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THE EMPIRICAL DETERMINATION OF THE DAMPING CONSTANTS OF FRAUNHOFER LINES

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Abstract—A systematic method is developed to determine the damping constant, γ, and its variation with optical depth, τ, in the solar photosphere, using the observed profiles of wings of Fraunhofer lines and their center-to-limb variation.

The method is applied to the $3s^3P^\circ$–$3p^3D$ multiplet of carbon at 10,700 Å.

The resulting $\gamma(\tau)$ curve with its mean errors is shown in Fig. 2. This result can be explained by the usual assumption $\gamma(\tau) = \gamma_{H,He}(\tau) + \gamma_e(\tau)$, where $\gamma_{H,He}$ is the contribution to the damping constant by collisional broadening by neutral particles; γ_e is due to the quadratic Stark effect of passing electrons. The "observed" values of the function $\gamma_{H,He}(\tau)$ agree perfectly with theoretical predictions. No theoretical predictions can yet be made for $\gamma_e(\tau)$ but the empirically derived values for this function are similar to values found from laboratory measurements for some lines of other elements, and look acceptable.

1. INTRODUCTION

RELATIVELY little attention has been given to the determination of the damping constants, γ, of Fraunhofer lines. The first determination of the damping constants in the sun, by MINNAERT and MULDERS,(1) showed γ to be ten to a hundred times larger than the classical value. This large value had to be ascribed to *collisional damping*, either by collisions with electrons, or with neutral hydrogen. Unlike early-type stars, giant stars or the solar chromosphere, *radiation damping* is unimportant in spectra of the solar photosphere. Around 1940 several theories on the damping constants were developed which allow us to predict γ in a semi-quantitative way. However, an exact theoretical computation of γ presents, for many atoms, severe difficulties of a quantum-mechanical nature. For this reason an empirical determination of γ and of its variation with depth in the solar photosphere appears to be important. That up to now such a determination has been made but rarely,(2,3) can only partly be due to the high observational accuracy required. It appears that the difficulties involved in separating the influence of the damping constant on the line profile from the influence of the model of the photosphere have been overestimated.

In this paper we give a simple systematic method for determining γ. The method is applied to the $3s^3P^\circ$–$3p^3D$ multiplet of carbon near 10,700 Å. We have taken this multiplet since we have already used it as a test example for the development of a systematic method for investigating Fraunhofer line profiles. Previously we showed already how to determine, empirically, four of the parameters that influence the line profiles:

$$\left.\begin{array}{l}\text{the convective velocity component, } v_c \\ \text{the microturbulent velocity component, } v_t\end{array}\right\}^{(4)}$$
$$\left.\begin{array}{l}\text{the abundance, } A_C \\ \text{the source function, } S(\tau)\end{array}\right\}^{(5)}$$

In this paper we will complete the series by describing a method for determining the damping constant γ. We intend to publish a descriptive review of our systematic method for analyzing line profiles.(6) We have furthermore started applying this method to a large number of solar line profiles that have been acquired during the last few years.(7)

With the five unknowns thus found, one should be able to compute the whole profile of the spectral lines and to compare it with the observations. This procedure would make possible a final checking of the results.

2. THE ABSORPTION COEFFICIENT IN THE WINGS OF FRAUNHOFER LINES

In well-known notation, the absorption coefficient $\kappa(v)$ at a wavelength $v = \Delta\lambda/\Delta\lambda_D$ in the region of a Fraunhofer line is given by

$$\kappa(v) = \kappa_c \mathrm{H}(\alpha, v), \tag{1}$$

where κ_c is the absorption coefficient in the line center/g. For large values of v, and for $\alpha \ll 1$:

$$\mathrm{H}(\alpha, v) = \frac{\alpha}{v^2(\pi)^{\frac{1}{2}}}.^{*} \tag{2}$$

Further

$$\alpha = \frac{\gamma}{2\Delta\omega_D} = \frac{\gamma\lambda^2}{4\pi c\Delta\lambda_D}, \tag{3}$$

$$v = \Delta\lambda/\Delta\lambda_D, \tag{4}$$

$$\begin{aligned}\kappa_c &= \frac{(\pi)^{\frac{1}{2}}e^2}{mc^2}\frac{\lambda^2\mathfrak{N}}{\Delta\lambda_D}[1-\exp(-c_2/\lambda T)] \\ &= \frac{(\pi)^{\frac{1}{2}}e^2\lambda^2}{mc^2\Delta\lambda_D} fg A_{el}\frac{b_{r,s}}{g}\frac{B}{4+B}\frac{1}{m_{\mathrm{H}}}[1-\exp(-c_2/\lambda T)]\end{aligned} \tag{5}$$

where $\mathfrak{N}$ is the number of classical oscillators per gram, A_{el} is the abundance of the element $= N_{el}/N_{\mathrm{H}}$, $b_{r,s}$ is the fraction of the atoms in the relevant ionization and excitation level, $B = N_{\mathrm{H}}/N_{\mathrm{He}}$, m_{H} is the mass of the hydrogen atom.

The other symbols have their usual meaning.

Combining equations (1)–(5), one obtains

$$\kappa(\Delta\lambda) = C_1\lambda^4\frac{\gamma fg A_{el}}{\Delta\lambda^2}\frac{b_{r,s}}{g}[1-\exp(-c_2/\lambda T)] \tag{6}$$

where

$$C_1 = \frac{e^2}{4\pi mc^3}\frac{B}{4+B}\frac{1}{m_{\mathrm{H}}} = 0{\cdot}260,$$

* This expression is based on the assumption of symmetrical broadening without appreciable line shift. This assumption is confirmed by the observations, which show the lines to be symmetric, at least at the large distances to the line center to which the present discussion refers.

if we assume $B = 5{\cdot}5$ (Unsöld mixture).

If we further introduce the "wing-parameter"

$$V = 0{\cdot}260\frac{\gamma f g A_{el}}{\Delta\lambda^2}, \tag{7}$$

equation (6) becomes

$$\kappa(V) = \lambda^4\frac{b_{r,s}}{g}[1-\exp(c_2/\lambda T)].V \tag{8}$$

This formula is applicable to all spectral lines provided that one confines oneself to that part of the line profile where the wing approximation $\kappa \propto \Delta\lambda^{-2}$ is correct.

3. THE EMPIRICAL DETERMINATION OF $\gamma(\tau_0)$

For an assumed value of V and for a given model of the photosphere, one can easily compute the corresponding selective optical depth by numerical integration:

$$\tau(V) = \int_0^{\tau_0}\frac{\kappa(V)}{\kappa_0}d\tau_0,$$

where κ_0 and τ_0 are the absorption coefficient and optical depth for $\lambda = 5000$ Å. Similarly, one finds the continuous optical depth τ_λ for the continuous spectrum at 10 700 Å. With

$$\kappa = \kappa_\lambda+\kappa(V),$$

and

$$\tau = \tau_\lambda+\tau(V),$$

one finds the relative line depression $d(V)$ with

$$d(V) = \frac{\int_0^\infty B(\tau_0)\exp(-\tau_\lambda\sec\theta)\sec\theta d\tau_\lambda - \int_0^\infty B(\tau_0)\exp(-\tau\sec\theta)\sec\theta d\tau}{\int_0^\infty B(\tau_0)\exp(-\tau_\lambda\sec\theta)\sec\theta d\tau_\lambda}. \tag{9}$$

In equation (9) we introduce B instead of S and, hence, assumed L.T.E. This assumption is validated by our finding[(5)] that $S \neq B$ only for $\tau_0 < 0{\cdot}2$, and (as will be shown further on) that the depression d is formed mainly in very deep layers, where $\tau_0 > 0{\cdot}6$ (see the average τ_0-values, denoted by τ_0^*, and given in the last two lines of Table 1).

It is important to have an impression of the average depth of formation τ_0^* of the line depression, which may be found as follows:

If we set

$$\Delta(\tau_0) = \frac{\kappa_\lambda}{\kappa_0}\exp(-\tau_\lambda\sec\theta)-\frac{\kappa}{\kappa_0}\exp(-\tau\sec\theta),$$

TABLE 1. K, $A_C f_\gamma$. γ^*, THE τ_0^* RANGE, AND THE MEAN ERROR μ_γ OF γ^*. THE TABLE CONTAINS $\gamma^* \times 10^{-9}$

	cos θ	1·00	0·75	0·60	0·50	0·40	0·30
	$K \times 10^{24}$	2·15	2·20	2·27	2·40	2·40	1·80
λ(Å)	$A_C f_\gamma$						
10691	$8{\cdot}6 \times 10^{-4}$	4·52	4·02	4·12	1·98	1·98	2·20
10683	4·8	5·40	4·96	3·41	2·95	3·64	4·10
10685	3·3	4·23	3·72	2·59	1·90	1·69	2·74
10730	1·6	7·45	5·76	5·56	2·84	3·66	4·32
10707	1·6	6·31	5·30	2·57	3·24	2·42	3·88
	$\gamma^* \times 10^{-9}$	5·58	4·74	3·66	2·59	2·68	3·45
	$\mu_\gamma \times 10^{-9}$	0·61	0·40	0·54	0·27	0·40	0·42
	τ^*-range	1·77	1·42	1·18	1·04	0·86	0·68
		1·96	1·53	1·26	1·08	0·90	0·71

then

$$\tau_0^* = \frac{\int_0^\infty \tau_0 \Delta(\tau_0) d\tau_0}{\int_0^\infty \Delta(\tau_0) d\tau_0}. \tag{10}$$

The procedure used in the investigation was as follows. First we computed the $d(V)$ and the $\tau_0^*(V)$ relations. The computations were made for the Utrecht Reference Model (1964) of the photosphere[8] separately for each of the three columns of this model.

We chose a small value of V and assumed a sec θ value. For these values the corresponding value of the depression d was computed with the IBM 1620 computer of the Uccle Observatory. If d appeared to be smaller than 10^{-3}, the computation was repeated for a new V-value that was three times larger than the previous one; if $d > 10^{-3}$ the resulting values of d and of τ_0^* were printed and the computations were repeated for a new value of V which was 1·4 times larger than the previous value.

This procedure was repeated until a value of d was reached that exceeded 0·08 (arbitrarily chosen upper limit).

The computations were made for the three columns of the URP 1964 model, and for six values of cos θ, ranging from 0·3 to 1·0.

In Fig. 1 we show, as an example, the resulting values of d and of τ_0^* as a function of V for the three elements, and for cos θ = 1·00. The differences between $d(\tau_0^*)$ values for the three columns are small but by no means negligible; at certain V-values the ratio between the extreme values is a factor 2. Since the relative contribution of the three kinds of photospheric columns is unknown we assumed them to be equally important and determined their straight mean value (solid curve) which we call $\bar{d}$. We do not show the results for other values of cos θ; these appeared to be nearly identical with those found

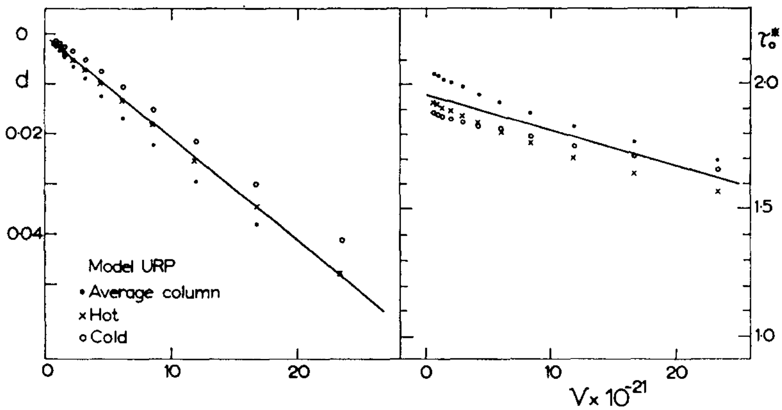

FIG. 1.

for the disc centre, at least for $d < 0{\cdot}05$. This is apparently due to the fact that the gradient of $B(\tau_\lambda)$ is nearly constant in the relevant parts of the photosphere. For $\bar{d}$-values smaller than about 0·05 it appears possible to write $\bar{d} = KV$, where K is a constant. For the various cos θ-values these K-values are given in the second line of Table 1.

In order to determine γ, the following procedure was used. The observed depressions d in the extreme line wings were represented by

$$d_{\text{obs}} = C(\Delta\lambda)^{-2}. \tag{11}$$

We know further that

$$\bar{d} = KV. \tag{12}$$

Here C and K are constants.

From the equations (11) and (12), setting $d_{\text{obs}} = \bar{d}$, we derive the expression

$$V = \frac{C}{K(\Delta\lambda)^2}. \tag{13}$$

Combining equation (13) with equation (7), one obtains

$$A_C\gamma = C/(0{\cdot}260 fgK). \tag{14}$$

Here all right-hand member quantities are known so that $A_C\gamma$ may be found.

The C-values have already been given previously[4]. The values of K, derived from the computations described in this paper are given in the second line of Table 1. In order to obtain an estimate of γ we assumed log $A_C = \bar{4}{\cdot}61$,[6] and so obtained the A_Cfg-values given in the second column of Table 1. Further, the Table gives the resulting γ-values.

We also give for each cos θ the average γ-value (γ^*) and its mean error. Furthermore, we give the range of τ_0^* values to which the γ^* values apply. This τ_0^*-range was found from the computed $\tau_0^*(d)$ relation, and from the extreme d-values where the linear relations (11) and (12) are still applicable.

The resulting $\gamma^* - \tau_0^*$ relation is shown in Fig. 2. It is clear that the mean error in γ is still rather large; this is due to the uncertainties in the determination of C.

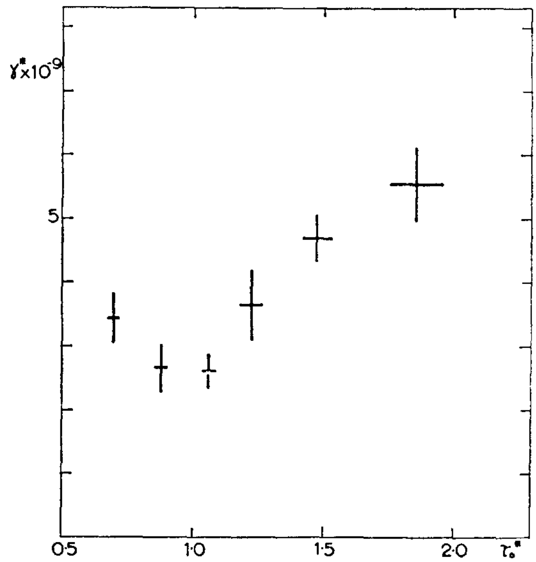

FIG. 2.

4. COMPARISON BETWEEN THEORY AND OBSERVATIONS

Although we want to postpone a thorough discussion of the results, when γ-determinations for a greater number of spectral lines are available, we will here draw attention to the shape of the $\gamma(\tau_0^*)$ curve which consists of a nearly horizontal part for $\tau_0^* < 1$, followed by a part where γ^* increases with depth. This latter part can easily be explained as being due to collisions with free electrons. For these collisions we have (ref. 8: eq. VII. 22, p. 66):

$$\log \gamma_e = \log K + \log P_e + 5/6 \log \Theta \approx \log K_1 + \log P_e. \quad (15)$$

Since $\log \Theta$ is nearly constant, K_1 is nearly constant too.

The $\gamma_e(\tau_0)$ gradient is virtually equal to the $\log P_e(\tau_0)$ gradient, which seems to apply to $\tau_0 > 1$.

In the less deep photospheric regions, where γ seems nearly constant, the effect of collisions with neutral H and He particles seems to dominate. For these we have

$$\log \gamma_{\mathrm{H,He}} = \log K_2 + \log P_g, \quad (16)$$

where K_2 is a nearly constant function of depth (ref. 8: eq. VII, 19 and VII, p. 20). Since also P_g changes only slightly with depth, $\gamma_{\mathrm{H,He}}$ is also nearly constant with depth. A rapid empirical determination of the values of K_1 and K_2 is made as follows:

We take two representative parts of the photosphere. At $\tau_0 = 0{\cdot}7$, where γ is still nearly constant, $\gamma = 3 \times 10^9$. At $\tau = 1{\cdot}85$, in the steeper part of the $\gamma(\tau_0)$ curve we have $\gamma = 5{\cdot}6 \times 10^9$. Since at these two depths $P_e = 48$ and 220, and $P_g = 1{\cdot}48 \times 10^5$ and

$1{\cdot}75-10^5$ respectively, we obtain two linear equations with the two unknowns K_1 and K_2. The solution is

$$K_1 = 1{\cdot}29\times10^7,$$

$$K_2 = 1{\cdot}61\times10^4.$$

Next, these two values are compared with theoretical predictions. For collisions with neutral H and He we have[8]

$$\log\gamma_{\mathrm{H,He}} = 3{\cdot}84+\log P_g+0{\cdot}7\log\Theta+\log(n_0/\Sigma n_r)_{\mathrm{H}} + \frac{2}{5}\log\left[\left(\frac{13{\cdot}6Z}{\chi_r-\chi_{r,s}}\right)^2_{\mathrm{upper}} - \left(\frac{13{\cdot}6Z}{\chi_r-\chi_{r,s}}\right)^2_{\mathrm{lower}}\right]$$

Since hydrogen is virtually neutral, $\log(n_0/\Sigma n_r)_{\mathrm{H}} = 0$. Inserting $\Theta = 0{\cdot}75$; $\chi = 11{\cdot}20$ eV; $\chi_{r,s} = 7{\cdot}45$ and $8{\cdot}60$ eV, we predict a theoretical value $K_2 = 1{\cdot}62\times10^4$, which is in better agreement with the "observed" value than one could expect from the inaccuracy of the observed γ values! This result proves that the observations confirm completely the theory on the contribution to the damping by collisions with neutral particles. There are no measurements or theories that would enable us to predict K_1 for the carbon lines studied by us. However, from the value of K_1 found above one can derive a value for the constant K occurring in equation (15); it is

$$\log K = 7{\cdot}21.$$

This is a very acceptable value, which is completely in the range of other values for this constant, measured for other lines (ref. 8: Table VII, 3, p. 66).

We conclude that there is fair agreement between theory and observations on the damping of neutral carbon in the solar photosphere.

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REFERENCES

1. M. G. J. Minnaert and G. F. J. Mulders, *Z. Astrophys.* **2**, 165 (1932).
2. P. ten Bruggencate and J. Houtgast, *Z. Astrophys.* **20**, 149 (1940).
3. H. H. Voigt, *Z. Astrophys.* **27**, 82 (1949).
4. C. de Jager and L. Neven, *Mem. Soc. Roy. Sci. Liège*, **9**, 213 (Liège Colloquium 1963) (1964).
5. C. de Jager and L. Neven, in *The Abundance Determination in Stellar Spectra*, Utrecht Symposium 1964 (1965).
6. C. de Jager and L. Neven, in preparation (1966).
7. C. de Jager and L. Neven, in preparation (1966).
8. C. de Jager and L. Neven, *Rech. Obs. Astr. Utrecht*, **13**, (4) (1957).
9. J. R. W. Heintze, H. Hubenet and C. de Jager, *Bull. Astr. Insts. Netherlds.* **17**, 442 (1964).