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SHELL MODEL CALCULATIONS OF NUCLEAR MAGNETIC MOMENTS AND M1 TRANSITION PROBABILITIES IN THE $2s_{\frac{1}{2}}$ $1d_{\frac{3}{2}}$ SHELL

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Abstract: Recently, shell model wave functions for low-lying nuclear states in the range ^{29}Si–^{40}Ca with intermediate coupling and configurational mixing, have been calculated. In this paper these wave functions are used to obtain numerical values for magnetic dipole moments and magnetic dipole transition rates.

Expressions are given for the matrix elements of the magnetic dipole operator in terms of single-particle matrix elements. The ground state magnetic moments and the widths for M1 transitions to the ground state are calculated ((i) with single-particle g values; (ii) with effective g values) for the nuclei in the range ^{29}Si–^{40}Ca. The results are tabulated.

1. Introduction

Simple calculations of the nuclear magnetic dipole moment, based on the extreme single-particle model, do not reproduce quantitatively the experimental values [1]). Yet the predictions are in agreement with the general trend.

Several reasons can be put forward to account for the discrepancy between the predictions of the extreme single-particle model and experimental results. Most important of these is the fact that this model is not able to account for the finer details of nuclear structure. The model should be replaced by the individual particle model, with intermediate coupling and configurational mixing. In particular exchange currents may be expected to modify the magnetic multipole moments [2,3]). There may also be some effect from velocity dependent forces [4]).

In this paper we propose to calculate nuclear moments from shell model wave functions. A recent calculation [5,6]) of nuclear states in the $2s_{\frac{1}{2}}$ $1d_{\frac{3}{2}}$ shell (^{29}Si–^{40}Ca), based on the individual particle model with intermediate coupling and configurational mixing, has supplied the necessary wave functions. In sect. 2 we shall give some definitions, and discuss the reduction of the matrix element of the M1 operator in a complex nuclear configuration to single-particle matrix elements. In sect. 3 we shall give expressions for the static magnetic dipole moments, as well as for the reduced M1 transition probabilities. Finally in sect. 4 we present the results of a numerical calculation and give a discussion.

2. Definitions and Reduction of the Transition Matrix Element

We consider nuclei in the range ^{29}Si–^{40}Ca. Any nucleus in this range is regarded as consisting of an inert ^{28}Si core of closed shells and $A-28$ nucleons which populate the $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ shells. Here A represents the mass number of the nucleus. Since the core has spin zero, it does not contribute to the nuclear moments. Hence nuclear moments of nuclei in the particular range are attributed to the configuration in the sd shell.

Let a configuration of a nuclear state with total spin J and isospin T consist of n_k particles in the s-shell, their spins and isospins coupled to J'_k and T'_k, and m_k particles in the d-shell, their spins and isospins coupled to J''_k and T''_k, respectively. The general wave function of this system, $\psi(JT)$, will be a linear combination of antisymmetrized products of the states $s^{n_k}(J'_k T'_k)$ and $d^{m_k}(J''_k T''_k)$. Thus one has

$$\psi(JT) = \sum_k a_k \phi\{s^{n_k}(J'_k T'_k)d^{m_k}(J''_k T''_k)|JT\} \equiv \sum_k a_k \phi_k, \tag{2.1}$$

where the summation is extended over all possible values of $J'_k T'_k$ and $J''_k T''_k$ that can couple to JT, and all values of n_k and m_k such that $n_k+m_k = A-28$. If there is more than one state with a particular set of JT values, additional quantum numbers are required. In order to keep the notation brief we suppress these additional quantum numbers and assume their summation implicitly where necessary. Likewise we have omitted the magnetic quantum numbers in eq. (2.1). The wave functions ϕ_k are orthonormal. The values of the coefficients a_k for all nuclear states in the sd shell have been calculated and are given in ref. [6]).

The static magnetic dipole moment of a nucleus in a state $|JMTM_T\rangle$ is defined as the expectation value of the z-component of the magnetic dipole operator $\boldsymbol{\mu}$ in the state with $M = J$. Here M and M_T represent the z-components of J and T, respectively. The operator $\boldsymbol{\mu}$ is given in terms of the nucleon (spin) g factors, $g_n^{(s)}$ and $g_p^{(s)}$, by

$$\boldsymbol{\mu} = \sum_{\sigma=1}^{n+m} \tfrac{1}{2}\{(1+\tau_{\sigma 3})\boldsymbol{l}_\sigma + g_p^{(s)}(1+\tau_{\sigma 3})\boldsymbol{s}_\sigma + g_n^{(s)}(1-\tau_{\sigma 3})\boldsymbol{s}_\sigma\}\mu_{\text{n.m.}}$$

or

$$\boldsymbol{\mu} = \sum_{\sigma=1}^{n+m} g_\sigma \boldsymbol{j}_\sigma \mu_{\text{n.m.}}, \tag{2.2}$$

with

$$g_\sigma = \tfrac{1}{2}\{(1+\tau_{\sigma 3})g_{p\sigma} + (1-\tau_{\sigma 3})g_{n\sigma}\}.$$

Here $\frac{1}{2}\tau_{\sigma 3}$, $\boldsymbol{l}_\sigma \hbar$, $\boldsymbol{s}_\sigma \hbar$ and $\boldsymbol{j}_\sigma \hbar$ represent the z-component of the isospin $\boldsymbol{t}$, the orbital angular momentum, the spin and the total angular momentum of the σth particle, respectively; $\mu_{\text{n.m.}} = e\hbar/2m_p c$ is the nuclear magneton. For the $2s_{\frac{1}{2}}$ shell the Schmidt values of the nucleon g factors are given by $g_{p\sigma} = g_p^{(s)} = 5.586$ and $g_{n\sigma} = g_n^{(s)} = -3.826$, and for the $1d_{\frac{3}{2}}$ shell by $g_{p\sigma} = \frac{1}{5}(6-g_p^{(s)})$ and $g_{n\sigma} = -\frac{1}{5}g_n^{(s)}$ (see e.g. ref. [7])).

For a system of $n+m$ particles, which may be protons and/or neutrons, the M1 transition operator becomes [8])

$$O(\mathrm{M1}) = \{3/4\pi\}^{\frac{1}{2}}\boldsymbol{\mu}. \tag{2.3}$$

Utilizing the M1 reduced transition probability

$$B(\mathrm{M1}) = \sum_{M_\mathrm{f},\, m} |\langle J_\mathrm{f} M_\mathrm{f} T_\mathrm{f} M_T | O(\mathrm{M1}m) | J_\mathrm{i} M_\mathrm{i} T_\mathrm{i} M_T\rangle|^2, \tag{2.4}$$

where m represents the z-component of the angular momentum removed by the photon, one obtains for the M1 radiative width [9])

$$\Gamma(\mathrm{M1}) = 11.6E^3 B(\mathrm{M1})\,\mathrm{meV}, \tag{2.5}$$

where $B(\mathrm{M1})$ is measured in units of [nuclear magnetons]^2 and E in MeV.

The matrix elements of $O(\mathrm{M1}m)$ for mixed states $\psi(JT)$, as given in eq. (2.1), are reduced first to a sum of matrix elements for pure configurations

$$\langle\psi(J_\mathrm{f} T_\mathrm{f})|O(\mathrm{M1}m)|\psi(J_\mathrm{i} T_\mathrm{i})\rangle = \sum_{\mathrm{f},\,\mathrm{i}} a_\mathrm{f} b_\mathrm{i} \langle\phi_\mathrm{f}|O(\mathrm{M1}m)|\phi_\mathrm{i}\rangle.$$

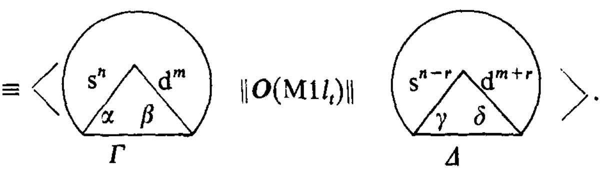
As the operator $O(\mathrm{M1}m)$ is not irreducible in isospace the Wigner-Eckart theorem yields

$$\langle\phi_\mathrm{f}|O(\mathrm{M1}m)|\phi_\mathrm{i}\rangle = [(2J_\mathrm{f}+1)(2T_\mathrm{f}+1)]^{-\frac{1}{2}}\langle J_\mathrm{i} M_\mathrm{i} 1m | J_\mathrm{f} M_\mathrm{f}\rangle \times \sum_{l_t} \langle T_\mathrm{i} M_T l_t 0 | J_\mathrm{f} M_T\rangle\langle J_\mathrm{f} T_\mathrm{f} \| \boldsymbol{O}(\mathrm{M1}l_t) \| J_\mathrm{i} T_\mathrm{i}\rangle,$$

where the summation over l_t refers to the scalar and vector parts in isospace.

In the notation for antisymmetrical wave functions as introduced by Macfarlane and French [10]), the reduced matrix elements are denoted by

$$\langle J_\mathrm{f} T_\mathrm{f} \| \boldsymbol{O}(\mathrm{M1}l_t) \| J_\mathrm{i} T_\mathrm{i}\rangle = I_r$$



(2.6)

Here Γ and Δ denote the JT values of the final and initial states, respectively. Similarly α and γ denote the JT values to which the s-particles couple in the final and initial states, respectively. Likewise β and δ for the d-particles.

As the operator $\boldsymbol{O}(\mathrm{M1})$ is taken to be a sum over single-particle operators, all I_r must vanish unless $r = 0, \pm 1$. For $r = 0, \pm 1$ the matrix elements I_r can be expressed in terms of single-particle matrix elements. This reduction requires the use of coefficients of fractional parentage $\langle\rho^n\alpha|\rho^{n-1}\beta\rangle$, defined for the direct-product JT space. For the $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ shells numerical values have been given in table 2 of ref. [5]). One thus obtains † for $r = 0$

$$I_0 = \{\Gamma, \Delta\} \Big[n\{\alpha, \gamma\}(-1)^{2\alpha+\beta+\Delta+s}\langle s\|O(M1l_t)\|s\rangle\delta_{\beta\delta} \times \sum_{\varepsilon}(-1)^{\varepsilon}\langle s^n\alpha|s^{n-1}\varepsilon\rangle\langle s^n\gamma|s^{n-1}\varepsilon\rangle \begin{Bmatrix} \alpha & \gamma & l \\ \Delta & \Gamma & \beta \end{Bmatrix}\begin{Bmatrix} s & s & l \\ \gamma & \alpha & \varepsilon \end{Bmatrix} + m\{\beta, \delta\}(-1)^{\alpha+\beta+\delta+\Gamma+d}\langle d\|O(M1l_t)\|d\rangle\delta_{\alpha\gamma} \times \sum_{\varepsilon}(-1)^{\varepsilon}\langle d^m\beta|d^{m-1}\varepsilon\rangle\langle d^m\delta|d^{m-1}\varepsilon\rangle \begin{Bmatrix} \beta & \delta & l \\ \Delta & \Gamma & \alpha \end{Bmatrix}\begin{Bmatrix} d & d & l \\ \delta & \beta & \varepsilon \end{Bmatrix}\Big]. \tag{2.7}$$

The matrix elements $I_{r=\pm1}$ can be reduced similarly. However, since the single-particle matrix elements $\langle s||O(M1l_t)||d\rangle$ vanish, the only contribution derives from $I_{r=0}$.

3. Nuclear Moments and Reduced Transition Probabilities

3.1. THE STATIC NUCLEAR MAGNETIC DIPOLE MOMENT

Applying the Wigner-Eckart theorem to the definition of the static magnetic dipole moment, and using eqs. (2.2), (2.3), (2.6) and (2.7), we obtain for a pure configuration (Γ represents $J = M_i = J_i = M_f = J_f$, T and M_T)

$$\mathcal{M}_{fi} \equiv \langle(s^n_\alpha d^m_\beta)_\Gamma|\mu_z|(s^n_\gamma d^m_\delta)_\Gamma\rangle = \{J(J+1)^{-1}\}^{\frac{1}{2}}\{J, T\} \times \Big[n\{\alpha, \gamma\}(-1)^{2\alpha+\beta+\Gamma+s}\sum_{\varepsilon}(-1)^{\varepsilon}\langle s^n\alpha|s^{n-1}\varepsilon\rangle\langle s^n\gamma|s^{n-1}\varepsilon\rangle \times \begin{Bmatrix} J_\alpha & J_\gamma & 1 \\ J & J & J_\beta \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ J_\gamma & J_\alpha & J_\varepsilon \end{Bmatrix}\delta_{\beta\delta}\Big[\frac{\sqrt{3}X_1(-1)^{\frac{1}{2}+2T_\alpha+T_\beta+T_\varepsilon+1}}{(2T_\alpha+1)\{2(2T+1)\}^{\frac{1}{2}}}\delta_{T_\alpha T_\gamma} + 3M_T X_2\{T(T+1)\}^{-\frac{1}{2}}\begin{Bmatrix} T_\alpha & T_\gamma & 1 \\ T & T & T_\beta \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ T_\gamma & T_\alpha & T_\varepsilon \end{Bmatrix}\Big] + m\{\beta, \delta\}(-1)^{\alpha+\beta+\delta+\Gamma+d}\sum_{\varepsilon}(-1)^{\varepsilon}\langle d^m\beta|d^{m-1}\varepsilon\rangle\langle d^m\delta|d^{m-1}\varepsilon\rangle \times \begin{Bmatrix} J_\beta & J_\delta & 1 \\ J & J & J_\alpha \end{Bmatrix}\begin{Bmatrix} \frac{3}{2} & \frac{3}{2} & 1 \\ J_\delta & J_\beta & J_\varepsilon \end{Bmatrix}\delta_{\alpha\gamma}\Big[\frac{\sqrt{15}X_3(-1)^{\frac{1}{2}+T_\alpha+2T_\beta+T_\varepsilon+T}}{(2T_\beta+1)(2T+1)^{\frac{1}{2}}}\delta_{T_\beta T_\delta} + \sqrt{90}M_T X_4\{T(T+1)\}^{-\frac{1}{2}}\begin{Bmatrix} T_\beta & T_\delta & 1 \\ T & T & T_\alpha \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ T_\delta & T_\beta & T_\varepsilon \end{Bmatrix}\Big]\Big]. \tag{3.1}$$

Here we have substituted explicit expressions for the Clebsch-Gordan coefficients, and introduced the abbreviations $X_1 = \frac{1}{2}(g_{p\sigma}+g_{n\sigma})$, $X_2 = \frac{1}{2}(g_{p\sigma}-g_{n\sigma})$ for a $2s_{\frac{1}{2}}$ particle, and $X_3 = \frac{1}{2}(g_{p\sigma}+g_{n\sigma})$, $X_4 = \frac{1}{2}(g_{p\sigma}-g_{n\sigma})$ for a $1d_{\frac{3}{2}}$ particle.

† In our direct-product notation we have denoted: $(-1)^\alpha = (-1)^{J_\alpha+T_\alpha}$, $(2\alpha+1) = (2J_\alpha+1)(2T_\alpha+1)$, $\delta_{\alpha\beta} = \delta_{J_\alpha J_\beta}\delta_{T_\alpha T_\beta}$, and similarly for the 6-j symbols. In the 6-j symbols l stands for $l = 1$ and $l_t = 0, 1$. The symbol $\{\alpha, \beta\}$ represents $\{(2\alpha+1)(2\beta+1)\}^{\frac{1}{2}}$.

The magnetic moment of a nucleus in a state described by eq. (2.1) is then calculated from

$$\mu = \sum_{\mathrm{f,i}} a_{\mathrm{f}} a_{\mathrm{i}} \mathscr{M}_{\mathrm{fi}}. \tag{3.2}$$

Numerical results will be given in the next section.

3.2. M1 REDUCED TRANSITION PROBABILITIES

In order to calculate the M1 radiative width, given in eq. (2.5), we must evaluate the reduced transition probability $B(\mathrm{M1})$. Performing the summation over magnetic quantum numbers in eq. (2.4), we obtain

$$B(\mathrm{M1}) = \{J_{\mathrm{i}}, T_{\mathrm{f}}\}^{-2} |\sum_{l_t} \langle T_{\mathrm{i}} M_T l_t 0 | T_{\mathrm{f}} M_T\rangle \langle J_{\mathrm{f}} T_{\mathrm{f}} \| O(\mathrm{M1} l_t) \| J_{\mathrm{i}} T_{\mathrm{i}}\rangle|^2 .$$

For pure configurations $(\mathrm{s}_\alpha^n \mathrm{d}_\beta^m)_{\Gamma \equiv J_{\mathrm{f}} T_{\mathrm{f}}}$ and $(\mathrm{s}_\gamma^n \mathrm{d}_\delta^m)_{\Delta \equiv J_{\mathrm{i}} T_{\mathrm{i}}}$ this becomes

$$\begin{aligned} B(\mathrm{M1}) = \frac{3}{4\pi} (2J_{\mathrm{f}}+1)(2T_{\mathrm{i}}+1) \Bigg[& n\{\alpha, \gamma\}(-1)^{2\alpha+\beta+\Delta+s} \sum_{\varepsilon} (-1)^{\varepsilon} \langle \mathrm{s}^n \alpha | \mathrm{s}^{n-1} \varepsilon\rangle \langle \mathrm{s}^n \gamma | \mathrm{s}^{n-1} \varepsilon\rangle \\ & \times \begin{Bmatrix} J_\alpha & J_\gamma & 1 \\ J_{\mathrm{i}} & J_{\mathrm{f}} & J_\beta \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ J_\gamma & J_\alpha & J_\varepsilon \end{Bmatrix} \delta_{\beta\delta} \Bigg[\frac{\sqrt{3} X_1 (-1)^{\frac{1}{2}+2T_\alpha+T_\beta+T_\varepsilon+T_{\mathrm{i}}}}{(2T_\alpha+1)\{2(2T_{\mathrm{i}}+1)\}^{\frac{1}{2}}} \delta_{T_{\mathrm{i}} T_{\mathrm{f}}} \delta_{T_\alpha T_\gamma} \\ & + 3\langle T_{\mathrm{i}} M_T 10 | T_{\mathrm{f}} M_T\rangle X_2 \begin{Bmatrix} T_\alpha & T_\gamma & 1 \\ T_{\mathrm{i}} & T_{\mathrm{f}} & T_\beta \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ T_\gamma & T_\alpha & T_\varepsilon \end{Bmatrix} \Bigg] \\ & + m\{\beta, \delta\}(-1)^{\alpha+\beta+\delta+\Gamma+\Delta} \sum_{\varepsilon} (-1)^{\varepsilon} \langle \mathrm{d}^m \beta | \mathrm{d}^{m-1} \varepsilon\rangle \langle \mathrm{d}^m \delta | \mathrm{d}^{m-1} \varepsilon\rangle \\ & \times \begin{Bmatrix} J_\beta & J_\delta & 1 \\ J_{\mathrm{i}} & J_{\mathrm{f}} & J_\alpha \end{Bmatrix} \begin{Bmatrix} \frac{3}{2} & \frac{3}{2} & 1 \\ J_\delta & J_\beta & J_\varepsilon \end{Bmatrix} \delta_{\alpha\gamma} \Bigg[\frac{\sqrt{15} X_3 (-1)^{\frac{1}{2}+T_\alpha+2T_\beta+T_\varepsilon+T_{\mathrm{i}}}}{(2T_\beta+1)(2T_{\mathrm{i}}+1)^{\frac{1}{2}}} \delta_{T_{\mathrm{i}} T_{\mathrm{f}}} \delta_{T_\beta T_\delta} \\ & + \sqrt{90} \langle T_{\mathrm{i}} M_T 10 | T_{\mathrm{f}} M_T\rangle X_4 \begin{Bmatrix} T_\beta & T_\delta & 1 \\ T_{\mathrm{i}} & T_{\mathrm{f}} & T_\alpha \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ T_\delta & T_\beta & T_\varepsilon \end{Bmatrix} \Bigg] \Bigg] . \end{aligned} \tag{3.3}$$

The extension to final and initial states with mixed configurations is obvious.

4. Numerical Results

With the use of eq. (3.2) numerical values for the static magnetic dipole moment of the ground states of nuclei in the range ^{29}Si–^{40}Ca have been calculated. The results are given in columns III and IV of table 1. It has been shown [11-13]) that intermediate coupling and configuration mixing can have a considerable effect on the value of the magnetic moment. As it turns out this is not so in our case. Although admixtures range from 50 % for nuclei consisting of 32 particles to some 10 % for nuclei with nearly complete shells, the effect of these admixtures is generally small. The values calculated with configuration mixing in general tend to be slightly better than the extreme single-particle values.

Two reasons can be put forward to explain the discrepancies which still exist between the calculated and experimental values. Firstly, in obtaining the wave functions which we have used in these calculations, the assumption was made that any nucleus in the range concerned consists of an inert ^{28}Si core and A–28 particles in the sd shell. This is not entirely correct. We have possibly disregarded some important contributions from the $1d_{\frac{3}{2}}$ shell, especially in the lower half of the $2s_{\frac{1}{2}}$ $1d_{\frac{3}{2}}$ shell, where the largest discrepancies occur. Secondly, we have used single-particle values for the nucleon g factors. The values that should be used, may be model dependent,

TABLE 1

Static magnetic dipole moment of ground states of nuclei in the range ^{29}Si–^{40}Ca

Nucleus	I	II	III	IV	V	VI
^{29}Si	s^1d^0	$\frac{1}{2}$ $\frac{1}{2}$	−1.91	−1.91	−0.557	−0.555
^{29}P	s^1d^0	$\frac{1}{2}$ $\frac{1}{2}$	2.79	2.79	1.338	
^{30}P	s^1d^1	1 1	0.31	0.44	0.535	
^{31}Si	s^2d^1	$\frac{3}{2}$ $\frac{3}{2}$	1.15	1.19	0.922	
^{31}P	s^3d^0	$\frac{1}{2}$ $\frac{1}{2}$	2.79	2.31	1.135	1.131
^{32}Cl	s^3d^1	1 1	1.06	0.90	0.770	
^{33}S	s^4d^1	$\frac{3}{2}$ $\frac{1}{2}$	1.15	1.13	0.908	0.643
^{33}Cl	s^4d^1	$\frac{3}{2}$ $\frac{1}{2}$	0.13	0.14	0.661	
^{35}S	s^4d^3	$\frac{3}{2}$ $\frac{3}{2}$	1.15	1.11	0.878	1.00
^{35}Cl	s^4d^3	$\frac{3}{2}$ $\frac{3}{2}$	0.26	0.20	0.670	0.821
^{35}Ar	s^4d^3	$\frac{3}{2}$ $\frac{1}{2}$	1.01	1.06	0.896	
^{36}Cl	s^4d^4	2 1	0.85	0.98	1.101	1.284
^{37}Cl	s^4d^5	$\frac{3}{2}$ $\frac{3}{2}$	0.13	0.09	0.648	0.683
^{37}Ar	s^4d^5	$\frac{3}{2}$ $\frac{1}{2}$	1.01	1.06	0.902	
^{37}K	s^4d^5	$\frac{3}{2}$ $\frac{1}{2}$	0.26	0.21	0.672	
^{38}K	s^4d^6	3 0	1.27	1.27	1.570	
^{39}K	s^4d^7	$\frac{3}{2}$ $\frac{1}{2}$	0.13	0.13	0.668	0.391

The units are nuclear magnetons
I : $s^n d^m$ configuration which makes the largest contribution;
II : spin J and isospin T of the ground state;
III : magnetic dipole moment for configuration I;
IV : magnetic dipole moment with configuration mixing and single-particle g factors;
V : same, but with effective g factors;
VI : experimental values.

so that it would not seem unreasonable to replace the single-particle g factors by values which produce the best agreement with experimental results. It is a simple matter to find effective g factors (assumed to be constant in the region considered) by a least-squares fit with the experimental data. A notable exception is the magnetic moment of the ^{32}P ground state. Here one may question the configuration † that was obtained from ref. [6]). As we could not obtain a proper fit to the magnetic dipole

† Recent calculations [14]) of ft values of the allowed transitions ^{32}Si → ^{32}P and ^{32}P → ^{32}S with the same wave functions yielded ft values far too low.

moment of the ^{32}P ground state, this case was not taken into account for the fitting procedure. The results are shown in tables 1 and 2.

It is seen from table 2 that a considerable difference exists between the single-particle and the effective g factors. The values of the magnetic moments calculated

TABLE 2

Single-particle and effective nucleon g factors

	Single-particle value	Effective value
$g_{p\sigma}$ for $2s_{\frac{1}{2}}$ shell	5.586	2.676
$g_{n\sigma}$ for $2s_{\frac{1}{2}}$ shell	−3.826	−1.115
$g_{p\sigma}$ for $1d_{\frac{3}{2}}$ shell	0.0830	0.442
$g_{n\sigma}$ for $1d_{\frac{3}{2}}$ shell	0.766	0.605

TABLE 3

Calculated level widths for M1 transitions to the ground state of nuclei in the range ^{29}Si–^{40}Ca

Nucleus	$J_f T_f$	$J_i T_i$	Energy (MeV)	$\Gamma(M1)$ [a] (meV)	$\Gamma(M1)$ [b] (meV)
^{29}Si	$\frac{1}{2}\ \frac{1}{2}$	$\frac{3}{2}\ \frac{1}{2}$	1.28	0	0
^{30}P	1 0	0 1	0.684	31.5	4.99
^{30}P	1 0	1 0	0.705	0.332	0.106
^{31}Si	$\frac{3}{2}\ \frac{3}{2}$	$\frac{1}{2}\ \frac{3}{2}$	0.76	0.0	0.0
^{31}P	$\frac{1}{2}\ \frac{1}{2}$	$\frac{3}{2}\ \frac{1}{2}$	1.265	0.300	0.0460
^{31}P	$\frac{1}{2}\ \frac{1}{2}$	$\frac{1}{2}\ \frac{1}{2}$	3.13	458.	86.0
^{31}S	$\frac{1}{2}\ \frac{1}{2}$	$\frac{3}{2}\ \frac{1}{2}$	1.1	0.320	0.0589
^{32}P	1 1	2 1	0.077	0.0581	0.0104
^{32}P	1 1	0 1	0.516	22.1	3.23
^{33}S	$\frac{3}{2}\ \frac{1}{2}$	$\frac{1}{2}\ \frac{1}{2}$	0.841	0.0234	0.0126
^{33}Cl	$\frac{3}{2}\ \frac{1}{2}$	$\frac{1}{2}\ \frac{1}{2}$	0.806	0.0631	0.0276
^{35}S	$\frac{3}{2}\ \frac{3}{2}$	$\frac{1}{2}\ \frac{3}{2}$	1.18	2.19	0.471
^{35}Cl	$\frac{3}{2}\ \frac{1}{2}$	$\frac{1}{2}\ \frac{1}{2}$	1.22	0.0927	0.0011
^{35}Ar	$\frac{3}{2}\ \frac{1}{2}$	$\frac{1}{2}\ \frac{1}{2}$	1.19	0.0866	0.0011
^{36}Cl	2 1	3 1	0.788	9.76	0.739
^{37}Cl	$\frac{3}{2}\ \frac{3}{2}$	$\frac{1}{2}\ \frac{3}{2}$	1.73	0.0	0.0
^{37}Ar	$\frac{3}{2}\ \frac{1}{2}$	$\frac{1}{2}\ \frac{1}{2}$	1.42	1.01	0.00799
^{37}K	$\frac{3}{2}\ \frac{1}{2}$	$\frac{1}{2}\ \frac{1}{2}$	1.46	1.49	0.0329
^{38}K	3 0	2 1	2.40	114.	6.48

([a]): with single-particle g factors; ([b]) with effective g factors).

with the effective g factors are shown in column V of table 1. There is a considerable improvement in most cases over the values calculated with single-particle g factors. The excellent agreement for ^{29}Si and ^{31}P is not surprising, since it turns out that the values for g_p and g_n for the $2s_{\frac{1}{2}}$ shell are almost exclusively determined by the magnetic moments of these two nuclei.

With the use of eqs. (2.5) and (3.3), level widths for M1 transitions to the ground states have been calculated for low-lying levels. Some numerical results are presented in table 3 (we have tabulated only either the transitions of less than 1 MeV or the lowest transition). It is seen that the value for the level width calculated from the single-particle g factors is in most cases much larger than the corresponding value obtained from effective g factors. Unfortunately there are very little experimental data available to compare the calculated results with. The only cases where M1 transitions to the ground state have been measured (to our knowledge) are ^{29}Si and ^{31}P.

The total E2–M1 radiation width of the 1.28 MeV level in ^{29}Si equals [17] 4.4 meV (± 35 %); the E2/M1 mixing ratio for the decay to the ground state is given by [15] $\delta = {}^{+0.21\pm0.03}_{-4.7\pm0.6}$. This yields for the M1 radiation width $\Gamma(\text{M1}) = 4.2$ meV or $\Gamma(\text{M1}) = 0.19$ meV. However, in our shell model calculations this transition is l-forbidden [16] and thus yields zero width.

The total E2–M1 radiation width of the 1.265 MeV level in ^{31}P is found [17] to be 3 meV (± 35 %). With the E2/M1 mixing ratio [18] $\delta = 0.28$ this yields for the M1 radiation width $\Gamma(\text{M1}) = 2.8$ meV. Our calculation with effective nucleon g factors gives 0.05 meV.

The lifetime of the 3.13 MeV level in ^{31}P has been determined [17] to be $\tau = 2\times 10^{-14}$ sec (± 35 %). This value was deduced from resonance scattering of bremsstrahlung with the spin assignment $\frac{3}{2}$ for this level. A more recent analysis [19, 20] has shown that the 3.13 MeV level in ^{31}P possesses the spin $\frac{1}{2}$ (parity undetermined), so that (i) the life time of this level, as it follows from Booth and Wright's measurements [17], reduces to $\tau \approx 10^{-14}$ sec, corresponding to a width $\Gamma \approx 60$ meV, and (ii) the transition to the ground state is pure dipole radiation. For the case of M1 radiation our calculation with effective g factors gives a width $\Gamma(\text{M1}) = 86$ meV.

It is seen that for a check of our results it is desirable that more experimental M1 transition rates become available.

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