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THE RADIANCE OF LUNAR OBJECTS NEAR OPPOSITION

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Abstract—The radiance of lunar objects at phase angles $|g| < 5°$ has been measured on plates taken at the Kirkwood and Yerkes Observatories during the lunar eclipse of 18 November 1956. The measurements have been combined on a uniform scale of brightness by comparison with photoelectric determinations of the radiance of the floor of Plato made for that purpose on the same night by Gehrels *et al.*[1] The lunation curves of Grimaldi, Copernicus and Tycho show the reality of a non-linear surge in radiance ("the opposition effect") for lunar objects. Between $g = -1{\cdot}4°$ and $g = -0{\cdot}7°$ there is a 10 per cent increase in the radiance. By linear extrapolation to $g = 0°$ we have found the normal albedo of 36 crater floors.

1. INTRODUCTION

The radiance of lunar objects at full phase cannot be observed from the Earth, because the full Moon is eclipsed at phase angle $g = 0°$. In the present paper we shall estimate values of the radiance (the brightness observed from the Earth) extrapolated to $g = 0°$.

Measurements of the radiance have usually been expressed in terms of the radiance of an ideal white screen. The radiance factor ρ is commonly used. It is defined as the observed radiance, divided by the radiance of a white screen normal to the incident light.

For a surface element on the lunar sphere, ρ is a function of the angle of incidence, i, the angle of observation, ε, and the phase angle, g; that is:

$$\rho = \rho_0 f(i, \varepsilon, g)$$

This function f is normalized to unity at $i = \varepsilon = 0$. The normal albedo ρ_0 is the value of ρ for an object near the centre of the full Moon[2]. Except for differences in the normal albedo all points of the disk of the Moon have the same radiance at full Moon[3,4]. This means that $f(i, \varepsilon, g) = 1$ if $i = \varepsilon$. In any case we may assume that the radiance ρ_0 of the individual lunar formations at full Moon corresponds to their normal albedo.

Several catalogues of normal albedos have been published[5,2,6,7], but the values in the catalogues do not correspond to the true full Moon, because this cannot be observed. Existing catalogues are strongly influenced by the photometric measurements of Fedorez[8], which contain an observation at $g = 1{\cdot}5°$. Even between $g = -4{\cdot}5°$ and $g = -1{\cdot}5°$ the mean increase of the radiance is still of the order of 15 per cent[9]. The correction for reducing observations near full Moon to the real normal albedo values is certainly not negligible and it may depend on the particular object. Orlova[10] tried graphical extrapolation toward $g = 0°$, and published a catalogue of improved values of the albedos. Better results, however, can be found by observing the variation of the radiance of selected lunar objects just before and after an eclipse and by investigating their lunation curves near full phase.

Laboratory measurements of the radiance of reflecting powders and ashes, made photoelectrically at the Utrecht Observatory with a mirror system, proved that the results for $i = 0°$ and $\varepsilon = 0°$ were some 10 per cent greater than those which would be obtained by extrapolation[2]. It is possible that the radiance of lunar objects near full Moon is underestimated. The observations of Götz[11] made at $g = -3°$ and $g = +4°$ are also suggestive. Comparison of 8 of his 55 points with those of other observers shows that his points generally lie far above the assumed mean curve[2]. For the asteroids a sharp increase in brightness was found close to opposition[12]. This increase was called the *opposition effect* and it may be present also in the lunation curves. The integrated radiance of the whole Moon obtained by Rougier[13] at all phase angles except the smallest appears to indicate an opposition effect. There are also physical arguments for the reality of this phenomenon which have been known in astronomy for a long time[14–20].

2. THE OBSERVATIONS

During the night of 18 November 1956, a total eclipse of the Moon enabled lunation curves of a number of lunar objects near $g = 0°$ to be investigated. The circumstances of the eclipse were as follows (Nautical Almanac, 1956):

Moon enters penumbra, 18 November	03^h	$59{\cdot}9^m$	U.T.
Moon enters umbra	05	02·6	
total eclipse begins	06	08·0	
middle of the eclipse	06	47·6	
total eclipse ends	07	27·3	
Moon leaves umbra	08	32·7	
Moon leaves penumbra	09	35·3	

During that night a series of 24 plates was taken at the Kirkwood Observatory of the Indiana University by E. C. Olson and C. T. Van Sant, with the 12-inch refractor equipped with an orange filter. In combination with the Eastern Super Orthopress plates the effective wave-length is about $\lambda 5500$. Weather and vision were good. Each plate contains two lunar images taken shortly after one another, numbered *a* and *b*. The data concerning these 24 plates are given in Table 1. The columns contain: plate number, date of the exposure, Universal Time and the phase angle g, corrected for parallax. The exposure time was less than 1 sec. The last columns contain a correction factor F for each separate lunar image which will be explained in section 3.

Seven other plates were taken at the Yerkes Observatory by G. P. Kuiper and A. Lenham. The weather impaired the quality of the plates. They were taken with the 40-inch refractor, combined with the parallax camera equipped with a yellow filter. Kuiper and Lenham reproduced the method and effective wave-length of a series of plates taken by Minnaert in 1946 and investigated photometrically by Van Diggelen[2]. The data concerning the seven plates are given in Table 2. The columns contain: plate number, Universal Time of the exposure and the phase angle g corrected for parallax.

The diameter of the lunar image on the Yerkes plates is 170 mm and on the Kirkwood plates 42·5 mm.

3. MEASUREMENTS AND REDUCTION METHODS

The Yerkes plates and some of the Kirkwood plates have been calibrated with a tube photometer. The transmission T was determined with the aid of the Utrecht microphotometer and as the intensity I of each step was known, we could make calibration curves for the plates from which the recorded transmission of a point on the lunar disk could be converted into intensity.

In order to combine the measures of different plates, the relative scale for each plate has to be reduced to a single uniform absolute scale of brightness. This could not be carried out by the method applied in a previous investigation[2] because the integrated radiance of the Moon is not exactly known at small phase angles.

TABLE 1. DATA OF THE KIRKWOOD PLATES

Plate number	Date 1956	U.T.	Phase angle (g) (deg.)	Correction factor for the two lunar images (F_{P_a})	(F_{P_b})
11	Nov 17	5^h10^m	−13·7	12·14	22·28
13		5 48	−13·3	8·40	10·20
15		7 07	−12·7	14·29	13·39
17		7 19	−12·6	11·64	13·28
19	Nov 18	2 20	−2·0	17·84	21·50
21		2 46	−1·8	8·04	11·95
23		2 48	−1·8	9·37	8·77
25		2 53	−1·8	10·08	9·10
27		3 18	−1·6	5·90	6·98
29		3 22	−1·6	8·22	5·90
31		3 25	−1·6	7·58	7·89
33		3 41	−1·5	8·68	5·61
35		3 44	−1·5	6·49	9·03
37		3 47	−1·4	6·92	11·78
39		3 56	−1·4	9·67	8·10
43		9 28	+0·7	13·71	11·94
47		9 37	+0·8	8·14	8·62
49		9 54	+0·9	10·50	7·30
51	Nov 19	3 32	+11·6	9·27	9·29
53		3 34	+11·6	11·33	5·13
55		3 42	+11·7	11·61	9·30
57	Dec 19	2 49	+19·5	10·51	10·20
59		2 51	+19·6	9·39	12·00
61		2 52	+19·6	10·32	11·90

TABLE 2. DATA OF THE YERKES PLATES

Plate number	U.T. Nov. 18 1956	Phase angle (g) (deg.)	Correction factor (R)
1	4^h07^m	−1·3	25·3
2	3 54	−1·4	11·15
3	3 53	−1·4	11·80
4	2 40	−1·8	11·75
5	9 33	+0·7	4·98
6	4 05	−1·3	5·74
7	9 35	+0·7	5·63

Direct comparison, however, can be made with photoelectric measurements taken just before the eclipse and on similar occasions during the following years by Gehrels, Coffen and Owings[1] and reduced by them to the absolute ρ-values of Van Diggelen, based on the integrated curve of Rougier. This was done for an interval of phase angles for which the photoelectric and the photographic lunation curves overlapped ($g = 2°$ to $g = 25°$). The values of ρ here and elsewhere are expressed consistently in units of 0·001. For the purpose of calibration two regions are recommended: they are the centres of Plato ($\lambda = -09°18'$, $\beta = +51°28'$) and of Copernicus ($\lambda = -20°08'$, $\beta = -10°11'$). The lunation curves of these calibration regions are known from many observations by different authors, and with the aid of the photoelectric measurements their shapes near $g = 0°$ could be obtained. They have been measured on all our Kirkwood plates together with the lunation curves of two other regions, the centre of Grimaldi ($\lambda = -67°49'$, $\beta = -5°20'$) and the centre of Tycho ($\lambda = -8°10'$, $\beta = -40°24'$). It appeared later that the last object was also measured by Gehrels. For each of the Kirkwood plates a correction factor F_P has been found by dividing the radiance of the floor of Plato at the corresponding phase, as given by Gehrels' photoelectric calibration curve, by the intensity measured on our plate. The value of this correction factor F_P is given for the plates in the last columns of Table 1.

In order to test whether a correction factor, determined for Plato, is also applicable to other crater bottoms, we investigated the F-values for the four different lunar areas in a range of g, where the lunation curves are sufficiently well known[(2)]. There is a satisfactory agreement between these F-values (cf. Table 3). We now feel justified in applying the correction factor F_P to all other crater bottoms near the opposition.

TABLE 3. *F*-VALUES AT LARGER PHASE ANGLES FOR THE KIRKWOOD PLATES

Plate number	Plato	Copernicus	Grimaldi	Tycho
11a	12·14	12·54	11·68	12·41
11b	22·28	22·87	21·90	22·08
13a	8·40	8·71	9·17	7·92
13b	10·20	11·30	9·48	10·18
15a	14·29	14·90	14·42	14·78
15b	13·39	12·38	11·30	15·62
17a	11·64	12·32	10·76	8·18
17b	13·28	13·68	12·75	12·84
51a	9·27	8·55	9·25	8·90
51b	9·29	9·32	9·25	8·24
53a	11·33	11·59	11·06	9·41
53b	5·13	4·70	5·32	5·52
55a	11·61	10·23	11·92	10·85
55b	9·30	9·14	9·55	8·03
57a	10·51	9·89	12·94	10·68
57b	10·20	10·01	10·40	10·08
59a	9·39	8·97	10·40	8·87
59b	12·00	11·68	12·30	11·97
61a	10·32	10·23	11·45	11·18
61b	11·90	10·12	13·01	11·88

Each measurement on a plate can now be converted into a uniform scale of intensity for all plates by multiplying it by the factor F_P. The four objects, Plato, Copernicus, Tycho and Grimaldi were chosen, because they could easily be measured even on the small-scale Kirkwood plates. The transmission measured in the centre of their floors was

TABLE 4. RESULTS OF THE KIRKWOOD PLATES USING THE *F* OF PLATO
ρ is given in units of 0·001

Phase angle = g (deg.)	Copernicus (ρ)	n	Grimaldi (ρ)	n	Tycho (ρ)	n
−13·1	74·1 ± 3·3	8	23·0 ± 1·2	8	104·3 ± 4·1	7
−2·0			52·2 ± 4·4	2	130·1	1
−1·8	121·2 ± 7·1	2	54·5 ± 1·7	6	135·8 ± 8·7	5
−1·6	125·8 ± 1·8	6	64·2 ± 2·0	6	143·6 ± 2·2	4
−1·5	135·5 ± 3·4	4	64·5 ± 1·9	4	150·0	1
−1·4	127·7 ± 2·8	2	68·3 ± 2·1	4	158·9 ± 6·5	3
+0·7	150·9 ± 9·8	2	84·7 ± 5·2	2	161·2	1
+0·8	133·6 ± 4·2	2	79·5 ± 5·1	2	161·0 ± 9·2	2
+0·9	121·6 ± 4·0	2	78·1 ± 5·5	2	149·2	1
+11·6	100·9 ± 5·6	6	57·3 ± 2·0	6	122·4 ± 8·7	6
+19·6	86·8 ± 2·5	4	45·6 ± 1·4	6	111·2 ± 9·0	6

converted into intensity. The results obtained after multiplying by the factor F_P have been given in Table 4. The number n in the table is the total number of measured transmissions of an object on different plates at the same phase angle which have been averaged in calculating the given value. In Fig. 1 the lunation curves near zero phase angle are shown. The reality of the opposition effect is apparent.

On each of the seven Yerkes plates we investigated the radiance of the floor of 38 craters. In our earlier work 39 craters including these 38 have also been measured. Only Clavius could not be identified on the plates, mainly because of the unfavourable libration. A mean correction factor R was determined by dividing the photoelectric radiance of Plato, Copernicus and Tycho by the measured intensity. The three values of R found were averaged (Table 2). The measured radiance of the crater floors on a given plate was multiplied by the correction factor of that plate (Table 5). They form an important addition to earlier published photometric measurements of the radiance of these objects at other phase angles.

4. DISCUSSION OF THE RESULTS

The reality of the opposition effect has already been demonstrated by the investigation of Gehrels *et al.*[1] used for our calibration. It is therefore not remarkable that the Copernicus, Tycho and Grimaldi results on the Kirkwood plates show an important deviation from a linear increase for $|g| \to 0°$, resulting in a clear jump at about $|g| = 2°$. The photometric measurements are smoothed by averaging different plates taken at about the same phase angle. The surge in the lunation curves, causing the opposition effect, was explained theoretically by Hapke[20].

The increase in brightness may have a considerable influence on the magnitude of objects at full Moon. We have tried to deduce the normal albedo of our 38 crater floors and of a number of lunar plains. The radiance on the Yerkes plates 4, 3, 2, 6 and 1 (all at about the same phase angle $|g| = 1{\cdot}4°$) was averaged and similarly the average radiance of the two other Yerkes plates taken at about $|g| = 0{\cdot}7°$ was calculated. The mean values are given in Table 5.

If we plot all determinations of the radiance of a given point (e.g. the floor of Tycho) at $|g| < 3°$ as a function of $|g|$ we get a series of points (Fig. 2) which do not lie on a curve. The scatter may be due to the lack of precision inherent in all photographic photometry, but it may also be due to luminescence effects. As shown by Gehrels *et al.*[1] there was a 10–20 per cent difference in radiance of the lunar surface, found by comparing results of 1956 with those of 1963. The effect was fairly constant from day to day. These effects may also have caused scatter in the points of Fig. 2. It would be desirable to combine only points of observations made at the same stage of the solar cycle. We have assumed that the relation between ρ and $|g|$ in this range is linear and may be extrapolated by this approximation in the same way to $|g| = 0°$. This extrapolation inevitably is somewhat uncertain. The same method has been used for all other crater floors. By extrapolating the radiance at $|g| = 1{\cdot}4°$ and at $|g| = 0{\cdot}7°$ (9th and 10th column of Table 5) we found the normal albedo given in the 11th column of Table 5.

The mean value of all objects is $\bar{\rho}(1{\cdot}4) = 104$; $\bar{\rho}(0{\cdot}7) = 114$ and $\bar{\rho}(0) = 125$. Between $|g| = 1{\cdot}4°$ and $|g| = 0{\cdot}7°$ the mean increase of the radiance is of the order of 10 per cent.

The radiance of the floor of some lunar plains has been determined on two plates at $g = -1{\cdot}4°$ (Table 6). For extrapolating the results to $g = 0°$ all measurements were multiplied by a correction factor, $\bar{\rho}(0)/\bar{\rho}(1{\cdot}4)$, obtained from Table 5. In doing so the lunation curves of all objects for $|g| < 2°$ were assumed to have about the same slope which

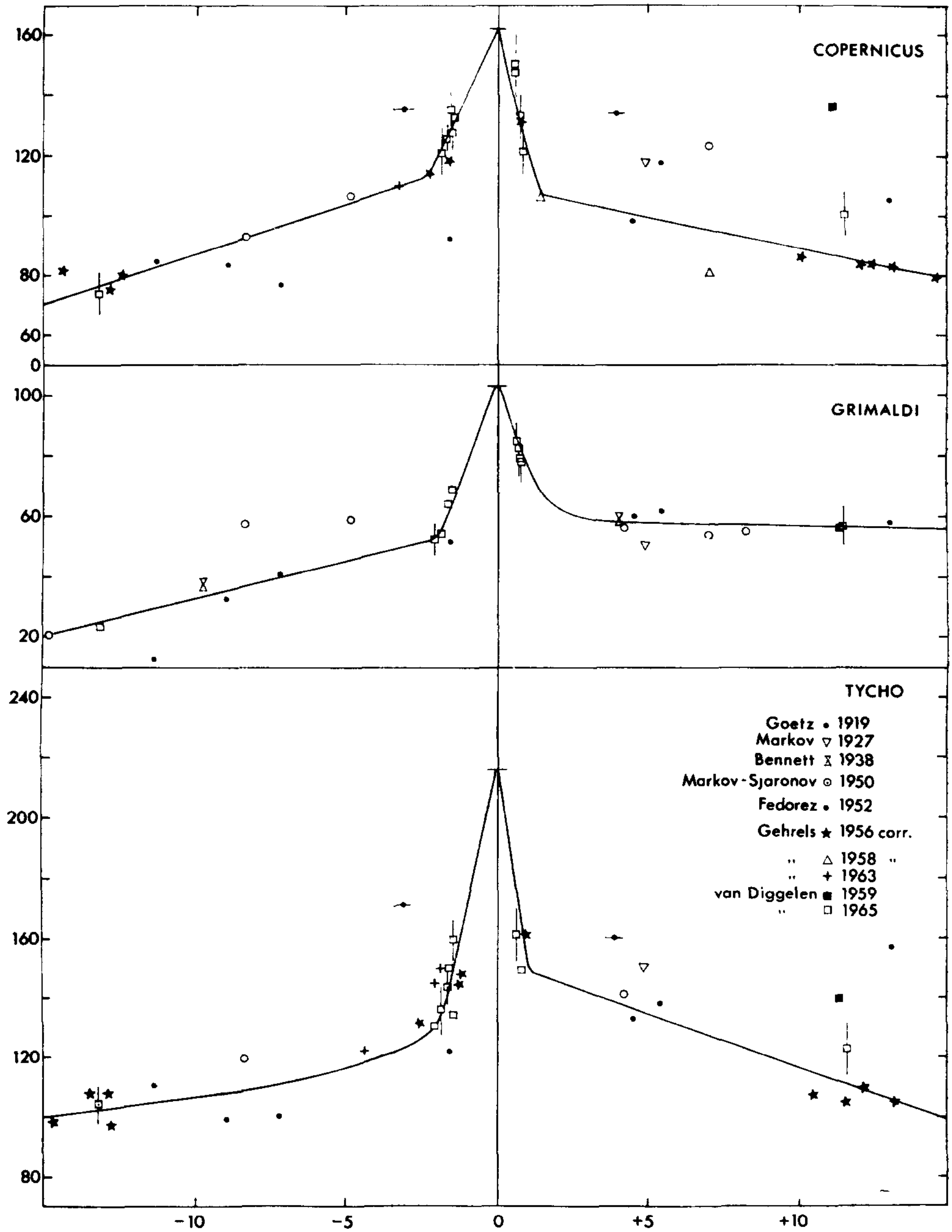

FIG. 1. LUNATION CURVES NEAR FULL MOON WITH OBSERVATIONS GIVEN IN THIS PAPER (SQUARES) TOGETHER WITH OLDER PHOTOGRAPHIC RESULTS AND THE PHOTOELECTRIC OBSERVATIONS OF GEHRELS *et al.*[1] RADIANCE ρ IN UNITS OF 0·001 AS A FUNCTION OF THE PHASE ANGLE g.

TABLE 5. RESULTS OF THE YERKES PLATES, MEAN VALUES OF THE RESULTS AT TWO PHASE ANGLES AND VALUES OF THE NORMAL ALBEDO ρ_0

Floor of crater	ρ = radiance on plates in units of 0·001									
	4	3	2	6	1	5	7	$\rho(1{\cdot}4)$	$\rho(0{\cdot}7)$	$\rho(0)$
Albategnius	109	106	114	107	95	120	117	106	119	132
Alphonsus	101	100	113	98	104	111	113	103	112	121
Archimedes	74	85	82	80	78	90	85	78	88	98
Aristarchus	175	180	185	174	180	206	230	179	218	257
Aristyllus	91	96	110	95	99	110	111	98	111	124
Bonpland	94	83	109	101	94	104	97	96	101	106
Billy	68	62	101	72	72	85	72	75	79	83
Bullialdus	93	123	131	124	107	126	131	116	129	142
Campanus	87	107	107	83	101	107	111	97	109	121
Cassini	103	126	141	103	137	113	123	122	118	?
Cleomedes	90	96	134	101	134	118	115	111	117	123
Copernicus	119	124	144	127	149	158	138	133	148	163
Cyrillus	94	127	144	100	112	124	123	115	124	133
Eratosthenes	94	120	125	89	111	116	113	108	115	122
Eudoxus	95	120	125	88	115	116	113	109	115	121
~~Gassendi~~	~~83~~	~~101~~	~~105~~	~~87~~	~~117~~	~~112~~	~~102~~	~~99~~	~~107~~	~~115~~
Grimaldi	63	61	71	51	75	79	89	64	84	104
Guericke	98	97	121	104	106	115	107	105	111	117
Hevelius	93	96	125	95	120	121	111	106	116	126
Landsberg	88	99	119	88	112	123	118	101	121	141
Maginus	123	136	138	130	134	140	139	132	140	148
Manilius	101	98	118	73	112	125	110	100	118	136
Marius	59	60	65	61	66	72	74	66	73	80
Maurolycus	103	94	119	108	119	105	103	109	104	?
Mercator	82	107	113	86	81	91	108	94	100	106
Petavius	101	91	105	89	75	112	104	92	108	124
Piccolomini	110	119	141	114	102	127	128	117	128	139
Pitatus	71	77	87	75	71	79	81	74	80	86
Plato	65	76	80	53	63	74	79	65	77	89
Plinius	92	101	105	102	84	106	102	97	104	111
Posidonius	102	98	100	95	97	106	103	98	105	112
Ptolemaeus	101	107	117	105	104	116	117	107	117	127
Pytheas	92	101	116	101	107	116	107	103	112	117
Reinhold	98	102	120	98	107	115	113	105	114	123
Stöffler	100	103	119	109	104	113	125	107	119	131
Thebit	89	88	95	77	92	89	87	88	88	88
Tycho	130	120	145	137	137	153	169	134	161	188
Walter	125	130	141	134	132	135	145	132	140	148

appears justified because the dark plains were connected with crater floors that resemble the plains in their properties. The results are in satisfying agreement with the photoelectric work of Gehrels *et al.*[1] in which the normal albedo was determined in a different way.

5. CONCLUDING REMARKS

This paper gives a series of measurements of the radiance of a number of lunar crater floors near full Moon. As a consequence of the large number of plates taken at Kirkwood during the eclipse night, each with two separate lunar images, a mean radiance was determined at the same phase angles from a number of plates (n). The values of n are given in Table 4. Comparison of the mean radiance of an object with the individual measurements gives an estimate of the scatter and of the precision of the results.

Though there may be a great deviation in earlier measurements it seems necessary to investigate first the possibility of explaining the scatter found by the lunar observers in

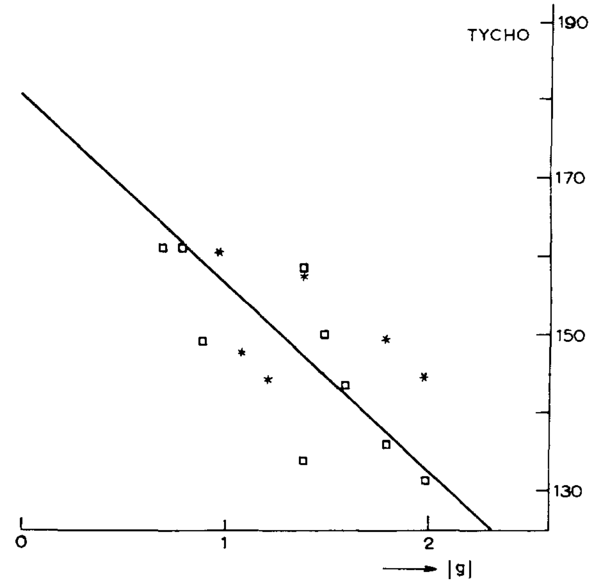

FIG. 2. THE OBSERVATIONS OF THE RADIANCE OF TYCHO FOR $|g| < 3°$ DO NOT DETERMINE A SMOOTH LUNATION CURVE. THE SYMBOLS ARE THE SAME AS IN FIG. 1.

We have assumed that the lunation curve may be approximated by a straight line and may be extrapolated in the same way to $g = 0°$.

TABLE 6. NORMAL ALBEDOS OF LUNAR PLAINS

Object	ρ on plate 3 (in units of 0·001)	$\rho(0)$
M. Serenitatis	70–73	84–88
M. Tranquillitatis	65–68	78–83
M. Crisium	77–85	93–103
M. Imbrium	81–98	98–118
M. Humorum	81	98
M. Nubium	79	96
O. Procellarum	86–91	104–110

the radiance of the same object during the last 50 years by solar influences. There are many indications that solar effects may cause considerable differences in the brightness of lunar objects.

In our discussion we have assumed that all lunation curves show a maximum at full Moon ($g = 0°$). Previously we had found some objects which reached their greatest radiance *after* full Moon[(2)]. This effect was chiefly found on Aristarchus, Copernicus, Kepler and Tycho. It was indicated, however, from lunation curves which were determined near $g = 0°$ by only a limited number of points. It is possible that luminescence effects have also influenced these observations. This may explain why some points just after full Moon gave indications of an increasing radiance. Moreover, there seems to be a much greater scatter in the observed points after full phase for Copernicus and Tycho than for Grimaldi (Fig. 1). The lunation curves of a selected number of lunar regions should be measured photoelectrically throughout a solar cycle.

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Резюме—Светимость лунных объектов при углах фаз в 5° измерялась на фотопластинках снятых обсерваториями "Кирквуд" и "Теркес" во время лунного затмения 18 ноября 1956г. Измерения были комбинированы на равномерной шкале яркости, по сравнению с фотоэлектрическими определениями светимости дна Платона, сделанного для этой цели в ту же ночь Гэрэлом и др. Лунационные кривые Гримальди, Коперник и Тихо указывают на подлинную сущность нелинейного напряжения в светимости ("оппозиционный Эффект") касательно лунных объектов. Между $g = 1°{\cdot}4$ и $g = 0°{\cdot}7$ существует 10-процентное возрастание в светимости. Посредством линейной экстраполяции к $g° = 0$, мы обнаружили нормальную отражательную способнось дна 36 кратеров.