EFFECTS OF LEVEL DEGENERACY AND SPONTANEOUS EMISSION ON DEFLECTION OF AN ATOMIC BEAM BY A STANDING LIGHT WAVE

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We present a general formalism for describing the diffraction of an atomic beam by a nearly resonant standing light wave, containing the correlation between the acquired transverse momentum, the number of emitted fluorescent photons and the internal polarization state of the atoms. The formalism is applied in some special cases to yield explicit results for the momentum distribution. At the same time it is shown to give a unified description of various previous results for two-state atoms.

1. Introduction

The deflection of an atomic beam in a resonant standing laser wave has recently received much attention. Apart from potential applications, the interest in the effect stems mainly from the fact that that it is an explicit demonstration of transfer of the quantized photon momentum on a macroscopic scale. Equivalently, the deflection of atoms by a standing wave may be viewed as a diffraction of matter by a periodic intensity distribution of radiation, which serves as a grating. Several recent experiments have demonstrated the effect [1–6]. Theoretical treatments so far have mainly considered atom models with two nondegenerate levels [7–14]. Furthermore, the effect of spontaneous emission is usually ignored, which basically restricts the interaction time T to values small compared with the spontaneous lifetime. This leads to an essential simplification, since it allows the evolution of the atomic state to be described by Schrödinger's equation for the atomic wave function, whereas inclusion of spontaneous emission requires a density matrix description, which doubles the dimension of the evolution matrix. Finally explicit calculations for the deflection profile are mainly carried out either for the case of exact resonance or in the limit of large detuning.

In this paper we consider beam deflection by a standing wave, while accounting for a possible level degeneracy and for sponaneous decay. Our starting point is a general evolution equation for the Fourier-transformed momentum distribution, and by expanding the solution in powers of the operator for spontaneous decay. This formalism is particularly advantageous when the number of spontaneous emission is limited. We derive explicit results for the deflection profile in several special cases.

2. Evolution of the density matrix

We consider at atom in a beam, moving with uniform momentum $\boldsymbol{p}_0$ in the z-direction. The ground level and the excited level have angular momenta J_g and J_e. The atomic beam crosses a standing wave of radiation with wave vector $\pm\boldsymbol{k}$ along the x-axis, and polarization $\boldsymbol{\epsilon}$. During the interaction the atom will acquire momentum in the x-direction, since it will pick up photon momentum at spontaneous and stimulated transitions. We assume that the gain in momentum of the atom in the x-direction is sufficiently small as to make the Doppler shift $\boldsymbol{k}\cdot\boldsymbol{p}/m$ negligible compared with the natural width A. Furthermore we assume that the motion of the atom

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in the x-direction during the interaction time is small compared with the wavelength of the radiation. These assumptions imply that in the evolution of the atomic density matrix the commutator with the kinetic-energy operator can be ignored [13], and only the evolution due to the internal energy states of the atom, and its coupling to the radiation field remains.

In order to study the momentum distribution of the atom, it is essential to evaluate the off-diagonal elements of the density matrix σ with respect to the position $\boldsymbol{r}$ of the atom. Hence it is convenient to study the evolution equation for the quantity

$$\sigma(\boldsymbol{r}, \boldsymbol{s}) = \langle \boldsymbol{r} + \tfrac{1}{2}\boldsymbol{s} | \sigma | \boldsymbol{r} - \tfrac{1}{2}\boldsymbol{s} \rangle , \tag{1}$$

which is still a matrix with respect to the internal states of the atom. The Fourier-transform of $\sigma(\boldsymbol{r}, \boldsymbol{s})$ with respect to $\boldsymbol{s}$ gives the Wigner distribution function [15]. The light beam is described by a classical standing wave and the coupling of the atomic transition to the field is proportional to the component of the reduced dipole operator $\boldsymbol{D}$ in the polarization direction. This dipole operator is defined by requiring the matrix elements of its spherical components

$$D_1 = -(D_x + \mathrm{i}D_y)/\sqrt{2} , \qquad D_{-1} = (D_x - \mathrm{i}D_y)/\sqrt{2} ,$$

$$D_0 = D_z \tag{2}$$

to be given by Clebsch-Gordan coefficients according to the relation

$$\langle J_e M_e | D_\sigma | J_g M_g \rangle = \langle J_e M_e | J_g M_g ; 1\sigma \rangle . \tag{3}$$

The coupling strength is measured by the effective Rabi frequency

$$\Omega = E_0 \langle e \| \mu \| g \rangle / \hbar (2J_e + 1)^{1/2} , \tag{4}$$

in terms of the reduced dipole matrix element, with E_0 the electric field amplitude at the antinodes of the standing wave. The component $\boldsymbol{\epsilon}\cdot\boldsymbol{D}$ of the operator $\boldsymbol{D}$ in the polarization direction is denoted by D. With these notational conventions, the evolution equations for the submatrices of $\sigma(\boldsymbol{r}, \boldsymbol{s})$ [13,14,16] are

$$(\partial/\partial t)\sigma_{ee} = -A\sigma_{ee} + \tfrac{1}{2}\mathrm{i}\Omega(c_+ D\sigma_{ge} - \sigma_{eg} D^+ c_-) ,$$

$$(\partial/\partial t)\sigma_{gg} = S\sigma_{ee} + \tfrac{1}{2}\mathrm{i}\Omega(c_+ D^+ \sigma_{eg} - \sigma_{ge} D c_-) ,$$

$$(\partial/\partial t)\sigma_{eg} = -[\tfrac{1}{2}A - \mathrm{i}(\omega - \omega_0)]\sigma_{eg}$$

$$+ \tfrac{1}{2}\mathrm{i}\Omega(c_+ D\sigma_{gg} - \sigma_{ee} D c_-) ,$$

$$(\partial/\partial t)\sigma_{ge} = -[\tfrac{1}{2}A + \mathrm{i}(\omega - \omega_0)]\sigma_{ge}$$

$$+ \tfrac{1}{2}\mathrm{i}\Omega(c_+ D^+ \sigma_{ee} - \sigma_{gg} D^+ c_-) . \tag{5}$$

Here A is the spontaneous-decay rate, ω is the light frequency, and ω_0 is the atomic transition frequency. The factors $c_\pm$ are defined by

$$c_\pm = \cos[\boldsymbol{k}\cdot(\boldsymbol{r} \pm \tfrac{1}{2}\boldsymbol{s})] . \tag{6}$$

The operator S describes the gain in the atomic ground state due to spontaneous emission, and the corresponding change in atomic momentum. It is defined by the equality [16]

$$S(\boldsymbol{s})\,\sigma = A \sum_{ij} M_{ij}(\boldsymbol{s})\, D_i \sigma_{ee} D_j , \tag{7}$$

with M_{ij} the elements of a cartesian matrix ($i, j = x, y, z$), with matrix element

$$M_{ij}(\boldsymbol{s}) = p_t(\zeta)\delta_{ij} + [p_l(\zeta) - p_t(\zeta)] s_i s_j / \boldsymbol{s}^2 . \tag{8}$$

The functions p_t and p_l are transverse and longitudinal retardation factors, given by [16]

$$p_l(\zeta) = -3(\cos\zeta/\zeta^2 - \sin\zeta/\zeta^3) ,$$

$$p_t(\zeta) = \tfrac{3}{2}(\sin\zeta/\zeta + \cos\zeta/\zeta^2 - \sin\zeta/\zeta^3) , \tag{9}$$

with $\zeta = \omega_0 s/c$. Note that we suppressed the arguments $\boldsymbol{r}$ and $\boldsymbol{s}$ in eq. (5). Furthermore the quantity $\sigma_{ee}(\boldsymbol{r}, \boldsymbol{s}, t)$ is still a matrix within the $(2J_e+1)$-dimensional manifold of substates of the excited level, and a similar statement holds for the other submatrices of σ.

The evolution equations (5) can be formally separated in the form

$$(\partial/\partial t)\sigma(\boldsymbol{r}, \boldsymbol{s}, t) = L(\boldsymbol{r}, \boldsymbol{s})\,\sigma(\boldsymbol{r}, \boldsymbol{s}, t)$$

$$= [L_0(\boldsymbol{r}, \boldsymbol{s}) + S(\boldsymbol{s})]\,\sigma(\boldsymbol{r}, \boldsymbol{s}, t) , \tag{10}$$

where the evolution operator L_0 is defined by the terms in eq. (5) other than S. Hence the operator L_0 describes the evolution of the density matrix for a fixed number of fluorescence photons. The operator L is simply a matrix for each value of $\boldsymbol{r}$ and $\boldsymbol{s}$, and it contains no derivatives with respect to $\boldsymbol{r}$ and $\boldsymbol{s}$. This indicates that the density matrix σ evolves independently for each value of $\boldsymbol{r}$ and $\boldsymbol{s}$. This feature is a con-

sequence of our assumption that free-flight effects and Doppler shifts are negligible during the interaction time.

The momentum distribution $Z(\boldsymbol{p}, t)$ of the atom at time t, regardless its position $\boldsymbol{r}$ and its internal state, is found from $\sigma(\boldsymbol{r}, \boldsymbol{s}, t)$, after taking the Fourier transform with respect to $\boldsymbol{s}$, integrating over $\boldsymbol{r}$, and taking the trace over the internal states. Hence we write

$$Z(\boldsymbol{p}, t) = (2\pi\hbar)^{-3} \int \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{s} \times \exp(-\mathrm{i}\boldsymbol{s}\cdot\boldsymbol{p}/\hbar)\, \mathrm{Tr}\, \sigma(\boldsymbol{r}, \boldsymbol{s}, t)\,. \tag{11}$$

At time zero, the atom is supposed to have the initial momentum $\boldsymbol{p}$ in the z-direction, accordingly the initial density matrix is

$$\sigma(\boldsymbol{r}, \boldsymbol{s}, 0) = \sigma_0(\boldsymbol{r}, \boldsymbol{s}) \equiv \sigma_g W(\boldsymbol{r}) \exp(\mathrm{i}\boldsymbol{s}\cdot\boldsymbol{p}_0/\hbar)\,, \tag{12}$$

with W a normalized distribution over the position $\boldsymbol{r}$, and σ_g specifying the normalized initial distribution over the ground-state sublevels.

3. Expansion of the momentum distribution

When the number of spontaneous emissions during the interaction time T remains limited, it is useful to expand $\sigma(\boldsymbol{r}, \boldsymbol{s}, T)$ in powers of the decay operator S. As we did in a previous paper for the case of atoms in a travelling wave [16], we introduce the combined probabilities $P_N(\boldsymbol{r}, \boldsymbol{s}, T)$, which are the contributions to $\sigma(\boldsymbol{r}, \boldsymbol{s}, T)$ pertaining to the case of exactly N spontaneous emissions during the interaction time T. Hence the Fourier transform of P_N gives the probability distribution for the event in which the atom has spontaneously emitted N photons in the interaction interval, and ends up in a specified internal state and momentum state at time T. The formal expression for P_N becomes very simple for the Laplace transform

$$\hat{P}_N(\boldsymbol{r}, \boldsymbol{s}, v) = \int_0^\infty \mathrm{d}T \exp(-vT)\, P_N(\boldsymbol{r}, \boldsymbol{s}, T)\,, \tag{13}$$

and we find [16]

$$\hat{P}_N(v) = [(v - L_0)^{-1} S]^N\, (v - L_0)^{-1}\, \sigma_0\,, \tag{14}$$

where we suppressed the dependence on $\boldsymbol{r}$ and $\boldsymbol{s}$ for notational convenience. This result (14) is simply the Nth order term in the expansion of the Laplace transform of $\sigma(t)$ in powers of S. The momentum distribution (11) after the interaction time can be expressed as

$$Z(\boldsymbol{p}, T) = \sum_{N=0}^{\infty} Z_N(\boldsymbol{p}, T)\,, \tag{15}$$

where

$$Z_N(\boldsymbol{p}, T) = (2\pi\hbar)^{-3} \int \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{s} \times \exp(-\mathrm{i}\boldsymbol{s}\cdot\boldsymbol{p}/\hbar)\, \mathrm{Tr}\, P_N(\boldsymbol{r}, \boldsymbol{s}, T)\,, \tag{16}$$

is the joint probability distribution that after the interaction time T the atom has emitted precisely N photons, and ends up in the momentum $\boldsymbol{p}$.

We are only interested in the final distribution of atomic momentum in the x-direction, which is the propagation direction of the standing wave. Then it is sufficient to take the vector $\boldsymbol{s}$ in the x-direction. Furthermore, the initial distribution W over position must be very wide, since the initial momentum $\boldsymbol{p}_0$ is well-determined. Since the operator $L_0(x, s)$ with x and s the x-component of $\boldsymbol{r}$ and $\boldsymbol{s}$ is periodic in these variables, it is sufficient to replace the $\boldsymbol{r}$-integration in (11) and (16) by an integration over x extending over one wavelength $\lambda = 2\pi/k$ of the radiation. The result for the Laplace transform of the distribution (16), integrated over p_y and p_z takes the form

$$\hat{Z}_N(p, v) = (2\pi\hbar)^{-1} \int_{-\infty}^{\infty} \mathrm{d}s \exp(-\mathrm{i}sp/\hbar) \times \frac{1}{\lambda} \int_0^\lambda \mathrm{d}x\, \mathrm{Tr}\, \hat{P}_N(x, s, v)\,, \tag{17}$$

with

$$\hat{P}_N(x, s, v) = \{[v - L_0(x, s)]^{-1} S(s)\}^N \times [v - L_0(x, s)]^{-1} \sigma_g\,. \tag{18}$$

The Laplace transform of the full distribution $Z(p,$

T) of x-momentum is again obtained by a summation of (17) over the photon number N.

4. Special cases

The general result (17) and (18) for the joint distribution over momentum and photon number is valid for arbitrary values of the angular momenta J_e and J_g, and for arbitrary polarization ϵ. In order to arrive at explicit results, we now specialize these results in several explicit cases.

4.1. $J_g=0$, $J_e=1$

We first consider the case that $J_g=0$ and $J_e=1$. For a single polarization vector ϵ, then only the excited sublevel

$$|e\rangle_0=\epsilon\cdot\boldsymbol{D}|g\rangle \tag{19}$$

is coupled to the nondegenerate ground state. Hence the system is effectively a two-level atom. From the explicit expression for L_0 and S as specified in (5) we can evaluate the N-photon contribution to the momentum distribution in Fourier-Laplace transform, and we find

$$\begin{aligned}\mathrm{Tr}\,\hat{P}_N(x,s,v)&=[\tfrac{1}{2}\Omega^2c_+c_-(v+\tfrac{1}{2}A)\,Ap_t(ks)/3]^N\,Q^{-(N+1)}\\&\times\{[(v+\tfrac{1}{2}A)^2+\Delta^2](v+A)\\&+\tfrac{1}{4}\Omega^2(c_++c_-)^2(v+\tfrac{1}{2}A)-\tfrac{1}{4}\mathrm{i}\Delta\Omega^2(c_+^2-c_-^2)\},\end{aligned} \tag{20}$$

where

$$\begin{aligned}Q=(v+A)\{&[(v+\tfrac{1}{2}A)^2+\Delta^2]v\\&+\tfrac{1}{4}\Omega^2(c_+^2+c_-^2)\,(v+\tfrac{1}{2}A)\}+\tfrac{1}{4}\mathrm{i}A\Delta\Omega^2(c_+^2-c_-^2)\\&+\tfrac{1}{4}\Omega^2(c_+^2+c_-^2)\,v\,(v+\tfrac{1}{2}A)+\tfrac{1}{16}\Omega^4(c_+^2-c_-^2)^2,\end{aligned} \tag{21}$$

and

$$\Delta=\omega-\omega_0 \tag{22}$$

is the detuning of the light frequency from resonance. In (20) the term $p_t(ks)/3$ accounts for spontaneous emission. The other s-dependent factors are periodic in s, and represent stimulated transitions. The summation of (20) over N is simple, but the remaining Laplace–Fourier inversion is difficult, and we resort to special limiting cases.

When the interaction time T is so small that spontaneous emission is unlikely ($AT\ll 1$), the main contribution to Z comes from Z_0. By Laplace inversion of (20) for $N=0$ we find

$$\begin{aligned}\mathrm{Tr}\,P_0(r,s,T)&=(1/D_+D_-)\,\{\Omega^2c_+c_-\sin(\tfrac{1}{2}D_+T)\sin(\tfrac{1}{2}D_-T)\\&+[D_+\cos(\tfrac{1}{2}D_+T)+\mathrm{i}\Delta\sin(\tfrac{1}{2}D_+T)]\\&\times[D_-\cos(\tfrac{1}{2}D_-T)-\mathrm{i}\Delta(\sin\tfrac{1}{2}D_-T)]\},\end{aligned} \tag{23}$$

where

$$D_\pm=(\Delta^2+\Omega^2c_\pm^2)^{1/2}. \tag{24}$$

The first term in (23) corresponds to the probability that the atom is in the excited state $|e\rangle_0$ at time T. In the momentum distribution Z_0 this term yields a series of peaks proportional to $\delta(p-m\hbar k)$ with m odd. Likewise the remaining term in (23) gives peaks in Z_0 displaced by an even number of photon moments, corresponding to the probability that the atom is in the ground state at time T.

Analytical results valid in two limiting cases, and that are known in the literature are easily recovered from (23). For excitation on resonance we may substitute $\Delta=0$, and we find for the momentum distribution [7]

$$Z_0(p,T)=\sum_{m=-\infty}^{\infty}J_m^2(\Omega T/2)\,\delta(p-m\hbar k), \tag{25}$$

in terms of Bessel functions. The main contribution to Z_0 arises from the terms with $m\leqslant\Omega T/2$. For larger m-values, the Bessel functions become negligible.

Another situation in which the result becomes simple occurs when the detuning Δ is large compared with the Rabi frequency Ω. Then we obtain either from (23) or from (20) the momentum distribution [8]

$$Z_0(p,t)=\sum_{m=-\infty}^{\infty}J_m^2(\Omega^2T/8\Delta)\,\delta(p-2m\hbar k). \tag{26}$$

The probability that the atom is excited is small at all times, and only the even peaks remain.

4.2. $J_g=1$, $J_e=0$

In the case that the excited state is non-degenerate, and the ground state has $J_g=1$, only the state

$$|g\rangle_0=\sqrt{3}\boldsymbol{\epsilon}^*\cdot\boldsymbol{D}^+|e\rangle \tag{27}$$

couple to the field [16]. Spontaneous decay can occur to all three sublevels of the ground state, and after a few spontaneous emissions, the atom has decayed with certainty to a state in which it does no longer couple to the field. Hence, even in the limit of very long interaction times, the number of spontaneous emissions remains limited. The long-time limit of the momentum distribution is related to its Laplace transform by

$$\lim_{t\to\infty} Z_N(p,t)=\lim_{v\downarrow 0} v\hat{Z}_N(p,v)\,, \tag{28}$$

and a similar equation holds for $\hat{P}_N$ and P_N. By an analogous method as applied before for a travelling wave [16], we obtain the explicit result

$$\mathrm{Tr}\,P_0(x,s,T\to\infty)=0\,,$$

$$\begin{aligned}\mathrm{Tr}\,P_N(x,s,T\to\infty)&=\{(\tfrac{1}{3}c_+c_-)^N\,p_t(ks)^{N-1}\,[p_t(ks)+p_l(ks)]\}\\&\times[\tfrac{1}{2}(c_+^2+c_-^2)+\tfrac{1}{12}(\Omega^2/A^2)(c_+^2-c_-^2)^2\\&+(\mathrm{i}\Delta/A)(c_+^2-c_-^2)]^{-N}\,.\end{aligned} \tag{29}$$

The factors p_t and p_l in (29) arise from spontaneous emission, whereas the factors $c_\pm$ reflect stimulated transitions. The case of precisely N spontaneous emissions must correspond to $N-1$ spontaneous decays to the state $|g\rangle_0$, which allows absorptive excitation back to $|e\rangle$, and finally the Nth spontaneous decay into a ground state orthogonal to $|g\rangle_0$.

In order to evaluate $Z_N(p,\,t\to\infty)$ from (29) we still must take the Fourier transform with respect to s, and the average over x, as indicated in (17). In the case of resonant excitation ($\Delta=0$) and moderate intensity ($A\gg\Omega$), we find that the lowest order contribution in Ω/A to (29) is independent of Ω and A.

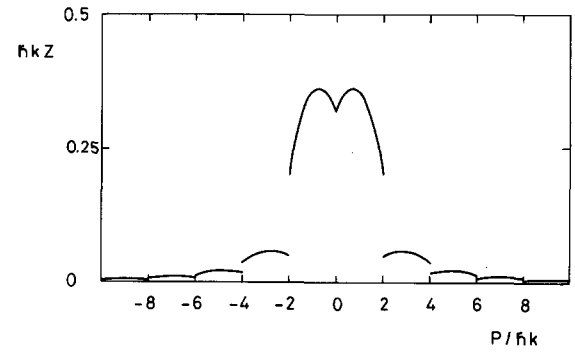

Fig. 1. Momentum distribution of atoms with $J_g=1$ and $J_e=0$, in the limit of long interaction time and low intensity.

Then an explicit evaluation of $Z_N(p)$ can be carried out analytically. Since the result is cumbersome, we do not reproduce it here. It is however easy to evaluate $Z(p)$, and we plot the result in fig. 1. It is obvious that the peaks in the momentum distribution are now broadened by the photon recoil at spontaneous emission. Surprisingly, even in this limit of low intensity, already the contribution from Z_1 contains peaks corresponding to more than one stimulated transition.

5. Conclusions

We have derived a general formalism giving the joint probability distribution of the momentum and the number of fluorescent photons for atoms crossing a standing light wave. The formal result is given in eq. (17), where P_N as given by eq. (18) also describes the internal state of the atom. These formal results hold for any value of the angular momenta J_e and J_g of the two levels of the driven transition, and for arbitrary polarization. For the case of $J_g=0$, $J_e=1$, the model constitutes effectively a two-state atom, and our formalism gives a unification and generalization of results which are known in the literature. For the case of $J_g=1$, $J_e=0$, we arrive at closed expressions for the Fourier-transformed distribution over momentum and photon numbers in the long-time limit, where all atoms have been pumped to substates of the ground level that are not affected by the field.

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