

LETTER TO THE EDITOR

Influence of the statistical factor on (d,p) cross sections

It has been noted by Schiffer¹⁾ that the cross section for the ground-state transition of the $^{43}\text{Ca}(\text{d}, \text{p})^{44}\text{Ca}$ reaction is roughly fifty times smaller than that for the ground-state transition of the $^{40}\text{Ca}(\text{d}, \text{p})^{41}\text{Ca}$ reaction. Schiffer's observation, from an experiment with 4.15 Mev deuterons, is confirmed by work done at the Massachusetts Institute of Technology with 6 Mev deuterons from an electrostatic generator²⁾. There, the cross-section ratio is found to be 40 ± 10 . In both cases the comparison was based upon the bombardment of a single target, enriched in ^{43}Ca , but containing a known percentage of ^{40}Ca . The protons were observed at 90° to the incident beam.

It is possible to give a simple explanation of this large difference in cross sections. The ground states of ^{40}Ca , ^{41}Ca , ^{43}Ca and ^{44}Ca have spins of 0^+ , $7/2^-$, $7/2^-$, and 0^+ , and both reactions involve the capture of a neutron in a $f_{7/2}$ orbit ($l_n = 3$). If the reactions proceed predominantly by a stripping mechanism, the angular distributions^{3) 4)} should be the same, except for small variations, caused by the difference in the momenta of the outgoing protons, the difference in the nuclear radii of ^{40}Ca and ^{43}Ca , and the difference in the reaction angles in the center of mass system, the angles being equal in the laboratory system. Taking then the angular distribution factors to be equal for the two reactions, the cross sections are governed by the statistical factor $(2J_f + 1)/(2J_i + 1)$, which is 8 for the $^{40}\text{Ca}(\text{d}, \text{p})^{41}\text{Ca}$, and $1/8$ for the $^{43}\text{Ca}(\text{d}, \text{p})^{44}\text{Ca}$ ground-state transition, thus yielding a factor of 64 for the cross-section ratio, which is not far from the observed factor of 40 or 50. In this discussion it has tacitly been assumed that the nuclear matrix elements of these reactions are equal. The good agreement between theory and experiment shows that their squares differ by at most a factor of 1.5.

From deuteron bombardment (at $E_d = 6.5$ Mev and $\theta = 90^\circ$) of targets enriched in ^{42}Ca and ^{44}Ca it has been observed³⁾ that the cross sections of the $^{42}\text{Ca}(\text{d}, \text{p})^{43}\text{Ca}$ and $^{44}\text{Ca}(\text{d}, \text{p})^{45}\text{Ca}$ ground-state transitions are 0.55 ± 0.05 and 0.4 ± 0.1 times that of $^{40}\text{Ca}(\text{d}, \text{p})^{41}\text{Ca}$. In these three reactions, the initial nucleus has spin 0^+ and the final nucleus presumably $7/2^-$. To explain the observed differences in cross sections, one has to take into account the degree to which the $f_{7/2}$ shell is filled in the target nucleus. In the case of the $^{40}\text{Ca}(\text{d}, \text{p})^{41}\text{Ca}$ reaction, the incoming neutron can occupy any of the eight empty magnetic substates. In ^{42}Ca only six substates are empty, and in ^{44}Ca only four. The cross sections of the (d, p) reactions on ^{42}Ca and ^{44}Ca would then be 0.75 and 0.5 times that of the $^{40}\text{Ca}(\text{d}, \text{p})^{41}\text{Ca}$ reaction, in reasonable agreement with the observed cross section ratios.

A similar comparison can be made between the cross sections of the ground-state transitions of the $^{24}\text{Mg}(\text{d}, \text{p})^{25}\text{Mg}$ and $^{25}\text{Mg}(\text{d}, \text{p})^{26}\text{Mg}$ reactions, both proceeding by capture of a $d_{5/2}$ neutron. The statistical factor is 6 for the first and $1/6$ for the second reaction, which yields a factor of 36 for the cross-section ratio. This reduces, however, to 12 because the $d_{5/2}$ subshell in ^{24}Mg is already for two third filled. The geometrical

mean of two experimental determinations ⁶⁾ (both at $E_d = 1.81$ Mev and $\theta = 90^\circ$, one using a natural Mg target, the other a target enriched in ²⁵Mg) is 13. The experimental error may have amounted here up to a factor of two.

For the ²⁸Si(d, p)²⁹Si and ²⁹Si(d, p)³⁰Si reactions, both involving capture of a $s_{1/2}$ neutron, the theoretical cross-section ratio is 4, whereas the geometrical mean of three experimental determinations ⁶⁾ (one at $E_d = 1.81$ Mev and two at $E_d = 2.00$ Mev, all at $\theta = 90^\circ$) amounts to 4.5, also subject to an experimental error of a factor of two.

Generalizing one may say that the cross sections of (d, p) reactions with even-even initial nuclei tend to be relatively large, especially so if the neutron number of the initial nucleus is magic. Another example of this rule was found recently ⁷⁾. The two most energetic proton groups from the ¹³⁸Ba(d, p)¹³⁹Ba reaction, observed at $\theta = 90^\circ$ from the deuteron bombardment of a BaCl₂ target both at $E_d = 7.0$ and 7.5 Mev, were more intense than any proton group from the ³⁵Cl(d, p)³⁶Cl or ³⁷Cl(d, p)³⁸Cl reactions, although Coulomb barrier penetration would greatly favour the latter reactions, and although the target contained twice as many Cl as Ba nuclei. The initial nucleus ¹³⁸Ba has a magic neutron shell, containing 82 neutrons.

In the past few years the Butler formalism has been extensively used, notably by Holt and Marsham ⁸⁾, and by Engle ⁹⁾, to explain the observed relative intensities of different proton groups from the same (d, p) reaction. This involves only the $(2J_f + 1)$ part of the statistical factor. It was the purpose of the present letter to point out that this formalism also successfully explains the observed relative intensities of proton groups from different (d, p) reactions, which necessitates the use of the full expression $(2J_f + 1)/(2J_i + 1)$, corrected for the partial filling of the relevant neutron shell.

It would be of interest to perform intensity measurements of higher accuracy for these and other analogous cases. The deviation from the rules given above might yield more information about the nuclear groundstate wave functions.

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Received 3-9-55.

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