Hall Effect in SN and SNS Junctions

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(Received 17 March 1997)

Hall effect in SN and SNS junctions is considered. It is shown that at low temperatures the Hall voltage is significantly suppressed as compared to its normal metal value. The time dependence of the Hall voltage in SNS junctions has a form of narrow pulses with the Josephson frequency. [S0031-9007(98)05980-8]

PACS numbers: 74.50.+r, 05.20.–y, 82.20.–w

Recent progress in microfabrication technology has rekindled the interest to low temperature proximity effect in superconductor–normal-metal (SN) junctions [1–10]. In this paper we consider the Hall effect in superconductor–normal-metal and superconductor–normal-metal–superconductor (SNS) junctions. At low temperatures $T \ll \Delta$, quasiparticles with energies $\epsilon \ll \Delta$ cannot tunnel from the metal into the superconductor. Here Δ is the value of the gap in the superconductors. On the other hand the tunneling of electron pairs, which is known as Andreev reflection, is possible. This gives rise to a coherence between electrons and holes inside the metal, which extends over distances of the order of $L_\epsilon = \sqrt{D/\epsilon} \gg l$. Here $D = \frac{1}{3} v_F l$ is the electron diffusion coefficient, l is the elastic mean free path, and v_F is the Fermi velocity. Consequently, the wave packets which carry the current in the metal are coherent superpositions of electron and hole wave functions. At $T \ll 1/\tau = v_F/l$ the packets' size is of order $L_T = \sqrt{D/T} \gg l$ and their effective charge is much smaller than the electron charge e. The above mentioned electron-hole coherence manifests itself in experimentally observable effects. For example, the density of states in the metal near the SN boundary $\nu(\epsilon, \vec{r})$ is significantly suppressed as compared with the bulk metal value $\nu_0 = m^2 v_F$ [8–10]; the low temperature resistance of the SN junction and the electric field distribution near the SN boundary turn out to be very sensitive to the phase breaking rate in the metal [4–7].

The suppression of the effective charge should also lead to a suppression of the Hall effect. We show below that the Hall voltage in SN junctions measured with the help of leads 1,2 shown in Fig. 1(a) is indeed significantly suppressed as compared to its bulk metal value. It is worth mentioning here that at zero temperature the resistance of a long SN junction is approximately the same as the resistance of the normal segment [4–7]. Therefore the suppression of the Hall voltage is not directly connected to the suppression of the electron density of states near the SN boundary [8–10].

We also show that the Hall voltage in SNS junctions exhibits oscillations of its amplitude with Josephson frequency and with no oscillations of the sign. It is different from the behavior of supercurrent through the junction which at a given voltage oscillates both in amplitude and in sign.

In this article we neglect the electron-electron interaction in metal and are interested only in effects linear in voltage applied to the junction. In the "clean limit" when $\Delta\tau, T\tau \gg 1$, the kinetic scheme describing Hall effect in superconductors was developed in [11,12]. In this limit, however, the above mentioned proximity effect leads only to small corrections to the value of the Hall voltage in metal. To describe the Hall effect in a metal near the SN boundary in the "dirty limit" when $T \ll \Delta, 1/\tau$ we use the following set of equations analogous to that in [13,14]:

$$\vec{J} = \vec{J}_s + \vec{J}_n + \vec{J}_H ,$$

$$\vec{J}_s = \frac{D}{4}\nu_0 \int d\epsilon f_0(\epsilon)\,\mathrm{Tr}(\sigma_z[G^R\nabla G^R - G^A\nabla G^A]), \qquad \vec{J}_n = \frac{D}{4}\nu_0 \int d\epsilon \nabla f_1(\epsilon)\,\mathrm{Tr}(1 - \sigma_z G^R \sigma_z G^A),$$

$$\vec{J}_H = \frac{\omega_c\tau}{16} D\nu_0 \int d\epsilon \vec{e}_b \times \nabla f_1 \times \mathrm{Tr}\{(1 + \sigma_z G^R\sigma_z G^R - G^A G^R - \sigma_z G^A \sigma_z G^R)(G^R\sigma_z - G^A\sigma_z)\}; \tag{1}$$

$$e\Phi(\vec{r}) = \int d\epsilon f_1(\epsilon, \vec{r})\nu(\epsilon, \vec{r}), \qquad \nu(\epsilon, \vec{r}) = \frac{1}{2}\mathrm{Tr}(\sigma_z G^R - G^A\sigma_z); \tag{2}$$

$$\epsilon\{G^R\sigma_z - \sigma_z G^R\} + D\partial\cdot(G^R\partial G^R) = I^R_{\mathrm{ph}}; \tag{3}$$

$$\nabla\{\mathrm{Tr}(1 - \sigma_z G^R\sigma_z G^A)\nabla f_1 + \mathrm{Tr}[\sigma_z(G^R\partial G^R - G^A\partial G^A)]f_0 + \omega_c\tau\vec{e}_b \times \nabla f_1\,\mathrm{Tr}[(1 + \sigma_z G^R\sigma_z G^R - G^A G^R - \sigma_z G^A\sigma_z G^R)(G^R\sigma_z - \sigma_z G^A)]\} = \mathrm{Tr}(\sigma_z I_{\mathrm{ph}}). \tag{4}$$

Here $\vec{J}_s, \vec{J}_n, \vec{J}_H$ are the supercurrent density, normal current density, and the Hall current density, respectively, $f_0 = \tanh(\epsilon/2kT)$ is the Fermi distribution function, $\omega_c = eH/mc$ is the cyclotron frequency; H is the magnetic field, and $\vec{e}_b$ is the unit vector in the direction of magnetic field; $\Phi = 1/2\partial_t\chi + \phi$, ϕ, and χ are the gauge invariant scalar potential, electrical potential, and phase of anomalous Green's function, respectively; σ_z is the Pauli matrix, $\partial X = \nabla X - i(\sigma_z X - X\sigma_z)\vec{A}$ is the covariant derivative, $\vec{A}$ is the vector potential of the magnetic field; $G^{R,A}$ are retarded and advanced Green's functions which are matrices in the Nambu space

$$G^{R,A}(\epsilon,\vec{r}) = \begin{pmatrix} g^{R,A} & F^{R,A} \\ -F^{R,A} & g^{R,A} \end{pmatrix}, \qquad (5)$$

$g^{R,A}(\epsilon,\vec{r})$ and $F^{R,A}(\epsilon,\vec{r})$ are normal and anomalous Green's functions, respectively. Thus the set of equations consists of electroneutrality condition Eq. (2), the Usadel equation for retarded and advanced Green's functions $G^{R,A}$ Eq. (3), and the kinetic equation Eq. (4) for the distribution function $f_1(\epsilon,\vec{r})$ which describes the imbalance of populations between the electron and hole branches of spectrum [13,14]. The scattering integrals $I^R_{\rm ph} \approx i\sigma_z\tau_{\rm in}$ and $\mathrm{Tr}(\sigma_z I_{\rm ph})$ describe the broadening of the spectrum and the imbalance charge relaxation due to inelastic processes. In Eqs. (3),(4), we take into account that in noninteracting metal the order parameter and the supercurrent are zero.

In the zeroth order approximation in the parameter $\omega_c\tau$, these equations were derived in [13]. However, to describe the Hall effect one has to keep terms linear in $\omega_c\tau$. The problem of the Hall effect in the mixed state of superconductors was addressed recently in [12,15,16]. We derive Eqs. (1),(3) which include the contributions linear in $\omega_c\tau$ using the same procedure as in [11,12,15,16].

Let us now consider the Hall effect in the SN junctions shown in Fig. 1(a). We assume that the magnetic field is smaller than the critical one and does not penetrate into the superconductor. Taking into account the normalization condition $[g^R(\epsilon,\vec{r})]^2 = F^R_2(\epsilon,\vec{r})F^R_1(\epsilon,\vec{r}) + 1$ one can express g^R, F^R, and Eqs. (1)–(4) in the form

$$\cos\theta(\epsilon,\vec{r}) = g^R(\epsilon,\vec{r}), \qquad \sin\theta(\epsilon,\vec{r}) = iF^R_1(\epsilon,\vec{r}), \quad (6)$$

$$\frac{D}{2}\nabla^2\theta(\epsilon) + \left(i\epsilon - \frac{1}{\tau_{\rm in}}\right)\sin\theta(\epsilon) - \frac{D}{4}\left(\frac{eH}{\hbar c}\right)^2 x^2 \sin 2\theta = 0, \qquad (7)$$

$$\nabla\cdot\{\cosh^2\theta_2\nabla f_1 + \tau\omega_c\vec{e}_b \times \nabla f_1 \cos\theta_1 \cosh^3\theta_2\} = 0,$$

$$\vec{J} = D\nu_0 \int d\epsilon\{\cosh^2\theta_2\nabla f_1 + \tau\omega_c\vec{e}_b \times \nabla f_1 \cos\theta_1\cosh^3\theta_2\}, \qquad (8)$$

where $\theta = \theta_1 + i\theta_2$ is the complex variable. We have chosen $\vec{A} = \vec{e}_y Hx$.

The boundary conditions for Eqs. (7),(8) have the forms [17]

$$D\vec{n}\cdot\nabla\theta(\epsilon,\vec{R}) = t\sin[\theta(\epsilon,0^+) - \theta(\epsilon,0^-)],$$
$$D\cosh\theta_2^2\cdot\nabla f_1 = tv_F\{f_1(0^-) - f_1(0^+)\} \times \cosh\theta_2(0^+)\cosh\theta_2(0^-) \times \cos[\theta_1(0^+) - \theta_1(0^-)],$$
$$f_1(0) = 0,$$
$$f_1(\epsilon, x=\infty) = -eV\partial_\epsilon f_0(\epsilon). \qquad (9)$$

Here $\vec{n}$ and t are the unit vector perpendicular to the boundary and the transmission coefficient of the boundary, respectively. Indexes $+,-$ indicate two sides of the SN or NN_p interface [Fig. 1(a)]. N_p represents lead 1,2. Inside the superconductor $\theta(\epsilon \ll \Delta) = \pi/2$ while inside the bulk metals $\theta = 0$.

Solving Eqs. (8),(9) in zeroth order approximation in $\omega_c\tau$ we get the expressions for the conductance $G_{\rm SN} = I_0/U_0$ of the junction shown in Fig. 1(a) [4–7] measured by the two probe method and the conductance G_p measured between lead 1 and lead 2. (Here I_0 is the current

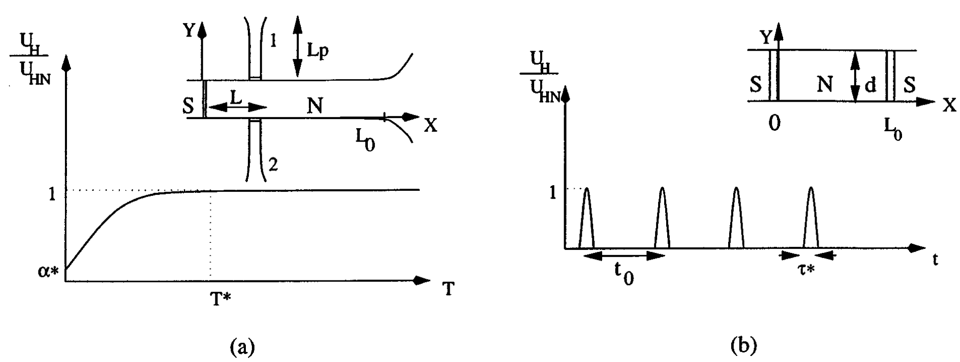

FIG. 1. (a) Temperature dependence of Hall voltage in SN junction. $\alpha^* = \max\{\frac{L_t}{L_0}, \frac{L}{L_0}\}$, $T^* = \min\{\frac{D_0}{L^2}, \frac{D_0}{L_t^2}\}$. (b) Time dependence of Hall voltage in SNS junctions with geometry shown in the inset, $\tau^* = t_0 T/E_c \ll t_0 = 1/2eU_0$. U_0 is applied along X direction and U_H is measured along Y direction in normal metals.

through the junction and U_0 is the voltage across the junction.) That is

$$G_{\rm SN}(T) = S_0\sigma_0 \int d\epsilon\,\partial_\epsilon f_0 \frac{1}{L_S(\epsilon)}\,, \qquad G_p(T) = S_p\sigma_p \int d\epsilon\,\partial_\epsilon f_0 \frac{1}{2L_p(\epsilon)}\,, \tag{10}$$

where

$$\begin{aligned} L_S(\epsilon) &= \frac{L_t}{\cosh\theta_2(\epsilon,0)\sin\theta_1(\epsilon,0)} + \int_0^{L_0} dx\,\frac{1}{\cosh\theta_2^2(\epsilon,x)}\,,\\ L_p(\epsilon) &= \frac{L_{tp}}{\cosh\theta_2(\epsilon,L)\cosh\theta_{2p}(\epsilon,0)\cos[\theta_1(\epsilon,L)-\theta_{1p}(\epsilon,0)]} + \int_0^{L_p} dy\,\frac{1}{\cosh\theta_{2p}^2(\epsilon,y)}\,, \end{aligned} \tag{11}$$

$S_{0,p}$ are the cross sections of the sample and the leads, $\sigma_{0,p}, D_{0,p}$ are the Drude conductivities and diffusion constants in the sample and leads; $L_t = D_0/t_0 v_F, L_{tp} = D_p/t_p v_F$ are the length characterizing SN and NN_p boundaries, t_0 and t_p are the transmission coefficients for SN and NN_p boundaries, $\theta_p(\epsilon,y)$ is the solution of Eq. (7) inside the leads.

In the first order approximation in $\omega_c\tau$ we get an expression relating the Hall current I_H and Hall voltage U_H measured by leads 1,2

$$I_H = G_p(T)U_H(T) = \tau\omega_c I_0 \frac{d}{G_{\rm SN}(T)} \int de\,\frac{S_p\sigma_p}{L_p(\epsilon)}\,\frac{1}{L_S(\epsilon)}\,\frac{\cos\theta_1(\epsilon,L)}{\cosh\theta_2(\epsilon,L)}\,\partial_\epsilon f_0\,. \tag{12}$$

Here d is the width of the junction (see Fig. 1). The values of θ_1, θ_2, and U_H (as well as $G_{\rm SN}$ and G_p) depend on the processes breaking the electron-hole coherence inside the sample and in the leads. Therefore, they are very sensitive to the ratio between parameters $L_0, L_T, L_t, L_{\rm tp}$ [4,5,7].

The solution of Eq. (7) in the metal for SN junctions when $L_0 \gg L_t$ with the corresponding boundary conditions in Eq. (9) is [7]

$$\theta(\epsilon,L) = \begin{cases} \frac{\pi}{2} - \sqrt{2}\,\frac{L_t+L}{L_0}\,, & L_t, L \ll L_0 \ll L_\epsilon\,;\\ \frac{\pi}{2} - (1-i)\,\frac{L_t+L}{L_\epsilon}\,, & L_t, L \ll L_\epsilon \ll L_0\,;\\ \frac{\sqrt{2}\,\pi(1+i)L_\epsilon}{4L_t}\exp[-(1-i)\,\frac{x}{\sqrt{2}L_\epsilon}], & L_\epsilon \ll L_t \ll L_0\,. \end{cases} \tag{13}$$

The most interesting results appear in the cases when the processes which break the electron-hole coherence are not effective and the value of $\theta_1(T,L)$ is close to $\pi/2$. At small T and large t

$$\frac{U_H}{U_{HN}} \propto \begin{cases} \max\{\frac{L_t}{L_0}, \frac{L_t}{L_T}\}, & L \ll L_t \ll L_T\,;\\ \max\{\frac{L}{L_0}, \frac{L}{L_T}\}, & L_t \ll L \ll L_T\,. \end{cases} \tag{14}$$

Here $U_{HN} = \omega_c\tau d I_0/S_0\sigma_0$ is the Hall voltage measured by the leads 1,2 in the absence of the proximity effect. The main feature of Eq. (14) is that U_H is significantly suppressed as compared with U_{HN} [see Fig. 1(a)].

At high temperatures when $L_T \ll L_t$ or L the proximity effect gives only small corrections to U_{HN}

$$\frac{U_{HN} - U_H}{U_{HN}} \propto \min\left\{\frac{L_T^2}{L_t^2}, \frac{L_T^2}{L^2}\right\} \ll 1\,. \tag{15}$$

The above results are obtained in the limit of low magnetic field when $L_H \ll L_T$. Here $L_H = \sqrt{\Phi_0/H}$ is the magnetic length, Φ_0 is flux quantum. In the opposite limit one should substitute L_T for L_H in Eqs. (14),(15). As we have mentioned the value of U_H is very sensitive to the nature of the leads. The requirement that the leads do not contribute to the breaking of the electron-hole coherence is $d \ll L_{tp}, L_t \ll L_0$.

Let us now turn to the discussion of the Hall effect in SNS junctions. In this case the ac Josephson effect causes the values of $\theta(\vec{r},\chi_0)$ and $\nu(\vec{r},\chi_0)$ to be time dependent. At small U_0 these dependences adiabatically follow the corresponding time dependence of the order parameter phase difference χ_0 across the junction: $\partial_t\chi_0(t) = 2eU_0$. For simplicity let us consider the case of a thin junction when $d \ll L_H$ when one can neglect any y dependence of χ_0 along the junction. It is well known that at $\chi_0 = 0$, the energy dependence of $\nu(\epsilon) = \cos\theta_1(\epsilon)\cosh\theta_2(\epsilon)$ exhibits a gap $E_g(\chi_0 = 0) \sim E_c = D/L_0^2$ [3]. The χ_0 dependences of θ, ν, and E_g were calculated in [10,18]. It was shown that the value of the gap E_g monotonically decreases with χ_0 and closes at $\chi_0 = \pi$. Hence $\cos\theta_1(\epsilon) = 0$ when $\epsilon \le E_g(\chi_0)$. In two limiting cases when χ_0 is close to π or 0 $E_g(\chi_0)$ is determined by [18]

$$E_g(\chi_0) = E_g(0)\begin{cases} (1 - C\chi_0^2), & \chi_0 \ll \pi\,;\\ C_1(\pi - \chi_0), & \pi - \chi_0 \ll \pi\,. \end{cases} \tag{16}$$

Here C and C_1 are of the order of unity. It follows from Eq. (12) that the Hall voltage U_H is a periodical function of time with a period $1/2eU_0$. At low temperatures $T \ll E_c$ the value of $U_H(t)$ is exponentially small except when the gap is small $E_g[\chi_0(t)] \sim T$. As a result $U_H(t)$

consists of short pulses of duration $\tau^* \sim T/E_c eU_0$ with maximums of the order of U_{HN} [see Fig. 1(b)]. Let t_n be the time when the nth maximum of $U_H(t)$ occurs. Then at $\tau^* \ll |t - t_n| \ll 1/2eU_0$,

$$U_H(t) \sim U_{HN} \exp\left(-\frac{|t - t_n|}{\tau^*}\right). \tag{17}$$

The main contribution to the Hall current averaged over the period of oscillations comes from the time intervals $|t - t_n| \sim \tau^* \ll 1/U_0$ and as a result

$$\langle U_H \rangle \propto \frac{T}{E_c} U_{HN} . \tag{18}$$

We would like to mention the difference in the temperature dependences in the SN and SNS cases. In the first case $U_H \sim \sqrt{T}$, while in the second case $U_H \sim T$. Let us note that the time dependence of the resistance of the junction exhibit peaks which are similar to the above considered peaks of the Hall voltage.

In addition to the quasiparticle's contribution to the Hall current in SNS junctions there is another contribution which can be associated with the supercurrent [15,16]. We believe, however, that in the limit when $d \ll L_H$, the supercurrent part of the Hall current does not contribute to the Hall voltage. In the case of SN junctions the supercurrent contribution to the Hall current does not exist.

In this paper we considered the limit $eU_0 \ll 1/\tau_\epsilon$ when it is possible to neglect nonequilibrium corrections to the quasiparticle distribution function inside the normal metal. In the opposite limit the quasiparticle distribution in the metal at $\epsilon < E_c$ becomes nonequilibrium and as a result the duration of the pulses τ^* becomes of the order of $(eU_0)^{-1}$.

The above results were obtained for the quantities averaged over realizations of random potential. We are planning to consider mesoscopic contributions in U_H elsewhere.

We acknowledge useful discussions with B. Altshuler, A. I. Larkin, and B. Pannetier. This work was supported by the Division of Materials Sciences, U.S. National Science Foundation under Contract No. DMR-9625370.

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