

## DECAY SCHEME, SPIN AND PARITY OF THE 3.13, 3.29, 3.41 AND 3.51 MeV LEVELS IN $^{31}\text{P}$

by H. A. VAN RINSVELT\*) and P. M. ENDT

Fysisch Laboratorium der Rijksuniversiteit, Utrecht, Nederland.

### Synopsis

Measurements of spectra, linear polarization, angular distributions and correlations of gamma rays from the  $^{30}\text{Si}(p, \gamma)^{31}\text{P}$  reaction at resonances below  $E_p = 2$  MeV provided information on the decay scheme, spin and parity of the levels  $^{31}\text{P}^* = 3.13, 3.29, 3.41$  and  $3.51$  MeV. The following branching percentages were observed in the decay:  $3.13$  MeV ( $\gamma_0$  100%),  $3.29$  MeV ( $\gamma_1$  82%,  $\gamma_2$  18%),  $3.41$  MeV ( $\gamma_1$  100%),  $3.51$  MeV ( $\gamma_0$  70%,  $\gamma_1 + \gamma_2$  30%). The mixing parameters of the gamma rays de-exciting these four levels have also been determined. Spins and parities are as follows:  $3.13$  MeV  $J = 1/2$ ,  $3.29$  MeV  $J^\pi = 5/2^{(+)}$ ,  $3.41$  MeV  $J^\pi = 7/2^{(+)}$ ,  $3.51$  MeV  $J^\pi = 3/2^{(+)}$ .

I. *Introduction.* With the  $^{31}\text{P}(p, p')^{31}\text{P}$  reaction Endt and Paris<sup>1)</sup> measured the excitation energy of thirteen  $^{31}\text{P}$  levels. The levels (3), (4), (5), and (6) with  $E_x = 3.133, 3.293, 3.414$ , and  $3.505$  MeV, all  $\pm 0.005$  MeV, are discussed in this contribution.

Most of the information about the decay scheme, spin and parity of these lower levels has been provided by measurements on the  $^{30}\text{Si}(p, \gamma)^{31}\text{P}$  reaction. Measurements of gamma-ray spectra, angular distributions and triple angular correlations of various gamma-ray cascades have been carried out by Hoogenboom<sup>2)</sup> and Broude *e.a.*<sup>3)</sup> at proton resonances up to 1 MeV.

This report discusses similar experiments performed over a broader energy range:  $E_p = 0.3$ – $2.0$  MeV. Some of the older measurements have been repeated. New results could be obtained due to modernized apparatus and a more elaborate method of analysis using an electronic computer. The large number of resonances in the energy range studied makes it possible to select the most suitable resonances in order to get more reliable information about specific lower levels. New results on the levels  $^{31}\text{P}^* = 3.29, 3.41$ , and  $3.51$  MeV are given. In addition, the properties of the  $3.13$  MeV level, that have been measured and briefly reported earlier<sup>4)</sup>, are further discussed. Information about the resonance levels can be found in references 5, 6 and 7.

Concurrently, similar experiments have been performed by Harris and Seagondollar<sup>8) 9)</sup> at proton resonances between 1.0 and 1.5 MeV.

\*) Boursier de l'Institut Interuniversitaire de Sciences Nucléaires de Belgique.

Recently, elastic and inelastic scattering of electrons by  $^{31}\text{P}$  has been studied by Kossanyi-Demay *e.a.*<sup>10)</sup>. Measurements on the  $^{30}\text{Si}(d, n)^{31}\text{P}$  reaction by Cujec *e.a.*<sup>11)</sup> lead to results in agreement with previous experiments.

II. *Experimental.* The experimental set-up has been described in an earlier publication<sup>6)</sup>. All the measurements were carried out using the two machines of the University of Utrecht: the 850 kV Cockcroft-Walton generator and the 3 MV Van de Graaff accelerator. In both cases, the accelerated protons were deflected through  $90^\circ$ , before hitting the  $^{30}\text{Si}$  target. Enriched  $^{30}\text{Si}$  targets of thickness  $25\text{ }\mu\text{g}/\text{cm}^2$ , supplied by the Atomic Energy Research Establishment, Harwell, England, were produced by evaporation onto 0.3 mm tantalum strips. Deterioration of the targets was not observed, even with 30 W beam power.

The gamma radiation was detected with two 10 cm diameter by 10 cm long NaI(Tl) crystals (resolution 8% for the 662 keV  $^{137}\text{Cs}$  line). The pulses were sorted by a 400-channel RIDL pulse-height analyser.

Four geometries, noted I, II, V and VI, were used in the triple angular correlation experiments. The distance between the target spot and the front of the two crystals, called A and B, was 10 cm. Coincidences between  $\gamma_1$  and  $\gamma_2$ , respectively the first and the second gamma ray of a cascade, were measured. Counter B was moving in the horizontal plane through the proton beam at angles between  $0^\circ$  and  $90^\circ$  with respect to the beam. Counter A was fixed perpendicular to the beam, either in the horizontal plane (geometries I and II), or vertically above the target (geometries V and VI). In geometries I and V,  $\gamma_1$  was detected in counter B, and in geometries II and VI,  $\gamma_1$  was detected in counter A. The use of four differential discriminators and two coincidence circuits made it possible to measure geometries I and II simultaneously in one run and geometries V and VI in another one.

III. *Analysis.* Gamma-ray spectra. The measured single and coincidence (resolution time  $\tau = 1\text{ }\mu\text{s}$ ) spectra corrected for background were analysed by the "peeling" method, using standard line shapes of monoenergetic gamma rays. The relative intensities of the gamma rays were then calculated from the relative heights of the peaks, appropriately adjusted for the detection efficiency of the crystals.

The determination of the branching ratios is still subject to errors due to 1) the possible presence of  $P_4(\cos\theta)$  terms in the angular distributions (single spectra measured at  $55^\circ$  and at 10 cm from the target), 2) summing effects (coincidence spectra measured at 1 cm from the target), and 3) different anisotropies in the angular correlations of the various cascades (coincidence spectra, in general).

Triple angular correlations. The  $^{30}\text{Si}$  ground state has  $J^\pi = 0^+$ .

Thus the channel spin can only have the value  $S = \frac{1}{2}$ . Consequently, mixing of proton orbital momenta does not have to be taken into account. However, both gamma rays of a cascade,  $\gamma_1$  and  $\gamma_2$ , can be mixed and the data must be analysed, for a given spin combination, in terms of their quadrupole/dipole mixing ratios,  $x$  and  $y$ . We assume that octupole radiation does not occur.

All the calculations were performed with programs written for the Utrecht University ZEBRA electronic computer.

In a first step the experimental data, corrected for background, eccentricity and chance coincidences, are least squares analysed in terms of the Legendre polynomials  $P_2(\cos \theta)$  and  $P_4(\cos \theta)$  where  $\theta$  is the angle between the moving counter and the proton beam. This analysis yields for each geometry the coefficients  $a_2$  and  $a_4$  in the expansion  $W(\theta) = A(1 + a_2P_2 + a_4P_4)$ , their errors and the correlation coefficient. In cases where theoretically the coefficient  $a_4$  should vanish, an expansion is used of the form  $W(\theta) = A(1 + aP_2)$ .

In a second step the quantity chi-squared is calculated; it is defined as:

$$\chi^2 = \frac{1}{M - R} \sum_{ij} (a_i - a_i^*)(a_j - a_j^*) X_{ij}^{-1}$$

where

- $M-R$  is the normalization factor equal to the number of free parameters,
- $M$  is the number of measured  $a_i$  coefficients,
- $R$  is the number of radiation mixing parameters used in the computation ( $R$  may be two, one or zero),
- $X_{ij}^{-1}$  is the inverted error matrix,
- $a_i$  are the experimental angular correlation coefficients,
- $a_i^*$  are the corresponding theoretical coefficients found with the tables given by Smith<sup>12</sup>) with the relevant solid angle attenuation factors taken into account.

The calculation of chi-squared for all possible spin sequences, varying  $\arctg x$  and  $\arctg y$  from  $-90^\circ$  to  $+90^\circ$  in steps of  $10^\circ$ , yields all possible solutions.

Finally, using the results just obtained, a program written by P. H. Vuister finds, for the best spin combination, the minimum of chi-squared, and consequently the corresponding values of the mixing parameters which best fit the experimental data.

In all chi-squared plots, a horizontal line is drawn at  $\chi^2 = c$ , such that the probability for  $\chi^2 > c$  is smaller than 0.1%<sup>13</sup>). Solutions with a  $\chi^2$  value above this "0.1% limit" are rejected.

IV. *The 3.13 MeV level.* Decay scheme. At the 1289 keV resonance (resonance spin =  $\frac{1}{2}$ , resonance strength  $(2J + 1)\gamma = 0.23$  eV, where  $\gamma = \Gamma_p\Gamma_\gamma/\Gamma^6$ ) this level is strongly excited. The  $(r) \rightarrow (3)$  intensity is 71%.

It is found to decay only to the ground state. From a coincidence spectrum measured with both crystals located 8 cm from the target spot, an upper limit of 1% could be estimated for possible transitions to the first or second excited states. This value must be compared with a 5% limit adopted by Broude *et al.*<sup>3)</sup> and by Harris and Seagondollar<sup>8)</sup>.

**Triple angular correlation measurements.** Triple angular correlation measurements on the  $(r) \rightarrow (3) \rightarrow (0)$  cascade were performed in geometries I, II, V and VI at the 498 keV resonance (resonance strength  $(2J + 1) \gamma = 0.18$  eV,  $(r) \rightarrow (3) : 13\%$ <sup>5)</sup>). The resonance level has  $J = 3/2$  ( $J = 1/2$  and  $5/2$  are excluded by the  $(r) \rightarrow (0)$  angular distribution:

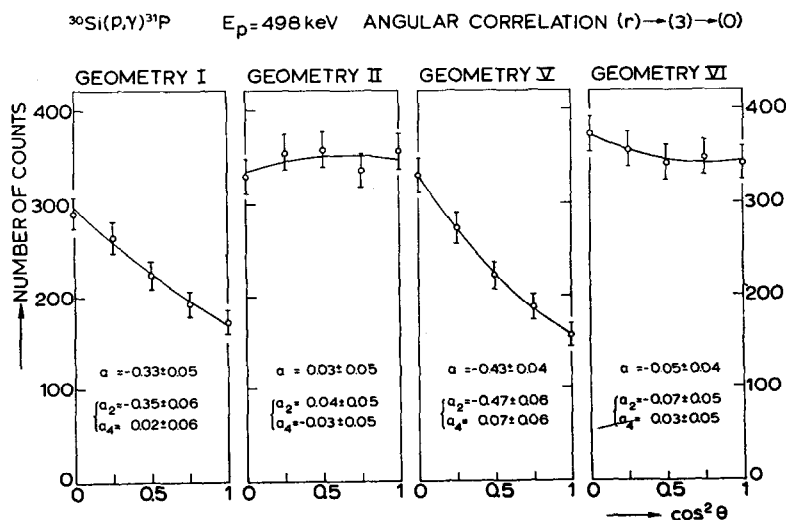


Fig. 1. Measured angular correlations of the  $(r) \rightarrow (3) \rightarrow (0)$  cascade in geometries I, II, V and VI at the 498 keV resonance. The quantities  $a_2$  and  $a_4$  are the coefficients of the Legendre polynomials in the expansion  $W(\theta) = A(1 + a_2 P_2(\cos \theta) + a_4 P_4(\cos \theta))$  which best fit the experimental data. The solid lines are the best fits in each case. The quantity  $a$  is the coefficient of the Legendre polynomial in an expansion of the form  $W(\theta) = A(1 + a P_2(\cos \theta))$ .

$a_2 = -0.39 \pm 0.01$ ,  $a_4 = -0.01 \pm 0.01$ ). The data are displayed in fig. 1. These data must be analysed for the spin sequence  $3/2 \rightarrow J(3) \rightarrow 1/2$  with  $J(3)$  as a parameter. The  $\chi^2$  analysis shown in fig. 2 easily excludes the highest possible value  $J(3) = 5/2$  (in this case,  $(3) \rightarrow (0)$  is assumed to be a pure quadrupole transition). For  $J(3) = 1/2$ , the  $\chi^2$  plot given in fig. 2 yields two solutions with different values of  $x$  (for this particular spin combination,  $(3) \rightarrow (0)$  is a pure dipole transition). The  $\chi^2$  contour plot presented in fig. 3 for the remaining possibility  $J(3) = 3/2$  gives four solutions with different values of  $x$  and  $y$ . The mixing parameters for the  $(r) \rightarrow (3)$  and  $(3) \rightarrow (0)$  transitions are given in table I for all possible solutions.

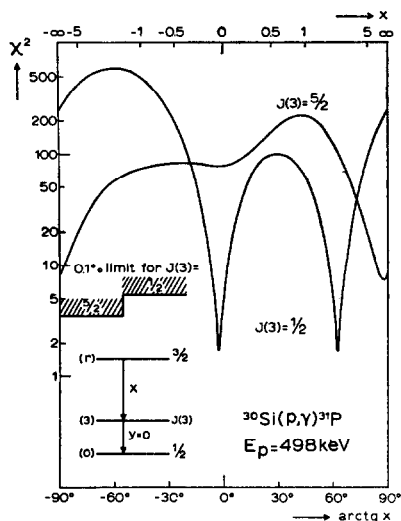


Fig. 2

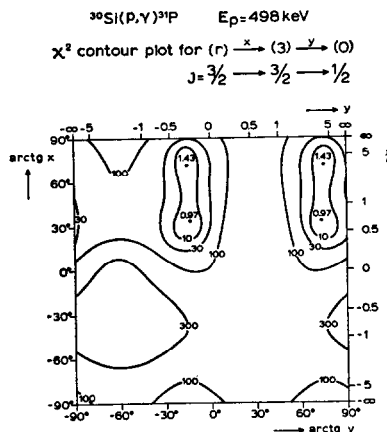


Fig. 3

Fig. 2. Chi-squared of the fit to the angular correlation measurements of fig. 1 plotted as a function of the mixing parameter  $x$  of the first gamma ray, with  $J(3)$  as a parameter. For  $J(3) = 1/2$  the  $(3) \rightarrow (0)$  transition is a pure dipole transition, for  $J(3) = 5/2$  it is assumed to be a pure quadrupole transition. The meaning of the 0.1% limit is discussed in the text.

Fig. 3. Resonance 498 keV. Chi-squared contour plot of the fit to the angular correlation measurements on the  $(r) \rightarrow (3) \rightarrow (0)$  cascade in geometries I, II, V and VI for  $J(3) = 3/2$ . The  $(x, y)$  combinations for the four solutions are given in table I.

TABLE I

Mixing parameters of the $(r) \rightarrow (3)$ and $(3) \rightarrow (0)$ transitions as measured at the 498 keV resonance. The spin $J(3)$ of the 3.13 MeV level is assumed to be $1/2$ or $3/2$		
$J(3)$	Mixing parameter of first gamma ray	Mixing parameter of second gamma ray
$1/2$	$x_1 = -0.05 \pm 0.02$ $x_2 = 1.94 \pm 0.09$	pure dipole
$3/2$	$x_1 = 0.66 \pm 0.05$ $x_1 = 0.66 \pm 0.05$ $x_2 = 3.1 \pm 0.4$ $x_2 = 3.1 \pm 0.4$	$y_1 = -0.24 \pm 0.02$ $y_2 = 3.3 \pm 0.3$ $y_3 = -0.27 \pm 0.02$ $y_4 = 3.7 \pm 0.3$

In fact, a second  $J^\pi = 1/2^+$  state in  $^{31}\text{P}$  at relatively low excitation energy (the ground state also has  $J^\pi = 1/2^+$ ), predicted by the Nilsson scheme<sup>3)</sup>, by the vibrational model<sup>14)</sup>, and by a recent shell model calculation<sup>15)</sup>, has not yet been identified.

For some time it was thought that  $J = 1/2$  should be assigned to the 3.13 MeV level. Angular correlation measurements by Hoogenboom<sup>2)</sup> on the  $^{30}\text{Si}(p, \gamma)^{31}\text{P}$  reaction gave  $J = 1/2$  as the most probable value,

without excluding, however,  $J = 3/2$ . The  $3/2$  assignment to this level resulted from the observed (small) anisotropy of the 3.13 MeV gamma-ray angular distribution at the 983 keV resonance in the same reaction<sup>3)</sup>. Recent angular correlation measurements by Harris<sup>9)</sup> gave no new information. As well as in the measurements described above, the data are in perfect agreement with  $J(3) = 1/2$  but never exclude  $J(3) = 3/2$ , in which case rather large mixing ratios are always needed for the  $(r) \rightarrow (3)$  and  $(3) \rightarrow (0)$  transitions.

Actually, one can prove by closer inspection of the theoretical expressions for the angular correlation, that, if the spin of level (3) is  $1/2$ , the spin determination by angular correlation measurements alone can never be unique. Exclusion of a  $J = 3/2$  assignment is impossible. Mixing ratios can always be found for the  $(r) \rightarrow (3)$  and  $(3) \rightarrow (0)$  transitions, such that the theoretical expressions fit all the experimental data\*).

This difficulty can be circumvented by direction-polarization correlation measurements.

**Direction-polarization correlation measurements.** *A. Experiment.* The measurement was carried out at the 498 keV  $^{30}\text{Si}(p, \gamma)^{31}\text{P}$   $J = 3/2$  resonance. The Compton scattering polarimeter was almost identical to that used by Suffert *et al.*<sup>16)</sup> in this laboratory. It incorporates a NaI scattering crystal (5 cm long and 5 cm diameter) below the target (with the proton beam coming in horizontally), and two NaI counters (10 cm long and 10 cm diameter) to detect the scattered radiation. The electronics used with the polarimeter was arranged according to a suggestion by Hazewindus<sup>17)</sup>. The pulses from the scattering crystal and either of the two detecting crystals are added in a summing circuit and stored in 100 channels of the 400-channel pulse-height analyser. The analyser gate only transmits pulses for storage if *a)* the pulses from the two crystals are in coincidence (resolution time  $\tau = 1 \mu\text{s}$ ), *b)* the pulse from the detecting crystal has an amplitude between  $E_\gamma = 0.6$  and 0.9 MeV. With a routing system the pulses of the two detecting crystals A and B, at "table" angles of  $0^\circ$  and  $90^\circ$ , respectively, are stored simultaneously, each in 100 channels (the angle between the proton beam direction and a detecting crystal is called "table" angle). These measurements are alternated with measurements where the counting system has been rotated over  $90^\circ$ , such that counter A is at  $-90^\circ$  and counter B at  $0^\circ$ . After the measurement the spectra at  $90^\circ$  and  $-90^\circ$ , and the two spectra at  $0^\circ$  are added separately. A conventional

\*) There are two exceptions to this rule. The  $J = 3/2$  possibility can be excluded if the  $a_2$  and  $a_4$  coefficients in a Legendre polynomial development of the  $(r) \rightarrow (3)$  angular distribution fulfill one of the following requirements: *a)*  $a_2$  smaller than  $-0.60$  (if the resonance spin is  $3/2$ ); *b)*  $a_4$  is measurably different from zero (if the resonance spin is  $5/2$ ). Such unusual coefficients have not yet been encountered in the  $^{30}\text{Si}(p, \gamma)^{31}\text{P}$  reaction.

gain stabilizer was used for the scattering counter. The stabilizer was adjusted on the photopeak of the 662 keV gamma ray from a  $^{137}\text{Cs}$  source positioned just above the target. A correction for eccentricity, only 2.2%, was determined by performing an analogous measurements at the  $E_p = 620$  keV,  $J = 1/2$  resonance, emitting isotropic and unpolarized radiation.

*B. Results.* The resulting spectra from a 60 hour run, together with the decay scheme of the 498 keV resonance, are given in fig. 4. In a preceding

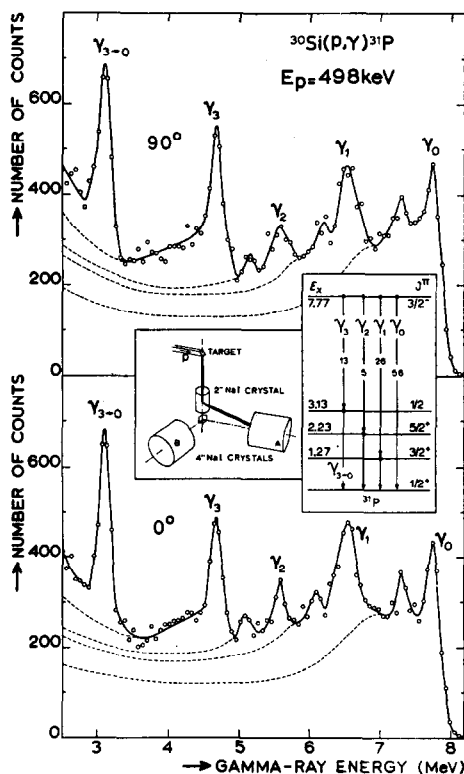


Fig. 4. Sum-pulse spectra (see text) at "table" angles of  $0^\circ$  and  $90^\circ$ , at the  $E_p = 498$  keV  $^{30}\text{Si}(p, \gamma)^{31}\text{P}$  resonance. The 4.64 MeV peak is more intense at  $90^\circ$  than at  $0^\circ$ , which points to negative polarization of  $\gamma_3$ . The  $\gamma_0$  transition has also negative polarization, while  $(3) \rightarrow (0)$  is unpolarized. The inserts show the polarimeter used in this experiment, and the decay scheme of the resonance level.

paper<sup>4</sup>) an analysis has been given of these polarization data. As pointed out by Dr. G. I. Harris, however, all polarizations in this paper were given with the wrong sign. The following analysis uses the correct signs.

*a)* The  $J^\pi = 3/2^-$  assignment to level (3) can first be excluded using the life-time of this level as measured by Booth and Wright<sup>18</sup>). Odd parity would entail an  $M2$  component in the  $(3) \rightarrow (0)$  transition of at least 40

Weisskopf units, even for the small value of the  $M2-E1$  mixing (see table I).

b) One can then prove that the resonance level has  $J^\pi = 3/2^-$ . The  $(\gamma) \rightarrow (1)$  and  $(1) \rightarrow (0)$  angular distribution measurements have been carried out by Hoogenboom<sup>2)</sup> and by Broude *e.a.*<sup>3)</sup>. The measured  $(\gamma) \rightarrow (1)$  angular distribution (average value  $a = 0.38 \pm 0.02$ ) leads to two values of the mixing ratio ( $x_1 = 0.02 \pm 0.01$  and  $x_2 = -4.1 \pm 0.3$ ) but only the small value  $x_1$  fits the  $(1) \rightarrow (0)$  angular distribution data ( $a = -0.17 \pm 0.02$ ). In the calculations use was made of the value  $y = 0.28$  for the  $E2-M1$  mixing ratio of the  $(1) \rightarrow (0)$  transition <sup>6)9)</sup>. The predicted  $(\gamma) \rightarrow (1)$  polarization is then  $P = 0.75 \pm 0.01$  if we assume odd parity for the resonance level. The predicted polarization has the opposite sign for even parity. The measured polarization is  $P = 0.71 \pm 0.32$ .

An analogous analysis of the 5%  $(\gamma) \rightarrow (2) \rightarrow (0)$  cascade (angular distribution  $(\gamma) \rightarrow (2)$ :  $a = 0.13 \pm 0.05$ ; angular distribution  $(2) \rightarrow (0)$ :  $a = 0.42 \pm 0.08$ <sup>2)</sup>) leads to the same conclusion, but the errors are larger. Here also the small value of the mixing ratio for the  $(\gamma) \rightarrow (2)$  transition has to be used. The predicted  $(\gamma) \rightarrow (2)$  polarization for  $3/2^- \rightarrow 5/2^+$  is  $P = -0.30 \pm 0.05$ , and  $P = 0.30 \pm 0.05$  for  $3/2^+ \rightarrow 5/2^+$ . The measured polarization is  $P = -0.42 \pm 0.67$ .

c) Finally a comparison between predicted and measured polarizations of the  $(\gamma) \rightarrow (3)$  transition for the three remaining  $J^\pi(\gamma) \rightarrow J^\pi(3)$  spin-parity combinations and for all mixing parameter possibilities is presented in

TABLE II

Comparison between predicted and measured polarizations of the $(\gamma) \rightarrow (3)$ transition			
$J^\pi(\gamma) \rightarrow J^\pi(3)$	Possible $x$ -values for $(\gamma) \rightarrow (3)$	Predicted polarization	Measured polarization
$3/2^- \rightarrow 1/2^-$	$-0.05 \pm 0.02$	$0.66 \pm 0.03$	$-0.63 \pm 0.27$
	$1.94 \pm 0.09$	$-0.65 \pm 0.03$	
$3/2^- \rightarrow 1/2^+$	$-0.05 \pm 0.02$	$-0.66 \pm 0.03$	
	$1.94 \pm 0.09$	$0.65 \pm 0.03$	
$3/2^- \rightarrow 3/2^+$	$0.66 \pm 0.05$	$0.64 \pm 0.02$	
	$3.1 \pm 0.4$	$0.23 \pm 0.03$	

table II. The data are consistent only with  $J(3) = 1/2$  and  $x = -0.05 \pm \pm 0.02$  for  $M2-E1$  mixing or  $x = 1.94 \pm 0.09$  for  $E2-M1$  mixing. It is still impossible with this experiment to determine the parity of level (3). Recent measurements on the  $^{30}\text{Si}(d, n)^{31}\text{P}$  reaction by Cujec *e.a.*<sup>11)</sup>, yielding  $l_p = 0$  for the neutron group to level (3), determine the parity as even.

C) Remarks. The  $(3) \rightarrow (0)$  transition is found to be unpolarized within the experimental error ( $P = 0.15 \pm 0.20$ ), corresponding to expectation if  $J(3) = 1/2$ . It is a pure dipole transition and one obtains



$|M|^2(M1) = 9 \times 10^{-2}$  using the life-time of level (3) as measured by Booth and Wright<sup>18</sup>) (the radiative widths are given in Weisskopf units<sup>21</sup>)).

The combination of the  $(r) \rightarrow (0)$  angular distribution ( $a = -0.39 \pm 0.01$ ) and its polarization ( $P = -0.56 \pm 0.26$ ) yields the mixing ratio  $x = -0.045 \pm 0.005$ . This transition certainly is an  $M2$ - $E1$  mixture. The  $M2$  strength is  $|M|^2(M2) = 1.2 \times 10^{-2}$  and the  $E1$  strength is  $|M|^2(E1) = 8 \times 10^{-5}$ .

The  $(r) \rightarrow (3)$  transition also may have  $M2$ - $E1$  mixing character. A pure  $E1$  transition would yield  $|M|^2(E1) = 9 \times 10^{-5}$ . If the  $M2$  component ( $x = -0.05 \pm 0.02$ ) is taken seriously it would lead to a strength  $|M|^2(M2) = 4.6 \times 10^{-2}$ .

The  $|M|^2$  values calculated here do not strongly deviate from the average transition strengths given by Van der Leun<sup>20</sup>).

V. *The 3.29 MeV level.* This level is strongly (35%) excited at the 1176 keV resonance. The resonance level has  $J = 7/2$  and does not decay to the other levels in the 3.0–3.5 MeV region nor to the first excited state. The resonance strength is  $(2J + 1) \gamma = 0.29 \text{ eV}^6$ )<sup>8</sup>).

Decay scheme. A coincidence spectrum was taken with both crystals at 10 cm from the target. The spectrum of the detector fixed at  $90^\circ$  with respect to the beam direction was measured in coincidence with the pulses from the second detector rotating in four steps from  $0^\circ$  to  $90^\circ$ . A narrow channel centred on the photo-peak of the  $(r) \rightarrow (4)$  transition in the spectrum of the moving detector provided the gating pulses. The result is presented in fig. 5.

It is seen that the 3.29 MeV level decays to the first and second excited

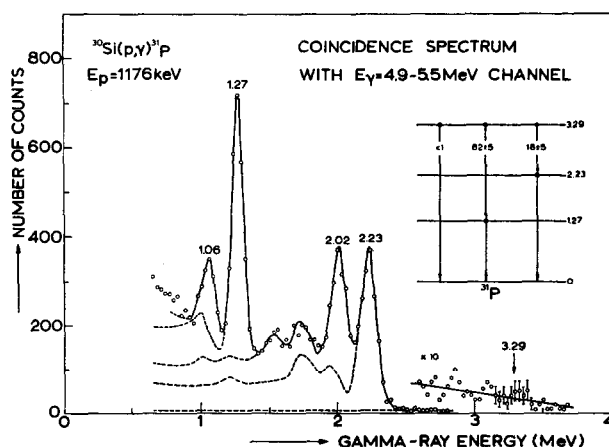


Fig. 5. Resonance  $E_p = 1176 \text{ keV}$ . Spectrum in coincidence with the transition to the 3.29 MeV level and final result for the decay of this level.

states (resulting in gamma rays of 2.02 and 1.06 MeV) and that the ground-state transition accounts for less than 1% of the decay. By combining the results obtained from this coincidence spectrum with those obtained from single spectra, the following decay modes could be established:  $(18 \pm 5) \%$  for the  $(4) \rightarrow (2)$  transition and  $(82 \pm 5) \%$  for the  $(4) \rightarrow (1)$  transition.

A previous experiment by Harris and Seagondollar<sup>8)</sup> indicated the presence of a ground-state transition. Recently Harris<sup>19)</sup> has repeated his measurements; the existence of a ground-state transition was not confirmed. For the branching ratio mentioned above, he reports the value 25 : 75 in good agreement with our results.

Spin and parity. The triple angular correlations of the transitions  $(\gamma) \rightarrow (4) \rightarrow (1)$  and  $(\gamma) \rightarrow (4) \rightarrow (2)$  were obtained simultaneously by measuring spectra coincident with the primary radiation. For each geometry, the spectra were taken at four angles ( $\theta = 0^\circ, 35^\circ, 55^\circ$  and  $90^\circ$ ) and each spectrum was stored in 100 channels of the 400-channel analyser. This experiment was carried out in geometries I, II, V and VI. The four spectra for geometry II are shown in fig. 6.

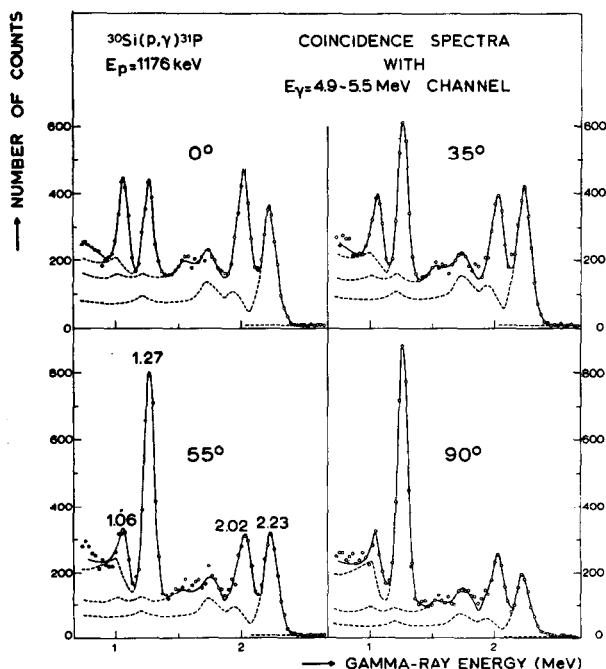


Fig. 6. Resonance  $E_p = 1176$  keV. Spectra in coincidence with the  $(\gamma) \rightarrow (4)$  transition detected at  $90^\circ$  with respect to the proton beam direction. The second counter was successively located at  $0^\circ, 35^\circ, 55^\circ$  and  $90^\circ$  with respect to the beam. The analysis of these spectra yields the triple angular correlations of both transitions  $(\gamma) \rightarrow (4) \rightarrow (1)$  and  $(\gamma) \rightarrow (4) \rightarrow (2)$  in geometry II.

TABLE III

Resonance $E_p = 1176$ keV. Legendre polynomial coefficients of the measured angular correlations of the $(\gamma) \rightarrow (4) \rightarrow (1)$ and $(\gamma) \rightarrow (4) \rightarrow (2)$ cascades, and values of the mixing parameters $x$ , $y_1$ and $y_2$ of the three gamma rays $(\gamma) \rightarrow (4)$ , $(4) \rightarrow (1)$ and $(4) \rightarrow (2)$ , respectively, for the solution $J(4) = 5/2$		
Decay studied	$(\gamma) \rightarrow (4) \rightarrow (1)$	$(\gamma) \rightarrow (4) \rightarrow (2)$
Geometry I	$a_2 = -0.32 \pm 0.03$	$a_2 = -0.04 \pm 0.13$
	$a_4 = 0.03 \pm 0.04$	$a_4 = -0.08 \pm 0.13$
	$a = -0.31 \pm 0.03$	$a = -0.08 \pm 0.10$
Geometry II	$a_2 = 0.36 \pm 0.04$	$a_2 = 0.49 \pm 0.09$
	$a_4 = 0.09 \pm 0.04$	$a_4 = 0.14 \pm 0.10$
	$a = 0.38 \pm 0.06$	$a = 0.53 \pm 0.10$
Geometry V	$a_2 = -0.37 \pm 0.05$	$a_2 = -0.62 \pm 0.09$
	$a_4 = 0.04 \pm 0.06$	$a_4 = 0.17 \pm 0.10$
	$a = -0.35 \pm 0.04$	$a = -0.52 \pm 0.09$
Geometry VI	$a_2 = 0.25 \pm 0.03$	$a_2 = 0.28 \pm 0.07$
	$a_4 = -0.06 \pm 0.03$	$a_4 = 0.06 \pm 0.07$
	$a = 0.23 \pm 0.04$	$a = 0.30 \pm 0.07$
Results for $J(4) = 5/2$		
Mixing parameter of first gamma ray	$x = 0.01 \pm 0.01$	$x = 0.04 \pm 0.03$
Mixing parameter of second gamma ray	$y_1 = -0.37 \pm 0.02$	$y_2 = -0.05 \pm 0.06$

The sixteen spectra were "peeled off" and the intensities of the radiation in the photo-peaks of the 1.06 and 2.02 MeV gamma rays as a function of the angle  $\theta$  were analysed into Legendre polynomials. The results are given in table III. The measurements on the  $(\gamma) \rightarrow (4) \rightarrow (1)$  cascade were analysed with  $J(4)$  as a parameter. For  $J(4) = 3/2$  or  $7/2$ , one of the gamma transitions can be assumed to have pure quadrupole character. The resulting chi-squared plot is presented in fig. 7. Both these spins can be excluded. For  $J(4) = 5/2$ , both gamma rays can be mixed. The chi-squared contour plot shown in fig. 8 yields very low  $\chi^2$  values. The  $5/2$  spin assignment to the 3.29 MeV level is in agreement with various other experiments<sup>2) 3) 9)</sup>. The minimalization of  $\chi^2$  gives the mixing ratios  $x$  and  $y_1$  for the primary radiation and the 2.02 MeV transition, respectively (see table III). As a comparison, the  $\chi^2$  plot for  $J(4) = 5/2$  is also given in fig. 7 assuming  $x = 0.01$ , the best value of this parameter. The measurements on the  $(\gamma) \rightarrow (4) \rightarrow (2)$  cascade were then computed for the spin sequence  $7/2 \rightarrow 5/2 \rightarrow 5/2$ . The mixing ratios  $x$  and  $y_2$  for the primary radiation and the 1.06 MeV transition are given in table III. They were obtained by minimalization of  $\chi^2$ . Both computed  $x$  values are small and equal within the experimental error. The  $y_1$  average value observed by Harris and Seagondollar<sup>9)</sup> is  $-0.47 \pm 0.03$ . The agreement with the value found in the present work,  $y_1 = -0.37 \pm 0.02$ ,

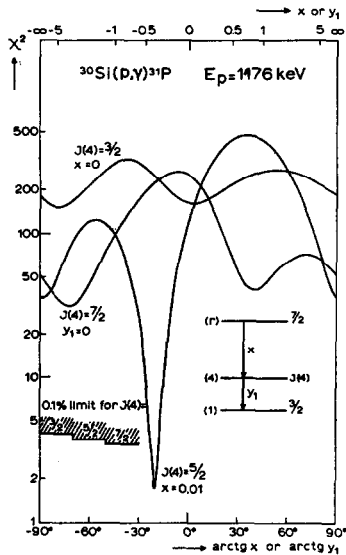


Fig. 7

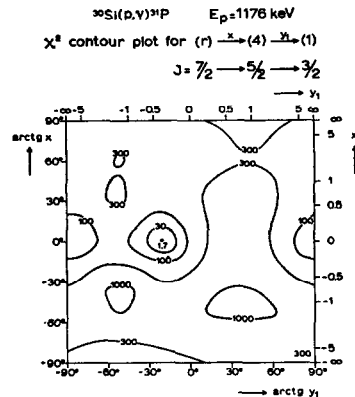


Fig. 8

Fig. 7. Resonance 1176 keV. Chi-squared of the fit to the angular correlation measurements on the  $(r) \rightarrow (4) \rightarrow (1)$  cascade in geometries I, II, V and VI, with  $J(4)$  as a parameter. For  $J(4) = 3/2$  and  $7/2$ , one of the gamma rays is assumed to have pure quadrupole character. The resulting chi-squared is plotted as a function of the quadrupole/dipole mixing ratio of the second one. For  $J(4) = 5/2$ , chi-squared is plotted as a function of  $y_1$  for  $x = 0.01$ , as obtained in fig. 8. The meaning of the 0.1% limit is discussed in the text.

Fig. 8. Resonance 1176 keV. Chi-squared contour plot of the fit to the angular correlation measurements on the  $(r) \rightarrow (4) \rightarrow (1)$  cascade in geometries I, II, V and VI for  $J(4) = 5/2$ . The minimum value of chi-squared (1.7) is reached for  $x = 0.01 \pm 0.01$  and  $y_1 = -0.37 \pm 2$ .

is not very good. In more recent work, Harris<sup>19)</sup> gives two possible  $y_2$  values,  $-0.39$  and  $-1.17$ , without further discussion. Both values are in disagreement with the unique value found in the present work,  $y_2 = -0.05 \pm 0.06$ .

A survey of  $M2$  strengths evaluated for all available results could not completely exclude the odd parity assignment to level (4). The even parity is strongly favoured by theoretical predictions<sup>3) 14) 15)</sup> and by the presence of a large quadrupole/dipole mixture ( $y_1 = -0.37 \pm 0.02$ ) in the  $(4) \rightarrow (1)$  transition. The 3.29 MeV level in  $^{31}\text{P}$  has thus  $J^\pi = 5/2^{(+)}$ , where the brackets indicate that the parity determination is probable but not quite certain.

VI. *The 3.41 MeV level.* Decay scheme. The spectrum measured in coincidence with the  $(r) \rightarrow (5)$  transition at the 1510 keV resonance is shown in fig. 9 (the distance between the target and both crystals was 10 cm).

As earlier obtained at the 1348 keV resonance<sup>6)</sup>, the 3.41 MeV level decays only to the 1.27 MeV level. The  $(\gamma) \rightarrow (2)$  decay mode is weaker than 2%. The ground-state intensity is below 1%. This is in agreement with the results reported in a recent publication of Harris<sup>19)</sup>, which supersedes an earlier report<sup>9)</sup> giving a 10% ground-state transition.

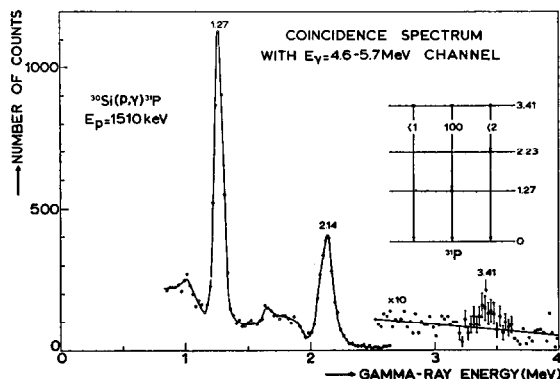


Fig. 9. Gamma spectrum at the 1510 keV resonance, in coincidence with the 5.34 MeV  $(\gamma) \rightarrow (5)$  transition. The weak peak at about 3.41 MeV is due to sum pulses. The decay modes of the 3.41 MeV level in  $^{31}\text{P}$  are also shown.

Spin and parity. The  $7/2$  spin assignment to the 3.41 MeV level has recently been established by triple angular correlation measurements at the 1348 keV resonance<sup>6)</sup> and at the 1510 keV resonance<sup>9)</sup>.

The 1510 keV resonance level has  $J^\pi = 5/2^+$  (odd parity is very improbable since it would entail an  $M2$  component in the  $(\gamma) \rightarrow (1)$  transition of 3 Weisskopf units<sup>6) 9)</sup>). The resonance strength is  $(2J+1)\gamma = 2.5\text{ eV}$ . Measurements by Harris and Seagondollar<sup>9)</sup> on the 29%  $(\gamma) \rightarrow (5) \rightarrow (1)$  cascade yield the quadrupole-dipole mixing ratio of the  $(\gamma) \rightarrow (5)$  transition:  $\alpha = 0.13 \pm 0.02$ . Odd parity assignment to level (5) would yield an  $M2$  width of 3.1 Weisskopf units for the  $(\gamma) \rightarrow (5)$  transition. The parity of the 3.41 MeV level in  $^{31}\text{P}$  is thus very probably even (the  $(5) \rightarrow (1)$  transition is then a pure  $E2$  transition). This  $7/2^+$  assignment is in agreement with several theoretical predictions<sup>3) 14) 15)</sup>.

Transition probabilities. If  $J^\pi = 5/2^+$  and  $7/2^+$  are assigned to the resonance level and the level (5), respectively, four  $|M|^2$  values can be calculated.

For the  $(\gamma) \rightarrow (1)$  transition the  $E2$  strength is  $|M|^2(E2) = 9.2 \times 10^{-2}$  Weisskopf units and the  $M1$  strength is  $|M|^2(M1) = 3 \times 10^{-2}$ . For the  $(\gamma) \rightarrow (5)$  transition the  $E2$  strength is  $|M|^2(E2) = 9.5 \times 10^{-2}$  and the  $M1$  strength is  $|M|^2(M1) = 3.8 \times 10^{-2}$ .

VII. *The 3.51 MeV level.* Spin and parity. At the strong 1770 keV resonance ( $J^\pi = 5/2^+$ ,  $(2J + 1)\gamma = 2.0 \text{ eV}^7$ ), the sixth excited state in  $^{31}\text{P}$  at 3.31 MeV is populated through a 21%  $(r) \rightarrow (6)$  transition. This level decays to the ground state, and to either (1) or (2). Angular correlation

TABLE IV

Legendre polynomial coefficients of the measured angular correlations of the $(r) \rightarrow (6) \rightarrow (0)$ cascade at the 1770 keV resonance, and values of the mixing parameters $x$ and $y$ of the $(r) \rightarrow (6)$ and $(6) \rightarrow (0)$ gamma rays, respectively, as obtained for the solution $J(6) = 3/2$			
Geometry	$a_2$	$a_4$	$a$
I	$-0.05 \pm 0.09$	$0.03 \pm 0.09$	$-0.03 \pm 0.07$
II	$0.25 \pm 0.06$	$-0.05 \pm 0.07$	$0.24 \pm 0.06$
V	$-0.12 \pm 0.07$	$0.18 \pm 0.08$	$-0.05 \pm 0.09$
VI	$0.23 \pm 0.06$	$0.02 \pm 0.07$	$0.23 \pm 0.06$
Results for $J(6) = 3/2$			
Mixing parameter $x$ of first gamma ray		$-0.15 \pm 0.02$	
Mixing parameter $y$ of second gamma ray		$-0.43 \pm 0.03$ or $8.2 \pm 1.8$	

measurements on the  $(r) \rightarrow (6) \rightarrow (0)$  cascade were performed in the four standard geometries. The results are summarized in table IV. These data were analysed for the spin combination  $5/2 \rightarrow J(6) \rightarrow 1/2$  with  $J(6)$  as a discrete parameter.

For  $J(6) = 5/2$ , the  $(6) \rightarrow (0)$  transition is assumed to have pure quadrupole character; the resulting  $\chi^2$  plot is given in fig. 10. For  $J(6) = 1/2$ , the first gamma ray of the cascade is assumed to be a pure quadrupole transition and the second one is a pure dipole transition; the value of  $\chi^2$  for this case is also given in fig. 10. It is seen that both the spins  $5/2$  and  $1/2$  can be excluded. For  $J(6) = 3/2$ , both gamma rays can be mixed quadrupole/dipole transitions. The corresponding  $\chi^2$  contour plot shown in fig. 11 yields two solutions, with  $x = -0.15 \pm 0.02$  in both cases and with  $y = -0.43 \pm 0.03$  or  $8.2 \pm 1.8$  (see table IV); here  $x$  and  $y$  are the mixing parameters of the  $(r) \rightarrow (6)$  and  $(6) \rightarrow (0)$  transitions, respectively. The  $3/2$  spin assignment to level (6) and the value of the mixing parameter  $y$  are in agreement with the results of several other authors<sup>2) 3) 9)</sup>.

The 3.51 MeV level probably has even parity since odd parity would entail an  $M2$  component in the  $(r) \rightarrow (6)$  transition of 2.1 Weisskopf units. If even parity is assumed, the  $E2$  strength is  $|M|^2(E2) = 6.4 \times 10^{-2}$  and the  $M1$  strength is  $|M|^2(M1) = 2.0 \times 10^{-2}$ .

Decay scheme. The insert of fig. 12 shows the three possible decay modes of the 3.51 MeV level, as obtained from coincidence spectra measured

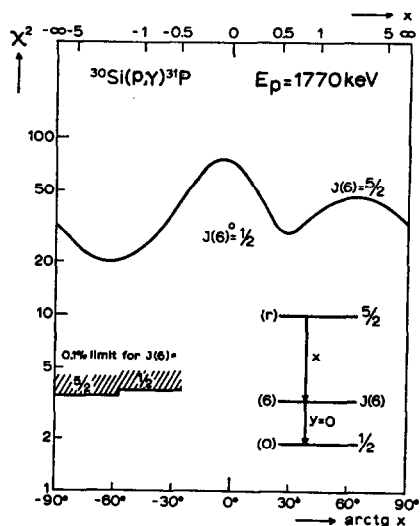


Fig. 10

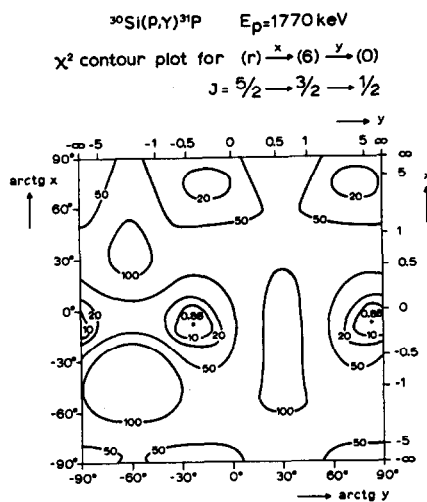


Fig. 11

Fig. 10. Resonance 1770 keV. Chi-squared of the fit to the angular correlation measurements on the  $(r) \rightarrow (6) \rightarrow (0)$  cascade in geometries I, II, V and VI for  $J(6) = 1/2$  and  $5/2$ . These two spin assignments to level (6) can be excluded since the resulting values of  $\chi^2$  are larger than the 0.1% limit.

Fig. 11. Resonance 1770 keV. Chi-squared contour plot of the fit to the angular correlation measurements on the  $(r) \rightarrow (6) \rightarrow (0)$  cascade in geometries I, II, V and VI for  $J(6) = 3/2$ . The values of the mixing parameters  $x$  and  $y$  for the two solutions ( $\chi^2 = 0.88$ ) are given in table IV.

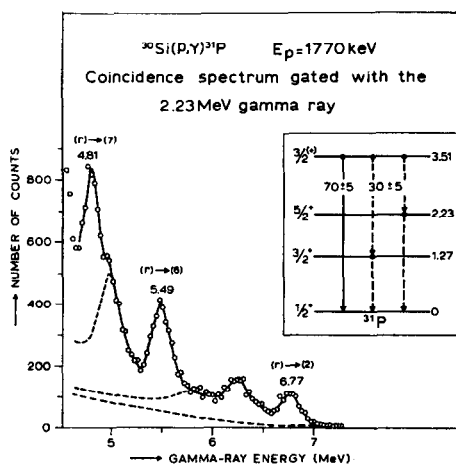


Fig. 12. Resonance  $E_p = 1770$  keV. Spectrum in coincidence with the 2.23 MeV gamma ray (both crystals were at 10 cm from the target spot). As explained in the text, this spectrum is the sum of the eight spectra used for geometries II and VI in the  $(r) \rightarrow (6) \rightarrow (1)$  and  $(r) \rightarrow (6) \rightarrow (2) \rightarrow (0)$  angular correlation measurements. The insert shows the different decay modes of level (6) at 3.51 MeV.

at resonances rather strongly decaying through this level. From the analysis of single spectra, the following decay modes could be established:  $(70 \pm 5)\%$  for the  $(6) \rightarrow (0)$  transition and  $(30 \pm 5)\%$  for the sum of the  $(6) \rightarrow (2) \rightarrow (0)$  and  $(6) \rightarrow (1) \rightarrow (0)$  cascades. From intensity measurements only, it is impossible to find the relative intensities of the latter two cascades, since the energies of the gamma rays involved in these cascades are almost identical (2.23 and 2.24 MeV, 1.27 and 1.28 MeV).

It was tried to determine the branching ratio by means of angular correlation measurements. At the 1770 keV resonance, spectra were measured in coincidence with the 2.23 MeV gamma ray, at sixteen angle combinations. In each spectrum, the intensity of the radiation in the photo-peak of the 5.49 MeV  $(r) \rightarrow (6)$  transition was determined. The complete analysis of the data in the four standard geometries yielded the sum of the following angular correlations:  $(r) \rightarrow (6) \rightarrow (1)$  and  $(r) \rightarrow (6) \rightarrow (2) \rightarrow (0)$  where the  $(6) \rightarrow (2)$  transition is not observed (the coincidence spectrum shown in fig. 12 was obtained by adding the eight spectra measured in geometries II and VI).

In the first step of the analysis, it was assumed that the 3.51 MeV level

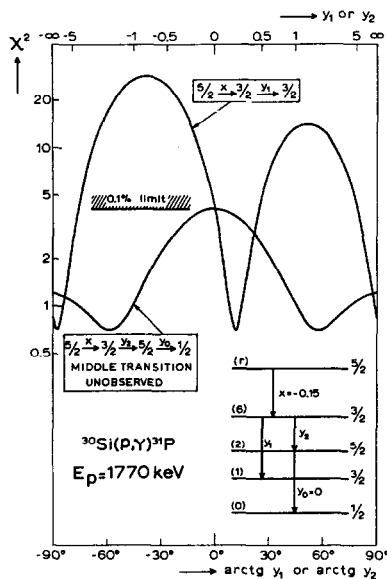


Fig. 13. Resonance  $E_p = 1770$  keV. Chi-squared of the fit to the angular correlation measurements on the  $(r) \rightarrow (6) \rightarrow (1)$  and  $(r) \rightarrow (6) \rightarrow (2) \rightarrow (0)$  cascades in geometries I, II, V and VI. The curve labelled  $5/2 \rightarrow 3/2 \rightarrow 3/2$  represents  $\chi^2$  values plotted as a function of the mixing parameter  $\gamma_1$  of the  $(6) \rightarrow (1)$  transition. The curve labelled  $5/2 \rightarrow 3/2 \rightarrow 5/2 \rightarrow 1/2$  represents  $\chi^2$  values plotted as a function of the mixing parameter  $\gamma_2$  of the  $(6) \rightarrow (2)$  transition. In both cases, the value  $x = -0.15$  was used for the mixing parameter of the  $(r) \rightarrow (6)$  transition. The  $(2) \rightarrow (0)$  transition was assumed to have pure quadrupole character. The meaning of the 0.1% limit is discussed in the text.



does not decay through the 2.23 MeV level. The data then pertain to the  $(r) \rightarrow (6) \rightarrow (1)$  cascade. They were analysed for the spin sequence  $5/2 \rightarrow \rightarrow 3/2 \rightarrow 3/2$ , using the earlier obtained value  $x = -0.15 \pm 0.02$  for the mixing parameter of the  $(r) \rightarrow (6)$  gamma ray. Chi-squared was calculated as a function of the mixing parameter  $x_1$  of the  $(6) \rightarrow (1)$  gamma ray; the  $\chi^2$  plot shown in fig. 13 yields two solutions with  $y_1 = 0.21 \pm 0.06$  or  $y_1 \approx -23$ . In the second step, it was assumed that the  $(6) \rightarrow (1)$  transition does not exist. With the  $x$  value given above, the data were then analysed with the spin sequence  $5/2 \rightarrow 3/2 \rightarrow 5/2 \rightarrow 1/2$  for the  $(r) \rightarrow (6) \rightarrow (2) \rightarrow (0)$  cascade. In this case, the second gamma ray (mixing parameter:  $y_2$ ) is not observed and the third one is assumed to have pure quadrupole character (mixing parameter:  $y_0 = 0$ ). In fig. 13, the resulting values of  $\chi^2$  are plotted as a function of  $y_2$ . Two broad minima appear for  $|y_2| \approx 1.6$ . Therefore one has to conclude that these measurements do not give any additional information on the branching ratio discussed here. From an analogous measurement<sup>19)</sup>, Harris concludes that the  $(6) \rightarrow (2)$  transition is weak relative to the  $(6) \rightarrow (1)$  decay.

VIII. *Conclusions.* From the measurements discussed here, the spins of the  $^{31}\text{P}$  levels at 3.13, 3.29, 3.41 and 3.51 MeV were uniquely established as  $J = 1/2, 5/2, 7/2$  and  $3/2$ , respectively. These four levels very probably have even parity; it would be interesting to check these assignments with polarization experiments or with measurements on the  $^{30}\text{Si}(d, n)^{31}\text{P}$  reaction. The decay scheme of each level and the values of the mixing parameters of the gamma rays were also determined; for the 3.51 MeV level, however, there are still two possibilities for the mixing parameter of the ground-state transition. The branching ratio  $\{(6) \rightarrow (1)\}/\{(6) \rightarrow (2)\}$  could not be determined.

The measurement of a small  $M2$  component in the  $(r) \rightarrow (0)$  transition at the 498 keV resonance, together with similar observations at other resonances in the reaction discussed here<sup>7)</sup> and also in the reaction  $^{26}\text{Mg}(p, \gamma)^{27}\text{Al}^{22})$ , is of considerable interest. It sheds doubt on many previous parity assignments that were based on the assumption that any observed quadrupole/dipole mixing in an odd  $A$  nucleus indicates  $E2/M1$  mixing. Therefore many of the older parity assignments have to be reconsidered. In future work the utmost care ought to be taken in the assignment of parities on the basis of observed quadrupole/dipole mixing.

In fig. 14, the first seven experimental levels in  $^{31}\text{P}$  are compared with the levels up to 4.15 MeV predicted by shell model calculations recently performed by Glaudemans *et al.*<sup>15)</sup>. It is seen that the experimental results and the theoretical predictions are in agreement. The absence of the 2.23 MeV level from the theoretical results and the shift of the 3.41 MeV level could be explained in terms of excitation of the  $^{28}\text{Si}$  core (see reference 15).

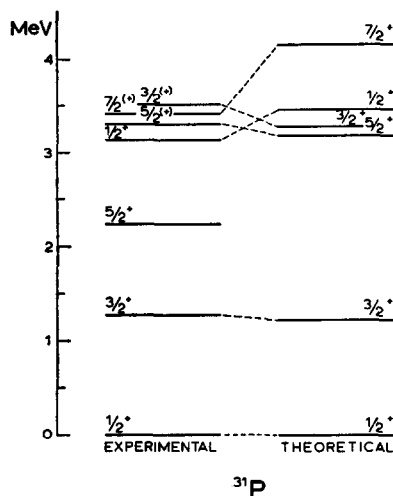


Fig. 14. Comparison between experimental and theoretical  $^{15}\text{P}$  levels in  $^{31}\text{P}$  below 4.15 MeV. The brackets indicate that the even parity is not yet quite certain.

Using the shell model wave functions obtained by Glaudemans *et al.*<sup>15)</sup>, Wiechers and Brussaard<sup>23)</sup> have calculated the life-time of the 3.13 MeV level, assuming pure  $M1$  decay to the ground state. Their result is in good agreement with the experimental value<sup>18)</sup>.

**Acknowledgements.** One of the authors (V.R) would like to thank Professors P. M. Endt and M. Demeur (University of Brussels) who have made his stay in Utrecht possible, and the "Institut Interuniversitaire des Sciences Nucléaires de Belgique" for the necessary financial support.

We have to thank many of our colleagues for active help during the measurements, and Dr. C. van der Leun and F. C. Ern  for stimulating discussions in all stages of this work.

This investigation was partly supported by the joint research program of the "Stichting voor Fundamenteel Onderzoek der Materie" and the "Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek".

Received 9-6-65

#### REFERENCES

- 1) Endt, P. M. and Paris, C. H., Phys. Rev. **106** (1957) 764.
- 2) Hoogenboom, A. M., Thesis Utrecht (1958).
- 3) Broude, C., Green, L. L. and Willmott, J. C., Proc. Phys. Soc. **A 72** (1958) 1097, 1115 and 1122.
- 4) Van Rinsvelt, H. A. and Endt, P. M., Physics Letters **9** (1964) 266; erratum, **12** (1964) 360.
- 5) Endt, P. M. and Van der Leun, C., Nucl. Phys. **34** (1962) 1.
- 6) Van Rinsvelt, H. A. and Smith, P. B., Physica **30** (1964) 59.
- 7) Van Rinsvelt, H. A. and Endt, P. M., preceding paper.

- 8) Harris, G. I. and Seagondollar, L. W., Phys. Rev. **128** (1962) 338.
- 9) Harris, G. I. and Seagondollar, L. W., Phys. Rev. **131** (1963) 787.
- 10) Kossanyi-Demay, P., Lombard, R. M. and Bishop, G. R., Nucl. Phys. **62** (1965) 615.
- 11) Cujec, B., Davies, W. C., Dawson, W. K., Grandy, T. B., Neilson, G. C. and Ramavataaram, K., Physics Letters **15** (1965) 266.
- 12) Smith, P. B., Chapter V in "Nuclear Reactions II", edited by Endt, P. M. and Smith, P. B., North-Holland Publ. Co., Amsterdam (1962).
- 13) Wapstra, A. H., Nijgh, G. J. and Van Lieshout, R., "Nuclear Spectroscopy Tables", North-Holland Publ. Co., Amsterdam (1959).
- 14) Thankappan, V. K. and Pandya, S. P., Nucl. Phys. **19** (1960) 303.
- 15) Glaudemans, P. W. M., Wiechers, G. and Brussaard, P. J., Nucl. Phys. **56** (1964) 529 and 548.
- 16) Suffert, M., Endt, P. M. and Hoogenboom, A. M., Physica **25** (1959) 659.
- 17) Hazewindus, N., Thesis Delft (1964).
- 18) Booth, E. C. and Wright, K. A., Nucl. Phys. **35** (1962) 472.
- 19) Harris, G. I., Proceedings of the Lawrence (Kansas) symposium on "The Structure of Low-Medium Mass Nuclei", (1964) 39.
- 20) Van der Leun, C., Proceedings of the Lawrence (Kansas) symposium on "The Structure of Low-Medium Mass Nuclei", (1964) 109.
- 21) Wilkinson, D. H., in "Nuclear Spectroscopy B", Academic Press, New York (1960) 852.
- 22) Van der Leun, C., private communication.
- 23) Wiechers, G. and Brussaard, P. J., Nucl. Phys. **73** (1965) 604.