

## SHELL MODEL CALCULATIONS OF $ft$ VALUES IN THE $2s_{\frac{1}{2}}$ AND $1d_{\frac{3}{2}}$ SHELLS

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**Abstract:** The  $ft$  values of allowed beta transitions between even parity states in the region between  $^{29}\text{Si}$  and  $^{40}\text{Ca}$  are computed with the use of wave functions that were calculated earlier in a  $j$ - $j$ -coupling model with configuration interaction taken into account.

On the whole, there is good agreement between theoretical and experimental  $\log ft$  values (the average absolute difference amounts to 0.5), with the exception of the decays of  $^{32}\text{Si}$  and  $^{32}\text{P}$ .

These results are applied in a calculation of (i) the cross section of  $^{37}\text{Cl}$  for solar neutrinos, (ii) the half-life of  $^{37}\text{Ca}$  and (iii) the Gamow-Teller matrix element for  $^{35}\text{Ar}$  decay.

### 1. Introduction

Recently, shell model calculations<sup>1,2)</sup> have made available the wave functions of nuclei in the  $2s_{\frac{1}{2}}$  and  $1d_{\frac{3}{2}}$  shells. These wave functions were determined so as to reproduce the energies of ground states and low-lying levels, with configuration mixing being taken into account. The first 28 nucleons, filling the  $1s_{\frac{1}{2}}$ ,  $1p_{\frac{3}{2}}$ ,  $1p_{\frac{1}{2}}$  and  $1d_{\frac{3}{2}}$  states, were assumed to constitute an inert, spherically symmetric  $^{28}\text{Si}$  core. The residual interaction of the extra-core nucleons was taken to be a sum of two-body interactions.

In this paper we shall, in the same model, calculate the  $ft$  values of allowed beta transitions between ground states and low-lying levels in the region  $^{29}\text{Si}$ – $^{40}\text{Ca}$ , employing the wave functions that were derived in refs. <sup>1,2)</sup>. The isospin is assumed to be a good quantum number, so that we can avail ourselves of the concise isospin formalism.

The model excludes the possibility of finding contributions of the inner core nucleons to the beta transition probabilities. It will be seen, however, that the numerical results for the  $\log ft$  values, where a comparison can be made, are in good agreement with the experimental values, except for the transitions involving the  $^{32}\text{P}$  ground state.

In sect. 2 we evaluate the transition matrix elements in terms of single-particle matrix elements. The calculated  $ft$  values are listed in sect. 3. In sect. 4 these results are applied to derive (i) the absorption cross section of  $^{37}\text{Cl}$  for solar neutrinos and the half-life of  $^{37}\text{Ca}$  and (ii) the Gamow-Teller matrix element for  $^{35}\text{Ar}$  decay. Sect. 5 contains some concluding remarks.

**2. Evaluation of the Matrix Elements**

The wave functions to be used are linear combinations of antisymmetrized products of the states  $s^{n_k}(J'_k T'_k)$  and  $d^{m_k}(J''_k T''_k)$ :

$$\Psi_{JT} = \sum_k a_k \psi \{ s^{n_k}(J'_k T'_k), d^{m_k}(J''_k T''_k); JT \} \equiv \sum_k a_k \psi_k. \tag{2.1}$$

The summation is extended over all possible values of  $J'_k T'_k$  and  $J''_k T''_k$  that can couple to spin and isospin  $JT$ , and all values of  $n_k$  and  $m_k$  such that  $n_k + m_k = A - 28$  (= the number of extra-core nucleons). The amplitudes  $a_k$  have been tabulated <sup>2)</sup> for the ground states and low-lying levels in the region  $^{29}\text{Si} - ^{40}\text{Ca}$ .

For the calculation of the beta decay matrix elements between states  $\Psi_{JT}$  one first has to determine the matrix elements between the pure configurations  $\psi_k$ . As we restrict our treatment to allowed transitions, we have to evaluate the Fermi and Gamow-Teller matrix elements only. These can both be reduced to summations over single-particle matrix elements, since the appropriate operators are sums over single-particle operators. Grayson and Nordheim <sup>3)</sup> have explicitly derived these expressions. We shall therefore forego the detailed reductions and only state the results for the square of Fermi and Gamow-Teller matrix elements between a final state

$$\sum_k a_k |s^{n_k} \alpha_k, d^{m_k} \beta_k; J_f T_f \rangle, \tag{2.2a}$$

and an initial state

$$\sum_l b_l |s^{n_l} \alpha_l, d^{m_l} \beta_l; J_i T_i \rangle. \tag{2.2b}$$

The isospin operator  $\tau_y$ , matrix form  $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ , will be used to comprise  $\beta^-$  and  $\beta^+$  decay. The vector operator  $\sigma$  will be the customary Pauli spin vector operator. The Greek symbols  $\alpha, \beta, \gamma, \delta$ , will denote  $J$  and  $T$  values together.

The squares of the Fermi matrix element

$$|M_F|^2 = \left| \int 1 \right|^2 = \frac{1}{2J_f + 1} \sum_{M_i M_f} \left| \sum_{kl} a_k b_l \langle s^{n_k} \alpha_k, d^{m_k} \beta_k; J_f T_f | \sum_{r=1}^{n+m} \tau_y(r) | s^{n_l} \alpha_l, d^{m_l} \delta_l; J_i T_i \rangle \right|^2 \tag{2.3}$$

and of the Gamow-Teller matrix element

$$|M_{GT}|^2 = \left| \int \sigma \right|^2 = \frac{1}{2J_f + 1} \sum_{M_i M_f} \left| \sum_{kl} a_k b_l \langle s^{n_k} \alpha_k, d^{m_k} \beta_k; J_f T_f | \sum_{r=1}^{n+m} \sigma(r) \tau_y(r) | s^{n_l} \alpha_l, d^{m_l} \delta_l; J_i T_i \rangle \right|^2 \tag{2.4}$$

between the states (2.2a) and (2.2b), can be given in one expression (where  $A = 0$  denotes the Fermi case,  $A = 1$  the Gamow-Teller case):

$$|M_A|^2 = \langle T_i, T_{zi}, 1, T_{zf} - T_{zi} | T_f, T_{zf} \rangle^2 \frac{(2J_f + 1)(2T_i + 1)}{9(2A + 1)^2} \times \left[ \sum_{k,l} a_k b_l \{ n_k A_A(k, l) + m_k B_A(k, l) \} \right]^2. \tag{2.5}$$

Here we have introduced the following abbreviations (the indices  $k$  and  $l$  are suppressed in the right hand members):

$$A_A(k, l) = \left\{ \frac{(2J_\alpha + 1)(2T_\alpha + 1)(2J_\gamma + 1)(2T_\gamma + 1)}{(2J_\beta + 1)(2T_\beta + 1)} \right\}^{\frac{1}{2}} (-1)^{1+J_\beta+T_\beta+J_\gamma+T_\gamma} \\ \times U(J_\alpha J_\gamma J_\beta J_i; AJ_\beta) U(T_\alpha T_\gamma T_\beta T_i; 1T_\beta) \langle s_{\frac{1}{2}} || \Omega_A || s_{\frac{1}{2}} \rangle \\ \times \sum_{\eta} (-1)^{J_\eta+T_\eta} \{ (2J_\eta + 1)(2T_\eta + 1) \}^{\frac{1}{2}} U(\frac{1}{2} J_\alpha J_\gamma; AJ_\eta) \\ \times U(\frac{1}{2} T_\alpha T_\gamma; 1T_\eta) \langle s^n \alpha | s^{n-1} \eta \rangle \langle s^n \gamma | s^{n-1} \eta \rangle, \quad (2.6)$$

$$B_A(k, l) = \left\{ \frac{(2J_\beta + 1)(2T_\beta + 1)(2J_\delta + 1)(2T_\delta + 1)}{(2J_\alpha + 1)(2T_\alpha + 1)} \right\}^{\frac{1}{2}} (-1)^{J_\alpha+T_\alpha+J_\beta+T_\beta+J_\delta+T_\delta+J_i+T_i} \\ \times U(J_\beta J_\delta J_\gamma J_i; AJ_\alpha) U(T_\beta T_\delta T_\gamma T_i; 1T_\alpha) \langle d_{\frac{1}{2}} || \Omega_A || d_{\frac{1}{2}} \rangle \\ \times \sum_{\eta} (-1)^{J_\eta+T_\eta} \{ (2J_\eta + 1)(2T_\eta + 1) \}^{\frac{1}{2}} U(\frac{3}{2} J_\beta J_\delta; AJ_\eta) \\ \times U(\frac{1}{2} T_\beta T_\delta; 1T_\eta) \langle d^m \beta | d^{m-1} \eta \rangle \langle d^m \delta | d^{m-1} \eta \rangle. \quad (2.7)$$

The non-vanishing reduced (i.e., reduced in  $J$ - and  $T$ -space<sup>4</sup>) single-particle matrix elements are given by

$$\langle s_{\frac{1}{2}} || \Omega_0 || s_{\frac{1}{2}} \rangle = \langle s_{\frac{1}{2}} || \tau || s_{\frac{1}{2}} \rangle = \sqrt{6}, \quad (2.8a)$$

$$\langle d_{\frac{1}{2}} || \Omega_0 || d_{\frac{1}{2}} \rangle = \langle d_{\frac{1}{2}} || \tau || d_{\frac{1}{2}} \rangle = \sqrt{12}, \quad (2.8b)$$

$$\langle s_{\frac{1}{2}} || \Omega_1 || s_{\frac{1}{2}} \rangle = \langle s_{\frac{1}{2}} || \sigma \tau || s_{\frac{1}{2}} \rangle = \sqrt{18}, \quad (2.8c)$$

$$\langle d_{\frac{1}{2}} || \Omega_1 || d_{\frac{1}{2}} \rangle = \langle d_{\frac{1}{2}} || \sigma \tau || d_{\frac{1}{2}} \rangle = -\sqrt{\frac{3}{5}}. \quad (2.8d)$$

The coefficients of fractional parentage  $\langle \rho^n \alpha | \rho^{n-1} \beta \rangle$  in the  $JT$  direct product space have been tabulated elsewhere<sup>1</sup>).

### 3. The $ft$ Values

The  $ft$  value of a transition is defined in terms of the matrix elements by

$$ft = \frac{2\pi^3 (\ln 2) \hbar^7}{m^5 c^4} \frac{1}{G_V^2 \{ |M_F|^2 + (C_A/C_V)^2 |M_{GT}|^2 \}} = \frac{6100}{|M_F|^2 + (C_A/C_V)^2 |M_{GT}|^2} s. \quad (3.1)$$

For the vector coupling constant the value<sup>5</sup>  $G_V = (1.403 \pm 0.002) \times 10^{-49}$  erg · cm<sup>3</sup> has been used. The ratio of the strengths of vector and axial vector interactions is given by<sup>5</sup>  $C_A/C_V = -1.18 \pm 0.05$ .

Employing eqs. (2) and (5)–(7) we have calculated the  $ft$  values according to eq. (3.1) for transitions between ground states and low-lying levels in the region <sup>29</sup>Si–<sup>40</sup>Ca. The numerical values of the amplitudes  $a_k(b_i)$  in eq. (2.5) were taken from ref.<sup>2</sup>.

The resulting log  $ft$  values are tabulated in table 1 and compared with experimental values. It is seen that the agreement is good (average of the absolute values of the

TABLE I

The log *ft* values computed with allowed matrix elements are given in the seventh column and are compared with the experimental values in the sixth column

Initial nucleus ( <i>JT</i> )	Decay and <i>Q<sub>m</sub></i> (keV)	Final nucleus ( <i>JT</i> ; <i>E<sub>x</sub></i> MeV)	Half-life	Branching ratio (%)	(log <i>ft</i> ) <sub>exp.</sub>	(log <i>ft</i> ) <sub>theor.</sub>
<sup>29</sup> P (½ ½)	β <sup>+</sup> , 4948 ± 9	<sup>29</sup> Si (½ ½; 0.0)	4.23 ± 0.05 s	98.8 ± 0.4	3.73 ± 0.01	3.07
		<sup>29</sup> Si (¾ ½; 1.277)		1.09 ± 0.10	4.94 ± 0.09	∞
<sup>30</sup> P (1 0)	β <sup>+</sup> , 4248 ± 10	<sup>30</sup> Si (0 1; 0.0)	2.55 ± 0.02 min	99.5	4.84 ± 0.06	4.05
		<sup>30</sup> Si (2 1; 2.23)		0.5	4.9 ± 0.1	3.86
<sup>31</sup> Si (¾ ¾)	β <sup>-</sup> , 14767 ± 46	<sup>31</sup> P (½ ½; 0.0)	157.3 ± 0.4 min	99.9	5.50 ± 0.02	5.69
		<sup>31</sup> P (¾ ½; 1.265)		0.07	5.2 ± 0.1	5.82
<sup>31</sup> S (½ ½)	β <sup>+</sup> , 5450 ± 17	<sup>31</sup> P (½ ½; 0.0)	2.61 ± 0.03 s	98.9	3.7 ± 0.1	3.19
		<sup>31</sup> P (¾ ½; 1.265)		1.1 ± 0.1	5.0 ± 0.1	5.94
<sup>32</sup> Si (0 2)	β <sup>-</sup> , 219 ± 15	<sup>32</sup> P (1 1; 0.0)	100–710 a	100	8.4 ± 0.4	4.19
<sup>32</sup> P (1 1)	β <sup>-</sup> , 1708.4 ± 2.0	<sup>32</sup> S (0 0; 0.0)	14.32 ± 0.02 d	100	7.9 ± 0.1	5.51
<sup>32</sup> Cl (2 1)	β <sup>+</sup> , 13000 ± 300	<sup>32</sup> S (2 0; 2.24)	0.306 ± 0.004 s	48 ± 15	4.6 ± 0.2	3.70
<sup>33</sup> P (½ ¾)	β <sup>-</sup> , 248 ± 2	<sup>33</sup> S (¾ ½; 0.0)	24.6 ± 0.2 d	100	5.0 ± 0.1	6.09
<sup>33</sup> Cl (¾ ½)	β <sup>+</sup> , 5575 ± 12	<sup>33</sup> S (¾ ½; 0.0)	2.53 ± 0.04 s	99.7	3.7 ± 0.1	3.56
		<sup>33</sup> S (½ ½; 0.841)				6.99
<sup>34</sup> P (1 2)	β <sup>-</sup> , 5100 ± 200	<sup>34</sup> S (0, 1; 0.0)	12.40 ± 0.12 s	75	5.1 ± 0.2	5.73
		<sup>34</sup> S (2 1; 2.13)		25	4.7 ± 0.3	3.91
		<sup>34</sup> S (2 1; 3.30)			> 5.6	6.24
		<sup>34</sup> S (2 1; 4.0)				4.07
		<sup>34</sup> S (0 1; 4.0)			≥ 0.2	≤ 4.9
<sup>34</sup> Cl (0 1)	β <sup>+</sup> , 5519 ± 21	<sup>34</sup> S (0 1; 0.0)	1.588 ± 0.014 s	100	3.5 ± 0.1	3.48
<sup>34</sup> Cl <sup>m</sup> (3 0)	β <sup>+</sup> , 5662 ± 21	<sup>34</sup> S (2 1; 2.13)	32.40 ± 0.04 min	27	6.1 ± 0.1	4.84
		<sup>34</sup> S (2 1; 3.30)		27	4.9 ± 0.4	4.84
		<sup>34</sup> S (2 1; 4.11)		0.4	5.4 ± 0.4	4.76
<sup>35</sup> S (¾ ¾)	β <sup>-</sup> , 167.34 ± 0.19	<sup>35</sup> Cl (¾ ½; 0.0)	86.73 ± 0.27 d	100	5.0 ± 0.1	4.49
<sup>35</sup> Ar (¾ ½)	β <sup>+</sup> , 5970 ± 30	<sup>35</sup> Cl (¾ ½; 0.0)	1.804 ± 0.012 s	93	3.8 ± 0.1	3.65
		<sup>35</sup> Cl (½ ½; 1.220)		5 ± 2	4.7 ± 0.4	6.34
		<sup>35</sup> Cl (¾ ½; 1.762)		2 ± 1	4.5 ± 0.5	4.50
<sup>37</sup> Ar (¾ ½)	EC, 816.0 ± 1.5	<sup>37</sup> Cl (¾ ¾; 0.0)	34.33 ± 0.15 d	K91; L9	5.0 ± 0.1	4.45
<sup>37</sup> Ca (¾ ¾)	β <sup>+</sup> , 11460 ± 100	<sup>37</sup> K	0.173 ± 0.004 s <sup>8,9)</sup>	see table 3		
<sup>37</sup> K (¾ ½)	β <sup>+</sup> , 6150 ± 50	<sup>37</sup> Ar (¾ ½; 0.0)	1.23 ± 0.02 s	98.0 <sup>13)</sup>	3.66 ± 0.01 <sup>13)</sup>	3.63
		<sup>37</sup> Ar (½ ½; 1.42)		< 0.20 ± 0.05 <sup>13)</sup>	≥ 5.7 <sup>13)</sup>	5.21
		<sup>37</sup> Ar (¾ ½; 2.79)		2.0 ± 0.4 <sup>13)</sup>		3.8 ± 0.1
<sup>38</sup> K (3 0)	β <sup>+</sup> , 5929 ± 11	<sup>38</sup> Ar (2 1; 2.16)	7.66 ± 0.03 min	100	5.0 ± 0.1	4.32
<sup>38</sup> K <sup>m</sup> (0 1)	β <sup>+</sup> , 6052 ± 14	<sup>38</sup> Ar (0 1; 0.0)	0.946 ± 0.005 s	100	3.5 ± 0.1	3.49
<sup>39</sup> Ca (¾ ½)	β <sup>+</sup> , 6490 ± 40	<sup>39</sup> K (¾ ½; 0.0)	0.877 ± 0.006 s	100	3.6 ± 0.1	3.52
		<sup>39</sup> K (½ ½; 2.53)		< 0.12	> 5.3	∞

The nuclear configurations used in the computation were all taken from ref. <sup>2)</sup>. For each initial state (ground state) we have used the lowest member of a *JT* multiplet; the final states are also lowest members of *JT* multiplets, with the exception of five instances of first excited states and two instances of second excited states (the <sup>37</sup>K(¾, ¾; 5.07) configuration belongs to the multiplet listed under <sup>37</sup>Cl(¾, ¾) in ref. <sup>2)</sup>, and in fact is given by the lowest member, with the energy corrected for the difference in Coulomb energies of <sup>37</sup>K and <sup>37</sup>Cl). The calculated infinite *ft* values (<sup>29</sup>P and <sup>39</sup>Ca decay) represent two cases of *l*-forbiddenness in our shell model calculations. All experimental data were taken from ref. <sup>7)</sup> except where indicated differently. No corrections for finite nuclear size and screening were made as these are outside the accuracy of our calculations.

deviations amounts to 0.5), with the exception of transitions involving the  $^{32}\text{P}$  ground state. For a calculation of ground state magnetic dipole moments a similar difficulty exists<sup>6</sup>); the configurations used do not give the correct magnetic moment of the  $^{32}\text{P}$  ground state, although for this nucleus five of the low-lying level energies are well reproduced.

This may be an indication that the assumption of a closed  $1d_{\frac{5}{2}}$  shell, being part of the inert core, is not quite realistic.

## 4. Applications

### 4.1. THE SOLAR NEUTRINO EXPERIMENT

The calculated matrix elements can be used to determine the cross section for the reaction  $^{37}\text{Cl} + \nu \rightarrow ^{37}\text{Ar} + e^-$ , which is of interest for the detection of solar neutrinos<sup>10, 11</sup>). Bahcall<sup>12</sup>) has discussed this reaction, which may proceed to either the  $^{37}\text{Ar}$  ground state or three of its excited states. Only the  $ft$  value of the ground state inverse reaction  $^{37}\text{Ar} \rightarrow ^{37}\text{Cl}$  is experimentally known. Assuming that the incomplete overlap of the single-particle radial wave functions of the  $^{37}\text{Cl}$  initial  $T = \frac{3}{2}$  and the three  $^{37}\text{Ar}$  final  $T = \frac{1}{2}$  states (i.e., ground state and two of its excited levels) is the same for all three transitions, Bahcall<sup>12</sup>) estimated the  $ft$  values. The fourth transition is superallowed and thus permits a theoretical prediction of its  $ft$  value. Averaging over the solar neutrino flux, Bahcall then predicted the number of neutrino captures to be  $(3.6 \pm 2) \times 10^{-35}$  per  $^{37}\text{Cl}$  atom per sec.

We have repeated his calculation, employing, however, the  $ft$  values as they were computed in the previous section and recent experimental data.

Some of the intermediate results are given in table 2, where a comparison is made with Bahcall's calculation. It is seen that the two sets of results appreciably differ, with the exception of the superallowed transition to the  $(\frac{3}{2}, \frac{3}{2})$  level. The averaging procedure over the solar neutrino spectrum attributes the largest weight to the latter transition. As a result we obtained for the number of neutrino captures  $8.7 \times 10^{-35}$  per  $^{37}\text{Cl}$  atom per sec.

TABLE 2

The solar neutrino absorption reaction by  $^{37}\text{Cl}$  may proceed to either of the four  $^{37}\text{Ar}$  levels listed above.

$^{37}\text{Ar}$ level ( $J, T$ )	Level energy (MeV)		$ft$ value ( $10^3$ s)	
$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	<i>115</i>	28.4
$\frac{1}{2}, \frac{1}{2}$	<i>1.42</i>	1.41	<i>30.4</i>	51.4
$\frac{5}{2}, \frac{1}{2}$	<i>1.61</i>	2.79	<i>21.7</i>	6.46
$\frac{3}{2}, \frac{3}{2}$	<i>5.15</i>	5.15	<i>1.89</i>	1.85

Intermediate results, necessary for a calculation of the cross section of this reaction, are given (i) according to Bahcall<sup>12</sup>) (in italics) and (ii) according to our calculation (in straight type). Since Bahcall's calculation was published, the  $(\frac{5}{2}, \frac{1}{2})$  level in  $^{37}\text{Ar}$  has been identified<sup>13</sup>) as the 2.79 MeV level.

The  $\beta^+$  decay of  $^{37}\text{Ca}$  is the mirror reaction of the neutrino capture by  $^{37}\text{Cl}$ . It is of interest therefore, to calculate the  $^{37}\text{Ca}$  half-life, since this half-life has been measured<sup>8,9</sup>). The intermediate results of such a calculation are given in table 3 and compared with Bahcall's results. Our calculation yields for the  $^{37}\text{Ca}$  half-life a value 79 ms. It is seen that this value is about two times smaller than the weighted mean  $172 \pm 3$  ms of the measured values<sup>8,9</sup>).

TABLE 3  
The  $\beta^+$  decay of  $^{37}\text{Ca}$  is the mirror reaction of the neutrino capture by  $^{37}\text{Cl}$ .

$^{37}\text{K}$ level ( $J, T$ )	Level energy (MeV)		$ft$ value ( $10^3$ s)		Half-life ( $10^3$ s)		Branching ratio (%)	
$\frac{3}{2}, \frac{1}{2}$	0.0	0.0	<i>115</i>	28.4	<i>1.04</i>	0.26	<i>12</i>	34
$\frac{1}{2}, \frac{1}{2}$	1.46	1.36	<i>30.4</i>	51.4	<i>0.57</i>	0.89	<i>23</i>	10
$\frac{5}{2}, \frac{1}{2}$	1.57	2.74	<i>21.7</i>	6.46	<i>0.43</i>	0.28	<i>31</i>	32
$\frac{3}{2}, \frac{3}{2}$	5.12	5.07	<i>1.89</i>	1.85	<i>0.39</i>	0.39	<i>34</i>	24

Four  $^{37}\text{K}$  levels to which the reaction may proceed are given with their energies, calculated corresponding  $ft$  values, half-lives and branching ratios (i) according to Bahcall<sup>12</sup>) (in italics) and (ii) according to our calculation (in straight type). Since Bahcall's calculation was published, the ( $\frac{3}{2}, \frac{1}{2}$ ) level in  $^{37}\text{K}$  has been identified<sup>13</sup>) as the 2.74 MeV level.

#### 4.2. GAMOW-TELLER MATRIX ELEMENT FOR $^{35}\text{Ar}$ DECAY

This matrix element for the transition to the  $^{35}\text{Cl}$  ground state has been calculated from the measured asymmetry for beta decay of polarized  $^{35}\text{Ar}$  nuclei by Calaprice, Commins and Dobson<sup>14</sup>) to be

$$\left| \int \sigma \right| = 0.1 \pm 0.05, \quad \text{or} \quad \left| \int \sigma \right|^2 \approx 0.01.$$

Starting from the calculated configurations, however, we obtained for the square of this Gamow-Teller matrix element a value of  $|\int \sigma|^2 = 0.6$ . This would imply a discrepancy of almost two orders of magnitude.

### 5. Concluding Remarks

The log  $ft$  values in the region  $^{29}\text{Si}$ - $^{40}\text{Ca}$ , calculated in the model of an inert  $^{28}\text{Si}$  core with extra-core nucleons filling the  $2s_{\frac{1}{2}}$  and  $1d_{\frac{3}{2}}$  shells only, are seen to be closely following the trend of the experimental values. The average of the absolute values of the deviations amounts to 0.5, so that one can expect that the calculated half-lives may differ by a factor three or four from the measured values. When the initial and/or final states are given by mixed configurations, there may be appreciable cancellation between the various terms that contribute to the cross section. In such cases the configurations are required to be known with high accuracy. This has been pointed out by Coussemant<sup>15,16</sup>), when explaining large  $ft$  values for allowed

transitions. The customary explanation of these large  $ft$  values, however, in terms of  $l$ -forbiddenness assumes rather pure configurations.

It seems that the calculation of half-lives where the interference of the various terms begins to be important, will require a better accuracy of the wave functions, than is obtained in the model of a  $^{28}\text{Si}$  core plus the  $2s_{\frac{1}{2}}$  and  $1d_{\frac{3}{2}}$  shells.

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