than the top due to the maximum of $I_M(\omega)$.

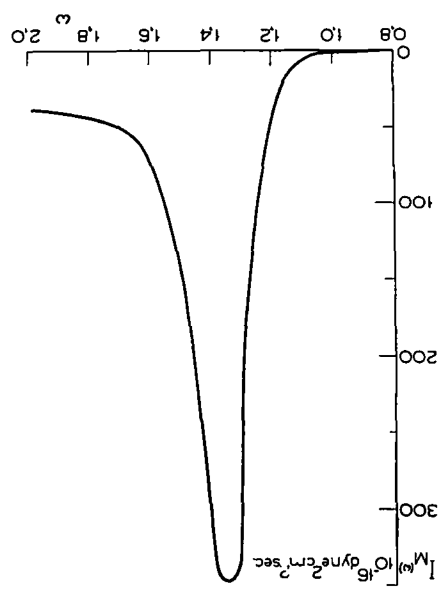

Fig. 5. The function $I_M(\omega)$ (explanation in text).

From the normalized function I_M we can compute the product of δ and the mean deviation due to the microseismic movements. With formula 2.5 we find

$$m^2\, \delta^2\, I_X(\omega) = I_M(\omega)/\omega^2$$

and by integrating

$$m^2\, \delta^2\, \overline{X}^2 = \int_0^\infty m^2\, \delta^2\, I_X(\omega)\,.\,\mathrm{d}\omega = 24.7 \cdot 10^{16}\, g^2\, \mathrm{cm}^4.$$

The mass of the system of the electrometer is $m = 7 \cdot 10^{-2}\,g$. De Bilt informed us, that on this day the mean amplitude of the microseismic movement was about 4 μ. We therefore find for the distance of the center of mass to the axis of rotation:

$$\delta = 1.8 \cdot 10^{-3}\ \text{cm}$$

which is an acceptable value.

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REFERENCES

1) Milatz, J. M. W. and van Zolingen, J. J., Physica **19** (1953) 181.
Van Zolingen, J. J., Thesis Utrecht 1948.

2) Schottky, W., Ann. Physik **57** (1918) 541.

3) Milatz, J. M. W., Ned. T. Natuurk. **8** (1941) 19.

4) Keller, K. J., Thesis Utrecht 1943.

5) Milatz, J. M. W., Wapstra, A. H. and Van Wieringen, J. S., Physica **19** (1953) 175.