J. Quant. Spectrosc. Radiat. Transfer. Vol. 6, pp. 675 689. Pergamon Press Ltd., 1966. Printed in Great Britain

COMPUTATION OF EMERGENT FLUX AND INTENSITIES FOR BLANKETING-EFFECT MODELS AND COMPARISON WITH OBSERVATION

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Abstract—The methods of constructing a model atmosphere and computing its theoretical spectrum are reviewed, attention being directed to the restrictions which are imposed and to the simplifications which are made in representing the interactions between radiation and matter. One important restriction is that it is assumed that Boltzmann's, Saha's and Kirchhoff's laws are valid throughout the atmosphere. However, observation indicates that these relationships are certainly not valid in those parts of the atmosphere where many of the strong lines which are used for classifying *OB* stars are formed. More accurate theories of line formation are required to predict the strengths and shapes of lines from the lowest levels and from metastable levels or levels connected to the metastable levels by strong transitions in the spectra of the light elements.

It is pointed out that the constraints radiative equilibrium and hydrostatic equilibrium do not define sufficiently accurately the parts of the atmosphere where many empirically selected classification lines are formed. Here the motion of the gases seems to play a significant role in determining the shape and strength of the absorption lines. Some examples are given of the differences between the line profiles predicted from blanketed and unblanketed early-type models and it is shown that these differences would not be readily detected from the usually observed spectrum in the range 3500 Å to 5000 Å. Although line blanketing lowers the effective temperatures of *OB* models by 2500° to 3000° from the values estimated by means of unblanketed models, the unblanketed models are adequate for interpreting the spectrum between 3500 Å and 5000 Å and deducing abundances from weak lines.

1. INTRODUCTION

ONE PURPOSE of making model atmospheres and computing the theoretical spectrum produced by models is to provide well-defined theoretical spectral-intensity distributions with which real stellar spectra may be compared. If the agreement in detail between the theoretical spectrum and the stellar spectrum is good, one may conclude that the temperature and pressure structure of the model represents the stellar atmosphere well and that the theory of spectrum formation and the abundances of the elements which have been adopted represent well the true state of affairs in the stellar atmosphere. This procedure suffers from one important defect, namely that unless the comparison is made using many spectral details of varying types, one cannot conclude that the accepted representation of the stellar atmosphere is unique.

There are four steps in the process and each will now be discussed with the particular intention of determining how the deductions that may be drawn from a comparison of theoretical and observed spectra may be affected by line blanketing. The four steps are these:

(a) construct a model atmosphere,
(b) calculate the spectrum emitted by the model,

(c) test that the model represents a star and if it does not, correct the model,
(d) identify a particular model with a particular star or spectral type.

2. CONSTRUCTION OF A MODEL ATMOSPHERE

A model atmosphere is constructed by postulating (i) a geometric configuration of layers, (ii) a law giving the change of pressure with depth in the atmosphere, and (iii) a law giving the variation of temperature with depth in the atmosphere. In order to interpret the spectra of sharp-line main-sequence stars one usually postulates plane parallel layers of gas of constant chemical composition (given by X, Y, and Z, the mass-fractions of hydrogen, helium and the other elements, respectively, in the atmosphere) which is in hydrostatic equilibrium. In models designed to represent main-sequence stars of type $B5$ and earlier, one should introduce a term in the equation of hydrostatic equilibrium to allow for the pressure gradient due to the radiation field. It is also possible to introduce a term to allow for the mechanical pressure due to a field of motion. If this is done, however, it is necessary to specify one more arbitrary factor when making the model and one must verify one's choice when demonstrating that the resulting model represents a selected star. The temperature law taken initially is somewhat arbitrary. Its precise form is adjusted to satisfy the particular criterion selected to indicate that the model represents a star.

Since the numerical methods of constructing a model atmosphere and the forms of the equations which are usually used are well known, they will not be discussed further here.

3. CALCULATION OF THE THEORETICAL SPECTRUM FROM A MODEL ATMOSPHERE

In order to compute the spectrum from a model atmosphere, one must adopt explicit expressions for the line and continuous absorption coefficients as functions of the abundances of the elements, the temperature, the electron pressure, and of wave length. One must also adopt a form for the equation of transfer compatible with the geometric form of the model and with the dominant interactions which occur between radiation, atoms, ions and electrons in the atmosphere.

Numerical methods exist in some simple cases for computing the monochromatic flux, F_ν, as a function of the monochromatic optical depth in the atmosphere. Often the Milne–Eddington transfer equation may be used. In this equation it is postulated that the emissivity is given by coherent, isotropic scattering and by re-emission as though the material were in thermodynamic equilibrium at the local temperature.

The chief sources of continuous absorption in early-type model atmospheres are H, H^-, He I, and He II; opacity is also produced by electron scattering. Resonance lines and strong lines from low-lying levels occur at wave lengths shorter than 1900 Å in the spectra of most of the atoms and ions expected to be abundant in early-type atmospheres. MORTON,[9] using model atmospheres has made a first estimate of the profiles and intensities of these lines in B-type spectra and has shown that these lines may be expected indeed to contribute a significant source of opacity in comparison to that from continuous spectra. Thus when determining the overall spectral distribution from a model, one should add these lines to the sources of opacity. The absorption and re-emission in these

lines affects significantly the redistribution of intensity of radiation with wave length which occurs at each level in the model.

The Milne–Eddington transfer equation gives a simple approximate method for predicting the spectrum of a star. In this case it is assumed that the emissivity due to the electrons may be represented by coherent, isotropic scattering while the emission resulting from absorption in the lines and continua of the abundant atoms and ions is given by a term of the form

$$(\kappa_\nu + l_\nu)\rho B_\nu, \tag{1}$$

where κ_ν is the continuous absorption coefficient at frequency ν, l_ν is the line absorption coefficient, ρ is the density and B_ν is the Planck function. This last assumption cannot always be justified, but detailed analysis suggests that it is a workable approximation for rather weak spectral features which arise from levels close to the ionization limit. It is not a good approximation for computing the true profiles of the strong lines which occur in *OB* spectra at wave lengths shorter than 1900 Å. These lines should be studied individually and appropriate equations of transfer found and solved, if it is desired to predict accurately the profiles and equivalent widths of these lines.

Calculations made with the approximate theory of the last paragraph have already indicated that in *OB* spectra the central parts of the resonance lines and of strong lines from low-lying levels of the light elements are formed very high in the model atmosphere. Here the density and the continuous opacity are so low that one may expect monochromatic radiative processes in the line spectrum to be dominant in fixing the exact form of the emissivity. Furthermore, one can expect that the field of motion in the equivalent high layers of a real star may have an important influence on the transfer of radiation, and thus on the line profile in the emergent spectrum. Consequently, calculations of the profiles of the resonance lines by the approximate theory should be regarded only as a rough guide to what the spectrum of a real star may look like at wave lengths shorter than 1900 Å. This approximate theory should be fairly reliable for weak subordinate lines at wavelengths greater than 2000 Å.

The lines of the He I spectrum which are particularly sensitive to dilution effects,* that is chiefly the lines arising from the 2^1S, 2^3S and 2^3P levels in stars of type *B*2 and earlier, also cannot be predicted reliably by means of the simple theory. Because of the large abundance of helium in normal stars and the rather great intrinsic strength of most of the He I lines which occur in the spectral region 4000 Å–7000 Å, one may readily show that the central parts of these He I lines will be formed high in the atmosphere. Here radiative processes dominate in the emissivity term and one is not justified in assuming expression (1).

In fact one must scrutinize carefully the transfer theory which is used to compute the profiles of lines from all atoms or ions which possess metastable levels. If it appears that the centres of the lines under discussion may be formed high in the stellar atmosphere, simple approximations to the true transfer problem such as the Milne–Eddington transfer equation may not be valid.

* Dilution effects were first recognized by STRUVE and WURM.[13] They are the spectroscopic consequences of the departures of the populations of the energy levels of an atom or ion from equilibrium populations. Conspicuous dilution effects are seen in spectra where lines from normal and from metastable levels are observed. They may result whenever the populations of the excited levels are dominantly controlled by specific radiative processes rather than by collisions.

4. TO DETERMINE THAT A MODEL REPRESENTS A STAR

In order that a model may represent a star it is necessary to put a constraint on the radiation field. The temperature law is then adjusted until the predicted radiation field meets the selected constraint. One of three constraints is used, namely

(a) that the temperature should vary with depth according to the theoretical expression valid for a grey body, or
(b) that the variation of limb darkening with wave length which is observed for the Sun should be reproduced, or
(c) that the total energy in the radiation field should remain constant at all depths in the atmosphere.

The first constraint may serve as a starting hypothesis when little or nothing is known about the properties of the radiation field from a certain type of star. In order to use this constraint, one has merely to decide how to define the characteristic optical depth, by means of which depth in the atmosphere is to be measured. For convenience only, the depth in an atmosphere is usually measured in terms of a characteristic optical depth defined by the equation

$$\tau = -\int_z^\infty \kappa\rho \, dz, \tag{2}$$

where z is the geometrical distance of the layer under consideration from the centre of the star, ρ is the density and κ is the factor, equivalent to a mass absorption coefficient, which is used to produce a folding of the geometric depth that resembles the folding produced by the monochromatic optical depth

$$t_\nu = -\int_z^\infty \kappa_\nu \rho \, dz = \int_0^\tau (\kappa_\nu/\kappa) \, d\tau. \tag{3}$$

The monochromatic optical depth is the function describing depth in the atmosphere which enters naturally into the equation of radiative transfer. In equation (3) the symbol κ_ν represents the monochromatic opacity at frequency ν due to all sources of opacity, line, continuum and electron scattering. Frequently, when the grey-body temperature law is used for a model atmosphere, the characteristic opacity coefficient κ is defined as a Rosseland mean absorption coefficient.

The second constraint on the temperature law can only be used for the Sun. The appropriate temperature law is found by trial-and-error comparison of computed limb darkening with the observed limb darkening. The resulting temperature law may be scaled and used for making models of stars of neighbouring spectral types. The temperature law found in this way has the advantage that it is an empirical quantity which does permit one to reproduce a certain number of observed data in the case of the Sun. It is known that the transfer of energy through stellar atmospheres of type G and thereabouts is not entirely by radiation. Some energy is transferred by convection. Thus it is not to be expected that the temperature law for a model representing a G-type star should be correctly given by applying the third constraint.

The third constraint is obtained by postulating that the transport of energy through the stellar atmosphere is entirely by radiation. Since, by hypothesis, there are no sources or sinks of energy in the atmosphere, the condition to be fulfilled at every depth τ in the model is

$$\int_0^\infty F_\nu(\tau)\,\mathrm{d}\nu = \text{const.} \tag{4}$$

The flux $F_\nu(\tau)$ is computed at a number of values of ν at each depth τ in the model atmosphere and the integration over frequency is performed. If the integral is not constant within a prescribed tolerance, the temperature law must be changed. When the monochromatic opacity is given by rather smooth functions such as the continuous absorption coefficients of H, H^-, He I, He II and by electron scattering, the integral of equation (4) may be evaluated with sufficient precision by taking less than 50 points. When the opacity due to lines is also considered, the function $F_\nu(\tau)$ becomes quite ragged, and it may require hundreds, if not thousands, of points for a precise evaluation of equation (4).

Clearly, if detailed information about the shape of $F_\nu(\tau)$ at many frequencies ν_i and at many depths τ_i in a model is required in order to determine whether the model satisfies the third constraint, radiative equilibrium, and thus represents a star, the problem of finding an appropriate model may become uneconomic. It is time to ask what are the minimum requirements that an arbitrary model will represent a star sufficiently closely that one may proceed with confidence to the next step of identifying the model with a particular star or spectral class. In other words, do the spectral details by which we recognise a particular star or spectral type have such a character that we must force our constraint radiative equilibrium to its limit in order to find a unique model for the star, or do we have at present insufficient information to select one from among a number of possible models of increasing complexity and greater physical reality?

Holweger and Unsöld[4] have looked into the question of how constant the integrated flux $F(\tau)$ must be if one is to tolerate a given error in the temperature law. They examined a grey model with a Rosseland mean absorption coefficient and found that if the constraint radiative equilibrium was to determine the temperature law to within one per cent, then at $\tau = 0{\cdot}01$ a tolerance of about 0·03 per cent in $F(\tau)$ was demanded; at $\tau = 0{\cdot}1$ the tolerance was 0·3 per cent while at $\tau = 1{\cdot}0$ the tolerance was 2·5 per cent. If line blanketing is to be considered and the constraint radiative equilibrium is to be used to fix the temperature law accurately in the layers where the lines causing the blanketing are formed, that is at $\tau \leq 0{\cdot}1$, it follows that the spectrum $F_\nu(\tau)$ will have to be determined with great accuracy. However, as mentioned in Section 3, it is precisely in the uppermost part of the atmosphere that the theory of radiative transfer which is used is most unreliable, particularly in the case of frequencies lying in strong lines. The only conclusion to draw is that it is impracticable to expect to use the constraint radiative equilibrium to establish a *very* accurate temperature law in the uppermost layers ($\tau \leq 0{\cdot}1$) of a model atmosphere.

For the purpose of establishing a preliminary model of a star it is not necessary to calculate the exact profiles of the strong lines which are causing the line blanketing. Rather crude representations which give the overall properties of the opacity due to the lines and the change of this opacity with depth in the model will suffice. Having obtained a model in a somewhat summary fashion, it is then necessary to examine rather closely

how well the predicted spectrum agrees with an observed spectrum and to see if there is indeed any observational reason for preferring a line-blanketed model rather than an unblanketed model as a representation of the star in question.

5. THE IDENTIFICATION OF A STAR WITH A MODEL

This identification is made by finding coincidences between observed spectral details, each of which is expressed as a relative intensity evaluated over a short wave-length range, with the equivalent computed values. The chief spectral details which have been selected empirically as spectral classification criteria fall in the range 3900 Å to 5000 Å. They are, usually, the relative intensities of a few spectral lines. It is important to realise that spectral classification, particularly for early-type stars, is based on the relative intensities of a few spectral lines and not on absolute energy distributions. Gradients over several thousand angstroms whether in the violet, blue or red spectral regions are rarely used for detailed spectral classification because of the difficulties of removing the distortions in the measured intensity distribution due to wavelength dependent extinction in interstellar space, in the earth's atmosphere and in the observing equipment.

In order to obtain a first estimate of the spectral type of a model of an early type star, one compares the apparent position and size of the discontinuity at the Balmer limit with observed values. Enough stars of type *A*0 and later are sufficiently close to us that for these stars one may use the general shape of the continuous spectrum as a first criterion for identifying a model with a star. However, even for nearby stars this criterion again fails for the stars of type *K* and later because the spectrum becomes so full of lines that the models available at present do not predict the emergent flux in adequate detail. Empirical corrections for line blanketing may be applied to the observed intensity distributions in order to derive an observed quantity like the computed intensity distribution in the case of few or no lines in the spectrum, but such corrections always lead to ambiguity in interpreting the observed spectra.

Whatever criteria are selected for identifying a star with a model, the identification will serve only to delineate a correspondence between certain parts of the stellar atmosphere and some layers of the model. Thus in the case of a *B* type star, if the identification is made by means of the Balmer jump and the shape and depth of the wings of Hγ, one has only shown that the moderately deep layers of the star and the model are equivalent. Here the weak spectral features are formed. One has confirmed almost nothing about the outermost layers of the model.

An empirical relationship between *D*, the Balmer jump, and the *MK* spectral type can be established by means of the measurements of *D* by CHALONGE and DIVAN.[1] The observed points for main-sequence stars are shown in Fig. 1 and a mean curve has been drawn through these points. It is clear that this relationship becomes rather flat at spectral types earlier than *B*3 and that because of the observational scatter in the plotted points, the spectral type of an early type model cannot be established with precision from *D* alone.

Somewhat different relationships between *D* and spectral type have been derived by others (see, for instance, MORTON and MIHALAS[8]) partly on the basis of photometric observations. The point is that this one parameter, *D*, is not sufficient to identify a model with a spectral type. One must also check that those stellar lines whose relative intensities

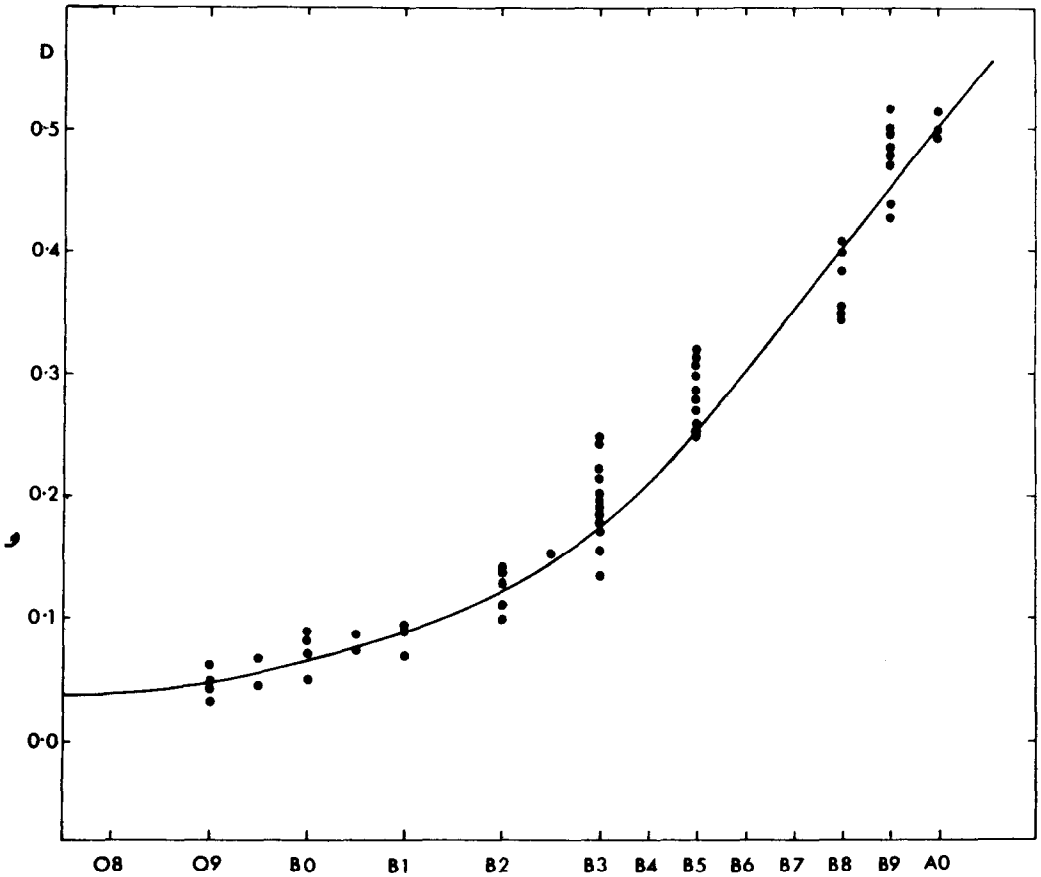

FIG. 1. The empirical relationship between D and MK spectral type derived from the observations of CHALONGE and DIVAN including a correction given by them.

are sensitive to spectral type are reproduced correctly by the model. In the case of early type stars, we are restricted to rather strong lines from the light elements and to the wings of the hydrogen lines as classification criteria. The wings of the hydrogen lines permit us to estimate rather closely the appropriate value of g for our model provided that the adopted theory of the Stark broadening of the hydrogen lines is sufficiently accurate and that a good guess has been made of the chemical composition of the atmosphere. The cores of the hydrogen lines and the central parts of the lines of He I cannot be used for classifying model atmospheres constructed to represent normal main-sequence O and B stars because, owing to the great abundance of H and of He, the central parts of these lines are saturated and are thus formed at such high levels in the model that the usually adopted theory of formation of spectral lines is not valid. It often appears that motion of the gas is the dominant factor in determining the observed strength of the cores of the strong lines.

The difficulties and ambiguities of identifying models with certain stars and spectral types may be illustrated by the pairs of models, all with $\log g = 4{\cdot}0$, which are listed in Table 1. The model by Guillaume and that by Underhill and Elst have been computed

TABLE 1. THREE PAIRS OF BLANKETED AND UNBLANKETED MODELS

Author	Model	T_{eff} (°K)	D	Equivalent Sp. Type	Remarks
MIHALAS[7]		24000	0·132	*B*1·5	No lines
MIHALAS and MORTON[8]		21914	0·130	*B*1·5	Detailed line blanketing
UNDERHILL[14]	63	25673	0·126	*B*1·5	No lines
GUILLAUME[2]	*B*13	23255	0·121	*B*1·5	Pseudo lines
UNDERHILL and DE GROOT[17]	*P*7	33965	0·036	*O*9	No lines
UNDERHILL and ELST[16]	*PB*24	30750	0·038	*O*9	Pseudo lines

by the program described by GUILLAUME.[2] Here the contributions to the opacity by the chief ultraviolet lines have been represented as rectangles, each with a relative weight which was selected empirically from a consideration of the results of MORTON.[9] The variation with depth of each rectangle is represented by the variation of the population

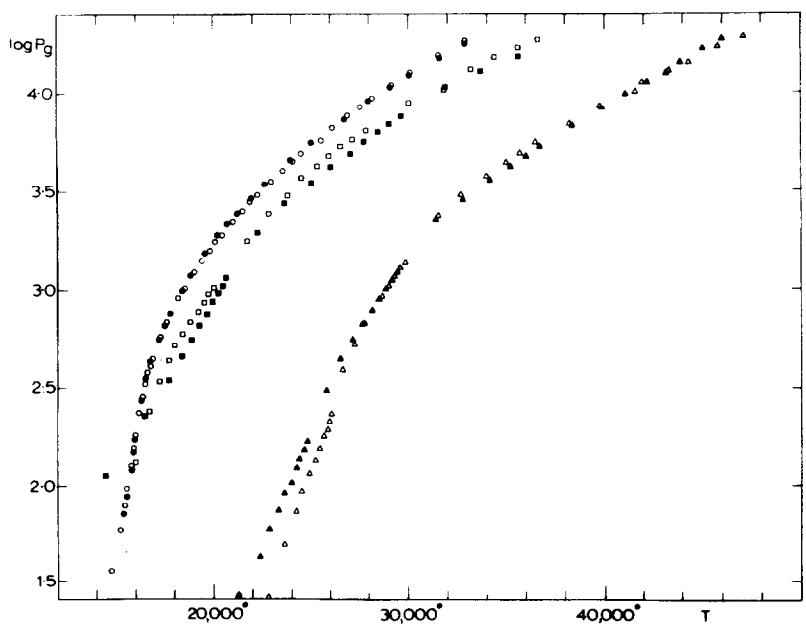

FIG. 2. The temperature–pressure structure in pairs of models which have the same spectral type. The filled symbols refer to models in which line blanketing is considered; the open symbols refer to models in which no line absorption is considered. Circles refer to the models of Mihalas and Morton, squares to the models of Underhill and Guillaume, triangles to the models of Underhill and Elst.

of the lower level of the chief line in the region, the population being calculated by the laws of Boltzmann and Saha. The line-blanketed model of MIHALAS and MORTON[8] was constructed allowing in detail for the shape of the absorption coefficient due to lines at wave lengths shorter than about 1500 Å.

These models may be compared by plotting temperature against $\log P_g$. This information for the outermost parts of the models is shown in Fig. 2. The characteristic common to each pair of models is that for a given value of the gas pressure the inclusion of line blanketing results in a lower temperature in the outermost parts of the model than what is found in the case of no line blanketing. However, at deeper layers where the chief contribution to the observed strength of weak features such as the Balmer jump and the wings of the hydrogen lines is made, the blanketed and unblanketed models have very nearly the same temperature–pressure structure. Accordingly, the first four models can be considered to represent main-sequence stars of type *B*1·5. The latter two models correspond to about type *O*9V.

GUILLAUME[2] and the author have computed the profiles of quite a few lines in Models 63 and *B*13 in order to see if one can detect the low temperature fringe on the line-blanketed model by means of the spectral lines usually used for spectral classification. We have considered that the line broadening was due to collisional or Stark broadening and to thermal Doppler effect and to a Doppler broadening four times this. The latter numerical experiment was made in order to simulate, roughly, the expected effects of microturbulence. The profiles of the hydrogen lines are computed by the approximate theory of Griem plus thermal Doppler broadening, while the collisional or Stark broadening of the other lines is approximated by assuming that the damping constant is $10\gamma_{el}$ and using the Voigt function. The line profiles of Hα and Hγ are shown in Fig. 3 for four models of rather similar characteristics which correspond about to type *B*1·5.

Models *P*11 and 63 have been constructed with H, H^-, He I, He II and electron scattering as the sources of opacity. The spacing in τ of the outer layers of Model 63 is a little too coarse for investigating the formation of strong spectral lines. Therefore Model *P*11 was constructed. Here the spacing is finer. The adopted $T(\tau)$ relation in *P*11 is essentially the same as that of Model 63 but the decrease of temperature as $\tau \to 0$ is followed in more detail than is done with Model 63. Models *B*13 and *LB*63 are made with the coarse spacing of the outer layers. Radiative equilibrium is enforced in the outer layers to within 0·5 per cent. Model *B*13 was constructed by GUILLAUME;[2] here the effects of the blanketing by the strong lines between 911 Å and 1500 Å are taken into account in an approximate manner. Model *LB*63 is a first attempt to explore the effects of line blanketing. In this case the model program was modified so that the absorption coefficient of hydrogen was maintained at its value for 1458 Å at all wave lengths between 911 Å and 1458 Å.

The wings of the hydrogen lines are practically the same in all these models at points more than 5 Å from the line centre. This result demonstrates that the changes in the pressure–temperature structure of the model caused by line blanketing are too small to affect the predicted intensity of weak spectral features, that is of features that are less than about 20 per cent deep. The absorption in the core of Hα is sufficiently strong that differences in the line profile do occur between one model and the next. It is clear that the exact temperature–pressure structure in the outermost layers of the star is important for predicting the central part of the Hα profile. The differences in the predicted profiles of Hγ

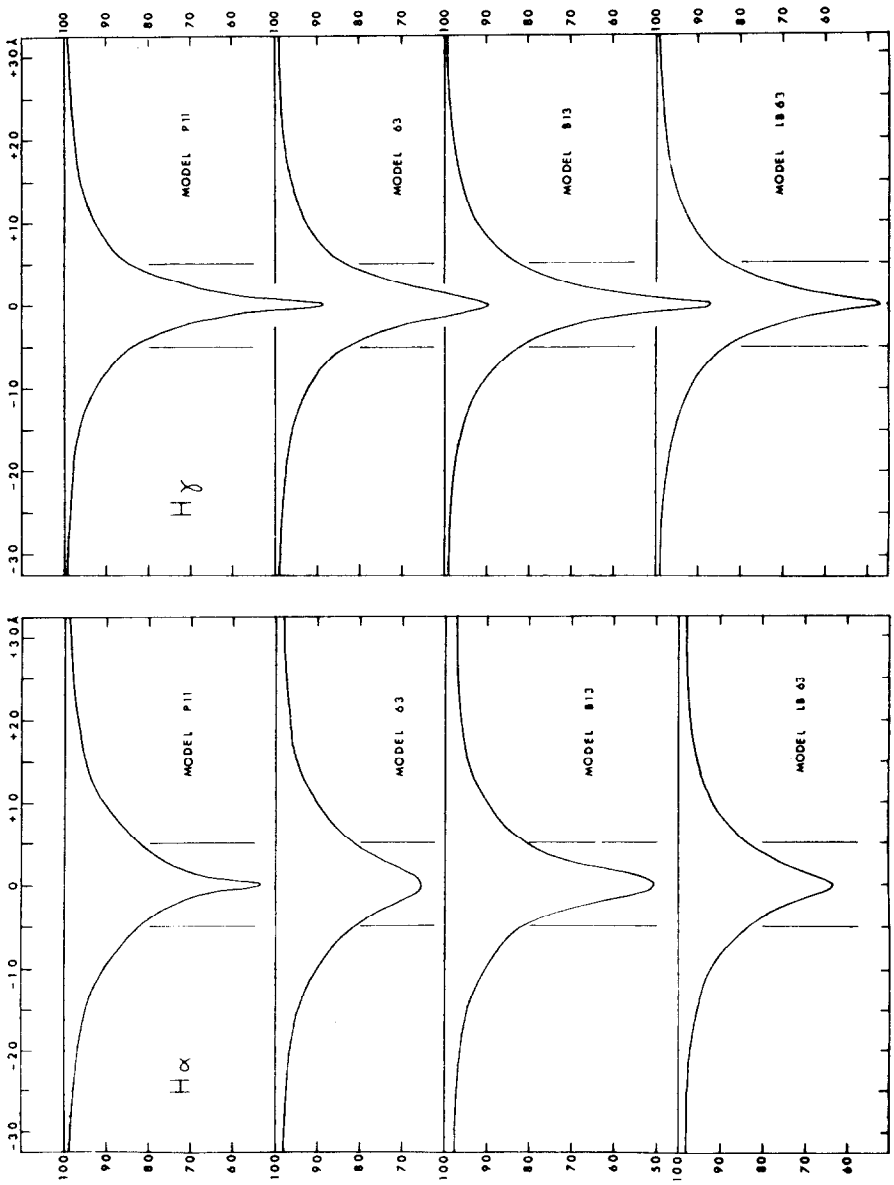

FIG. 3. Profiles of Hα and Hγ in four models corresponding to about the same spectral type but with different line blanketing characteristics; see the text.

are much less and it is doubtful if one could decide on the basis of agreement with observed line profiles which of the four models would give the most satisfactory representation of a stellar atmosphere of type about $B1{\cdot}5$.

The computed profile of Si III $\lambda 4552$ in Models 63 (full line) and $B13$ (broken line) is shown in Fig. 4. A fractional abundance by mass of silicon of $1{\cdot}206 \times 10^{-3}$ has been adopted. The line $\lambda 4552$ is observed to be strong at spectral type $B1{\cdot}5$. It may be strengthened by overpopulation of the lower level owing to dilution effects (UNDERHILL[14]). Figure 4

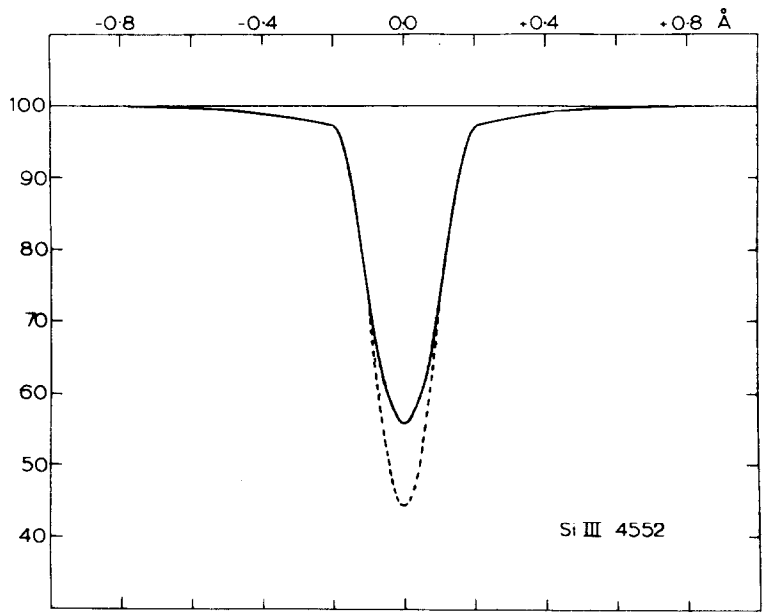

FIG. 4. A comparison of the effects of line blanketing on the predicted profile of Si III $\lambda 4552$ in models of about the same spectral type. The adopted fractional abundance by mass of silicon is $1{\cdot}206 \times 10^{-3}$.

clearly demonstrates that the central part of this line is formed sufficiently high in the atmosphere that the exact pressure–temperature structure in the outermost part of the atmosphere affects the strength of this line by a significant amount. It is obvious that were there a large field of motion in the outer part of the stellar atmosphere, the line Si III $\lambda 4552$ would be strongly broadened and, perhaps, displaced.

Similar effects are illustrated by the profiles of the Mg II doublet at 4481 Å shown in Fig. 5. In the upper part of the diagram the profile has been computed considering only thermal Doppler broadening and using a damping constant equal to $10\gamma_{cl}$. In the lower part of the diagram the broadening effects of microturbulence have been simulated by taking a Doppler width four times the thermal Doppler width. In this case the doublet structure has completely disappeared. In most so-called "sharp-lined" stars the Mg II doublet is not resolved even on spectrograms of adequate intrinsic resolving power. An observed profile of Mg II $\lambda 4481$ in the spectrum of γ Pegasi is shown in Fig. 6. The doublet should have been resolved if the lines were broadened by thermal Doppler effect only.

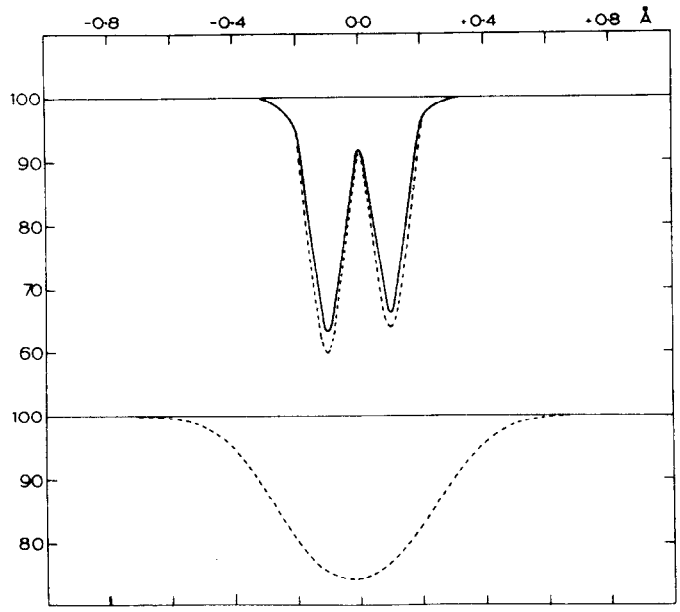

FIG. 5. A comparison of the effects of line blanketing and turbulence on the predicted profile of the Mg II doublet at 4481·228 Å.

The profile from the bottom of Fig. 5 is drawn in Fig. 6. It is evident that the observed shapes of the lines in the spectrum of γ Pegasi can be reproduced by using a Doppler width four times the thermal Doppler width. The Mg II doublet predicted by means of Models 63 and B13 and an adopted fractional abundance of magnesium by weight of

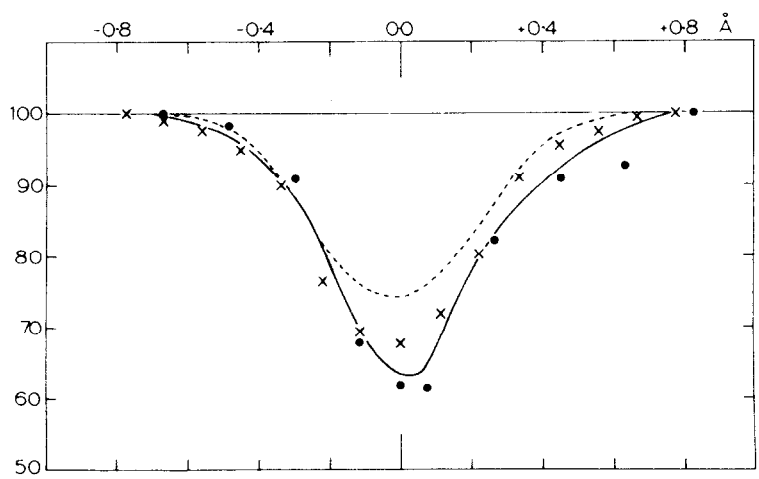

FIG. 6. The observed profile of Mg II λ4481 in γ Pegasi. The mean results from three spectrograms taken with the Litt GIII BL84 spectrograph of the Dominion Astrophysical Observatory are shown by crosses while those from the 3263 spectrograph of the 48-in. telescope at that observatory are shown by dots.

$3{\cdot}28 \times 10^{-4}$ is weaker than the line observed in γ Pegasi. Comparison between other observed and computed profiles suggests that Models 63 and *B*13 are too early in spectral type for representing accurately the atmosphere of γ Pegasi, although the observed Hγ profile of γ Pegasi matches well the Hγ profiles shown in Fig. 3.

Guillaume has compared computed profiles of some ultraviolet lines between 2000 Å and 3000 Å in Models 63 and *B*13. Her results are similar to those presented here. The cool outer fringe of the line-blanketed model can only be detected by means of the cores of the strongest lines. Microturbulence has a greater effect on the profiles and strengths of moderately strong lines than do refinements of model structure resulting from the inclusion of line blanketing in the construction of a model atmosphere.

The conclusion to be drawn from this work is that if one wishes to use a model atmosphere to interpret the equivalent widths and profiles of weak and moderately strong lines in the spectral range 2000 Å to 7000 Å of *O* and *B* stars in terms of abundances and state of motion of the atmosphere, an unblanketed model is adequate. Such lines normally carry very little information about conditions in the outermost fringes of the atmosphere, so it matters little that the models do not describe these parts of the star accurately.

If, however, one wishes to predict accurately the equivalent widths and shapes of the strong resonance lines at wave lengths shorter than 1900Å and the strong lines at longer wave lengths, it is necessary to have a model which is defined in detail in the outermost layers. At present the theory of line formation which is used is not correct for resonance lines, nor do we have any reason to believe that the temperature–pressure structure found by forcing the conditions of hydrostatic equilibrium and radiative equilibrium in these outermost layers leads to models which will necessarily represent the outermost layers of real stars. The problem of interpreting the strong ultraviolet lines in early type stars will remain until observed profiles are available. With observed profiles one could attempt to define a preliminary model for the outermost layers of the *OB* stars and then proceed further. One might begin this process by observing carefully those strong spectral features which occur in the ordinary spectral range and are formed in extended atmospheres such as the emission lines in early type supergiants and those lines which are strengthened by dilution effects and attempt to derive a preliminary model of the outer layers of the star.

The above discussion is based on results for *OB* stars and models. However, the same difficulties also occur with stars of later spectral type. Thus when one attempts to interpret the blue–violet spectral region of *G*, *K* and *M* stars one finds that the lines are so strong that models made neglecting the contribution from the lines have little, or no meaning, and that one must use the line spectrum with an appropriate theory of line formation, in order to deduce a model of those layers which are relevant for the formation of the observed spectrum.

6. THE USE OF MODEL ATMOSPHERES FOR INTERPRETING THE HR DIAGRAM

Theories of stellar evolution characterize the position of a star in an evolutionary sequence by two parameters, the effective temperature and the total luminosity. Stellar spectra at most give an estimate of the temperatures and pressures in the line-forming layers. The problem is to relate the latter pieces of information to the total radiation field of the star in a unique manner so that one can establish single-valued relationships

between spectral type (including luminosity class) and effective temperature and bolometric correction.

The research summarized in Section 5 has shown that in the case of early-type stars, at least, it is very difficult from an analysis of the stellar spectrum to decide upon the best model for representing the stellar atmosphere. An unblanketed model is adequate for many purposes. However, with an unblanketed model the effective temperature will be between 2500° and 3000° higher than for an equivalent blanketed model (see Table 1). Blanketed models clearly are necessary for representing early-type stars if one wishes chiefly to establish accurately the relationship between effective temperature and spectral type. At present it appears that the empirically selected classification criteria for *O* and early *B* stars do not give a unique determination of effective temperature. The observed spectra can be interpreted equally well by models of different effective temperature. To satisfy ourselves about the correct effective temperatures for the *OB* stars it will be necessary to obtain observations which permit us to establish the physical conditions in the highest atmospheric levels of the stars. The uncertainty in the estimated effective temperatures of the *OB* stars appears to be about ten per cent. It is likely that the effective temperatures are lower by this amount than the values estimated by means of unblanketed model atmospheres.

The estimate of stellar radii from visual absolute magnitudes making use of theoretical values of effective temperatures and bolometric corrections will not be seriously in error so long as the value of the bolometric correction predicted from a model is used with the effective temperature predicted by the same model. When the effective temperature is reduced, the bolometric correction is also reduced and these factors compensate each other in the calculation of radii (see UNDERHILL[15]).

The present uncertainty in the effective temperatures and absolute magnitudes of stars of type *B*2 and earlier are rather disconcerting because the theoretical tracks in the HR diagram found for massive stars, for instance by HAYASHI and CAMERON,[3] IBEN[5,6] and STOTHERS,[10–12] rise rather steeply when the star has just left the main-sequence. Consequently, if there is no independent information about the mass of the star, the star may be quite easily placed on the wrong evolutionary track when use is made of luminosities and effective temperatures estimated from an analysis of the spectrum. The result will be that one deduces a wrong value of the stage of evolution of the star.

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DISCUSSION

K. H. BÖHM: You have used the HOLWEGER–UNSÖLD result[4] in order to state the accuracy which can be achieved for the temperature stratification in rather high layers ($\bar{\tau} \approx 0{\cdot}01$) for a given error of the flux constancy. It seems to me that a number of authors have drawn much too pessimistic conclusions from the Holweger–Unsöld paper. We have to keep in mind that Holweger and Unsöld assume a rather special form for the depth dependence of the flux error. The assumed form of the flux error in their paper has the consequence that the gradient of the flux error is always very large. Since the error in temperature depends stronger on this gradient than on the flux error itself it is obvious that one must get a large temperature error from this assumed form of the flux error. Fortunately all the temperature correction procedures (like e.g. Lucy's method) which are used at present lead to a rather smooth dependence of the flux error on depth and consequently the situation is much better than a superficial look at the Holweger–Unsöld paper may indicate. That this is really true is clearly shown by the numerical results, which have been obtained at different places (see e.g. BÖHM and DEINZER, *Z. Astrophys.* **62**, 167 and **63**, 177 (1966)).

The more correct way to apply the Holweger–Unsöld result (as emphasised by Professor Unsöld in a private communication) would be the following: If you are interested in the temperature at—say $-\bar{\tau} = 0{\cdot}01$, consider only the change of flux from $\bar{\tau} = 0$ to $\bar{\tau} = 0{\cdot}01$ and not the total deviation from the correct flux.

R. CAYREL: It seems to me that a very clever remark made by Unsöld a few years ago, on the right way of handling the $\sigma_\nu J_\nu$ term in the transfer equation for *B* stars has been overlooked in most recent works. The point is that electrons scatter light with a Doppler shift of the order of 6 Å. Therefore in computing a line the term standing for scattering by free electrons should be replaced by $\sigma_\nu \int \phi_\nu J_\nu \, d_\nu$, where ϕ_ν is the distribution of the Doppler shifts. For weak or middle–strong lines this term is nearly equal to $\sigma_\nu J_\nu$ (in the continuum) and not to $\sigma_\nu J_\nu$ (in the line). Equivalent widths are obviously decreased by taking the effect into account.

In *B* stars the electron density N_e is about 100 times larger than in the solar photosphere. The ratio ε of collisionnal to radiative processes in the visible is of the order of 0·1 to 1 and therefore LTE is not a bad approximation for ā *B* star atmosphere.