SOME PROPERTIES OF THE RELAXATION MATRIX OF PHOTOCONDUCTORS OUT OF THERMODYNAMIC EQUILIBRIUM

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A three-level model of a photoconductor exhibiting a generation recombination noise spectrum with a local maximum for $\omega \neq 0$ is presented. In another model it is shown that the relaxation matrix can have complex eigenvalues.

It is well known [1, 2] that in semiconductors showing generation-recombination noise the noise density of the occupancy of the conduction band can be represented by

$$S(\omega) = \sum_{k=1}^{s-1} \frac{c_k}{1+\omega^2\tau_k^2},$$

where ω = angular frequency and c_k and τ_k are constants, in the general case that transitions between s levels affect the population of the conduction band. It is generally believed that in such a case the noise power decreases monotonously with frequency. This assumption implies the requirement that each τ_k is real and each c_k non-negative.

A necessary condition for the existence of a stable steady state is the requirement that the eigenvalues $1/\tau_k$ of the relaxation matrix A (cf. below) have positive real parts, or are positive real numbers. If, moreover, microscopic reversibility prevails, it can be shown that the imaginary part of each τ_k vanishes [3, 4] and each c_k is non-negative [4].

The question to what conditions τ_k and c_k will conform when one is going to consider e.g. an illuminated photoconductor far from thermodynamic equilibrium has not been studied as far as we know. Our object is to show theoretically on the basis of a simple three-level model (s = 3), that for an illuminated photoconductor is absence of microscopic reversibility the eigenvalues of A can indeed be complex and that the spectral noise power as a function of frequency can have a local maximum for $\omega \neq 0$, because of the circumstance that one of the coefficients c_k is negative.

We start from a simplified model of a photoconductor with one level in the forbidden band gap, which is a slight modification of Klasens' [5] model (see fig. 1).

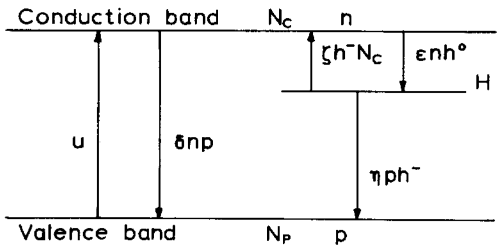

Fig. 1. n = density of electrons in the conduction band. p = density of holes in the valence band. N_c = effective density of states in the conduction band. h = density of H-levels. h^- = density of H-levels occupied by electrons. $h^0 = h - h^-$. U = rate of band-band excitation by light absorption. δ, ϵ, ζ and η are rate constants of relevant transitions.

It is now assumed that the bandgap is so large and the dark Fermi-level so far below the H-level and so far above the valence band that for $U = 0$, $n = p = h^- = 0$. Because of charge neutrality we have $p = n + h^-$. Finally, we also supposed $n \ll N_c$ for all values of U.

Considering n and p as the independent variables, and following the Langevin procedure by adding a "stochastic force" term to the linearized rate equations we obtain:

$$\frac{d\Delta n(t)}{dt} = [-2\epsilon n - \epsilon h - \zeta N_c + (\epsilon - \delta)p]\Delta n + [(\epsilon - \delta)n + \zeta N_c]\Delta p + \xi_1(t),$$

$$\frac{d\Delta p(t)}{dt} = (\eta - \delta)p\Delta n + [(\eta - \delta)n - 2\eta p]\Delta p + \xi_2(t),$$

where $\Delta n(t)$ and $\Delta p(t)$ are the deviations of $n(t)$

and $p(t)$ from their average values. In matrix notation these equations read:

$$\frac{d\alpha(t)}{dt} = -\vec{\vec{A}} \cdot \alpha(t) + \xi(t) \ ,$$

where $\alpha(t)$ is a column vector with components $\Delta n(t)$ and $\Delta p(t)$ and $\xi(t)$ a column vector with components $\xi_1(t)$ and $\xi_2(t)$.

By Fourier transformation and multiplying from the left by $(\vec{\vec{A}} + j\omega\vec{\vec{I}})^{-1}$ we obtain the corresponding equation in the frequency domain

$$\boldsymbol{a}(\omega) = (\vec{\vec{A}} + j\omega\vec{\vec{I}})^{-1}\, x(\omega) \ ,$$

where $\vec{\vec{I}}$ is the unit matrix and j the imaginary unit. Defining the noise density matrix $\vec{\vec{S}}_\alpha(\omega)$ by $\vec{\vec{S}}_\alpha(\omega) = \boldsymbol{a}(\omega)\boldsymbol{a}^\dagger(\omega)$, where the dagger (†) indicates the Hermitian conjugate, we find finally

$$\vec{\vec{S}}_\alpha(\omega) = (\vec{\vec{A}} + j\omega\vec{\vec{I}})^{-1}\ \vec{\vec{S}}_\xi(\omega)(\vec{\vec{A}} + j\omega\vec{\vec{I}})^{-1\dagger}$$

If we assume the transition rates to show full shot noise the elements of $\vec{\vec{S}}_\xi(\omega)$ are

$$\left(\vec{\vec{S}}_\xi(\omega)\right)_{n,m} = \begin{cases} \frac{2}{\pi}\sum\limits_{l=1}^{s} p_{l,m} \ , & \text{for } n = m \ , \\ -\frac{1}{\pi}(p_{n,m} + p_{m,n}) \ , & \text{for } n \neq m \ , \end{cases}$$

where $p_{n,m}$ is the absolute value of the average transition rate from level n to level m. If no detailed balance is assumed $p_{m,n}$ may differ from $p_{n,m}$ but we still have $\Sigma_m p_{n,m} = 0$ for any fixed n. From these expressions we calculated $\left(\vec{\vec{S}}_\alpha(\omega)\right)_{1,1}$, i.e. the noise density in the occupancy of the conduction band, for various selected more or less realistic sets of parameter values, by means of an electronic computer (Electrologica X1). The results of some of these computations as shown in fig. 2.

In these three cases the eigenvalues of the relaxation matrix appear incidentally to be real. In the figure their values are indicated by O. It is seen that for the case of highest excitation (curve III) the spectrum has its usual shape with two well-separated bumps. With decreasing exciting photon density the bumps shade off into each other, whereas in curve I there appears a peak in the spectrum, owing to the circumstance that one of the c_k's becomes negative. At still lower photon densities (not shown here) the spectrum regains gradually its usual shape. For another set of parameter values e.g.

$$N_c = 10^{19}, \quad h = 10^{18}, \quad n = 3\times10^{14}\ (\text{in cm}^{-3})\ ;$$

$$\delta = 2.7\times10^{-11}, \quad \epsilon = 10^{-13}, \quad \zeta = 4\times10^{-24},$$

$$\eta = 8.1\times10^{-11}\ (\text{in cm}^3\ \text{sec}^{-1}),$$

the eigenvalues of the relaxation matrix happen to be complex ($\tau_{1,2} = (8.5 \pm 1.2\,j)\times10^{-6}$ sec). Although one would expect the occurrence of complex eigenvalues to be favourable for the appearance of a peak in the noise spectrum we have as yet not succeeded in obtaining a peak in these cases.

We also computed for similar models the time response of the photoconductance to light pulses with rectangular shape. It turned out that under circumstances where the noise spectrum has a peak the response function showed an "overshoot". This overshoot and a peak in the noise spectrum was found also experimentally with CdSe photoconductors.

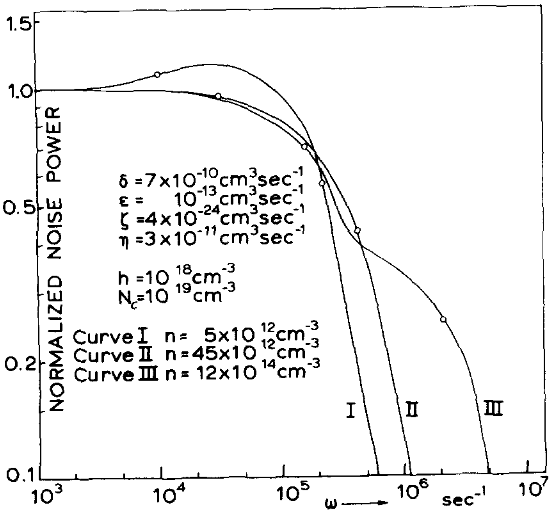

Fig. 2.

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