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HIGH RESOLUTION INVESTIGATION OF THE $^{30}Si(p, p)^{30}Si$ REACTION

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Synopsis

The differential cross section for elastic scattering of protons on ^{30}Si was measured with surface barrier counters at four angles. Thirty-six $^{30}Si(p, \gamma)^{31}P$ resonances are known in the $E_p = 1$–2 MeV region. Fifteen of these were also observed in the $^{30}Si(p, p)^{30}Si$ reaction, with natural widths varying from 10 keV down to 13 eV. For all fifteen resonances the proton orbital momenta, and thus the parities, could be determined unambiguously; the spins are known already from the (p, γ) work.

Partial widths for proton and γ-ray emission are computed from the widths and the (p, γ) yields. Upper limits are given for the widths of unobserved resonances.

1. *Introduction.* Recently, the $^{30}Si(p, \gamma)^{31}P$ reaction was studied extensively in this laboratory[1][2]. Altogether thirty-six (p, γ) resonances were observed in the $E_p = 1$–2 MeV region. From γ-ray angular distribution and $\gamma - \gamma$ angular correlation measurements the spins of all but the very weakest resonances could be uniquely determined. Generally, however, such work provides little or no information as to the resonance parity. The old supposition that γ transitions of mixed dipole-quadrupole character should proceed between states of equal parity, or, in other words, that $E1$–$M2$ mixtures are a rare exception, has been proven unfounded, not only in self-conjugated nuclei where dipole transitions between states of equal isotopic spin are suppressed, but also in odd-A nuclei. In the following it will be shown that the (p, p) reaction is an ideal tool for the determination of the missing resonance parities.

It is obvious that resonances, to be observable in elastic proton scattering, should have a relatively large proton width. In preceding $^{30}Si(p, p)^{30}Si$ work[3], three resonances were observed in the $E_p = 1$–2 MeV region, at $E_p = 1292$, 1514 and 1812 keV, with a total width given as $\Gamma = < 3$, 4 and 7 keV, which were broad enough for a meaningful analysis, resulting in $J^\pi = 1/2^+$, $1/2^+$ and $3/2^-$ assignments, respectively. Five other resonances (of which one in the present work was shown to be spurious) showed some interference structure, too weak to be analysed.

There are justified hopes, that one can do much better. The Utrecht

3 MeV HVEC Van de Graaff generator provides an energy resolution, using thin targets, down to, at least, 0.5 keV [2]). Moreover, the knowledge of the exact resonance energies, with errors between 0.6 and 1.2 keV, from the (p, γ) work [1]) [2]) mentioned above, makes it possible to concentrate the search for (p, p) resonances to narrow energy regions, which can be investigated with good statistics in small energy steps of e.g. 0.1 keV. The broad regions in between (p, γ) resonances were ignored altogether. The precision in the measurement of resonance energies is increased considerably by the simultaneous measurement of proton and γ-ray yields.

These improvements have led to the observation and to the determination of the parity and width of fifteen $^{30}Si(p, p)^{30}Si$ resonances in the $E_p = 1$–2 MeV region. The two narrowest of these resonances have a width of only 13 eV.

In itself, the parities and widths of resonance levels at high excitation energies do not seem to be very interesting. Combined, however, with the results of the (p, γ) work they are of importance in several respects. First, the detailed knowledge of the properties of the resonance levels is a prerequisite in the interpretation of $\gamma - \gamma$ angular correlation and polarization measurements, leading to J^{π} determinations of lower levels. Second, from the total width and the (p, γ) yield of a resonance, the partial widths for proton and γ-ray emission can be determined, which can be useful for statistical considerations on transition probabilities.

Experimental details are given in section 2, the theoretical and computational analysis is presented in section 3, and the final results and conclusions are to be found in sections 4 and 5.

2. *Experimental.* Protons were accelerated with a 3 MeV electrostatic generator and magnetically deflected over 90°. With 0.5 mm magnet entrance and exit slits and an orbit radius of 32 cm in the magnet, the energy distribution of the analysed proton beam, at $E_p = 1$ MeV, has a full width at half maximum of at most 160 eV [4]). The contributing factors are the energy spread at the ion source, and the high voltage fluctuations, the latter partly caused by fluctuations (1 part in 40 000) in the field of the deflecting magnet.

The target material was quartz enriched in ^{30}Si (^{28}Si 32%, ^{29}Si 3%, ^{30}Si 65%), obtained from Oak Ridge National Laboratory, U.S.A. Targets were prepared by evaporation in vacuo of the quartz from a tantalum boat at a temperature of about 1700°C onto thin carbon foils. The first foils were home-made, produced with the graphite evaporation technique [5]), but later foils, with a thickness of about 10 $\mu g/cm^2$, were obtained from the Yissum Research Dev. Cy., Israel.

The best targets proved to have a thickness of about 4 $\mu g/cm^2$, corresponding to an energy loss of 600 eV for 1.5 MeV protons. They stand a proton current of about 1 μA. A target of 600 eV is still rather thick in comparison with the beam energy spread of 160 eV. In principle, a thickness of, say, 200 eV would be preferable for the observation of extremely narrow resonances. Such thin targets, however, apparently contain a relatively rather large amount of tantalum, which made them less suitable (see also below).

The proton current was measured with a Faraday cup behind the target, equipped with a suppressor ring at negative potential to prevent secundary electron exchange between target and cup.

Elastically scattered protons were detected with four surface barrier counters at angles of 90.0°, 125.3°, 140.8° and 149.5° (c.m.), relative to the beam direction and at a distance of 5 cm from the target. Most of the work has been done with home-made[6]) counters of 1 mm^2 sensitive area and an energy resolution of about 40 keV for 5.5 MeV alpha particles. Some weak resonances were later remeasured with four ORTEC detectors, of 25 mm^2 sensitive area and 20 keV resolution.

Detector pulses were amplified up to a few volts with a charge sensitive tube preamplifier and a transistorized RC-shaping main amplifier. Each main amplifier leads to a single-channel pulse analyser which was set with the aid of a multi-channel analyser and a pulse generator.

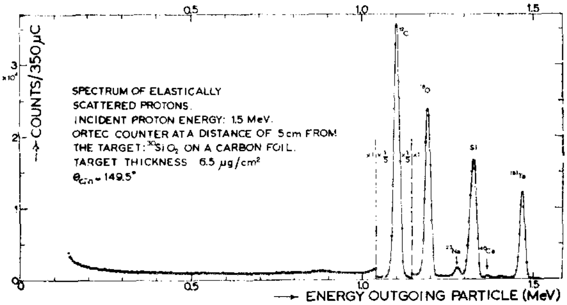

Fig. 1. Energy spectrum of elastically scattered protons. The peak marked Si contains contributions from the three isotopes ^{28}Si(32 %), ^{29}Si(3 %) and ^{30}Si(65 %).

In fig. 1 an energy spectrum is shown of elastically scattered protons at $E_p = 1.5$ MeV, $\theta = 149.5°$, as measured with one of the ORTEC counters. The strong groups result from scattering on ^{12}C, ^{16}O, Si and ^{181}Ta, and some weak groups might be due to various contaminants as Na, Cl, Ca and Fe or Cu. It should be noted that the spectrum of fig. 1 was taken under relatively favourable circumstances. The energy resolution of the home-made counters is worse than that of the counter used here, and at lower proton energies and at smaller angles the groups are much closer. The high amount of Ta on the target, resulting from the high temperature needed to evaporate the quartz, was something of an unpleasant surprise. The amount changes somewhat from target to target, but it proved impossible to get rid of it altogether.

Gamma rays were detected with a 5 cm × 5 cm NaI scintillation counter at a distance of 10 cm from the target. This setup is far from ideal, as compared with the equipment generally used in this laboratory for γ-ray detection (see e.g. refs. 1 and 2), such that weak (p, γ) resonances tend to get lost in the background.

The proton channels were set such that they just about covered the ^{30}Si peak in the pulse spectrum. Inevitably, the channel then also contains the ^{28}Si and ^{29}Si peaks, unresolved from the ^{30}Si peak, in addition to contributions from contaminant peaks and from the low-energy tail of the Ta peak. These non-^{30}Si contributions constitute the background. Before a run over a resonance region, say 10 keV wide, the channel was adjusted at the centre of this region. At the ends of the region a small fraction of the ^{30}Si peak might have run out of the channel, and also the background might be, to some extent, dependent on proton energy. In the analysis of the data, these effects were taken care of by treating the background, different for each of the four angles, as unknown, depending linearly, but with an unknown slope, on E_p. The background constants were then determined from a least-squares fitting program (see section 3).

3. *Analysis of the data.* The cross section for elastic scattering is given by Blatt and Biedenharn[7]). It can be described as a sum of three parts, a contribution from Rutherford scattering, a pure resonance term, and a term describing the interference between Rutherford and resonant scattering. A contribution from hard-sphere scattering has been computed but can be neglected for the present analysis.

For an even-even target nucleus, and for a single isolated resonance level, the proton orbital momentum l is uniquely determined in the resonance scattering process. The interference term is then proportional to the Legendre polynomial $P_l(\cos\vartheta)$. For $l = l_0 \neq 0$, the interference term thus vanishes at an angle, ϑ_{l_0}, where $P_{l_0}(\cos\vartheta)$ has a zero. By using counters at the four angles ϑ_1 through ϑ_4, one can determine l_0. The occurrence of interference manifests itself as a dip in the cross section. It drops below the Rutherford cross section, either below or above the resonance energy. This elegant way of determining l_0 and thus the resonance parity, is not always the most suitable one. The most common resonances are p-wave and d-wave, corresponding to $\vartheta_1 = 90.0°$ and $\vartheta_2 = 125.3°$, respectively. For weak resonances, the resonance structure at these relatively forward angles is often very little pronounced as compared to the much more beautiful structure observed at $\vartheta = 149.5°$. The reason is, of course, the $(\sin\vartheta/2)^{-2}$ dependence of the Rutherford scattering amplitude, such that the resonance over Rutherford amplitude ratio increases fast with angle. The conclusion is that for weak resonances the l_0 value is best determined from analysis of the data at the two most backward angles.

It has been suggested[8]) that the resonance scattering integral, evaluated at ϑ_{l_0}, should provide a simple way to determine the total width Γ of the resonance. The scattering integral is independent of the instrumental resolution. The same drawback as mentioned above for l_0 determinations, pertains to this method. Weak resonances are often hardly visible at 90° and 125°, and thus Γ should rather be evaluated from the data at the more backward angles.

For the analysis the instrumental resolution function has to be known. It is obtained from the experimental shapes of (p, γ) resonances which from the (p, p) work are known to have a small natural width, say $\Gamma < 30$ eV. It was found that the resolution function can be well approximated by a triangle, somewhat steeper on the low-energy than on the high-energy side. The full width at half maximum is almost equal to the target thickness, and thus it differs from target to target. It is almost independent of proton energy.

One also needs the phase difference between the resonance and Rutherford amplitudes, which is a function of l_0, E_p and ϑ_i, where $i = 1$ through 4 numbers the counter angles. It was calculated with an existing computer program[6]), using a nuclear radius $R = r_0(A^{\frac{1}{3}} + 1)$ with $r_0 = 1.45$ fm.

For all (p, p) resonances to be analysed, the spin J is already known from the (p, γ) work[1]) [2]). The analysis was only performed for the two l values compatible with J. For instance, for $J = 5/2$, one only needs to investigate the $l = 2$ and 3 possibilities.

The analysis now proceeds as follows.

1. The sum of the background and Rutherford contributions is subtracted from the measured points. This sum is obtained by fitting a straight line through the wings of the resonance, where resonance effects have virtually disappeared.

2. One then divides by the Rutherford contribution $CE_p^{-2}(\sin \vartheta_i/2)^{-4}$, where, for the moment, C is an arbitrary parameter.

3. The result is compared to the theoretical expression which is a function of E_p and ϑ_i, and contains as unknown parameters the resonance energy E_0, the orbital momentum l, and the total width Γ (or rather Γ_p^2/Γ, but because Γ_γ is much smaller than Γ_p one can approximate $\Gamma_p \approx \Gamma$). This expression is folded into the known resolution function.

4. At each angle ϑ_i, one computes a χ^2 value, χ_i^2, which is minimized by variation of the parameters E_0, Γ, C, a_i and b_i, where the constants a_i and b_i determine the background, $B_i(E_p) = a_i E_p + b_i$.

5. One then takes the results at all four angles together and constructs an overall χ^2 value, which is minimized by variation of E_0, Γ and C. The constants a_i and b_i are now known from the preceding fitting process.

6. Finally, the program computes the error in Γ. The minimum values of χ^2, obtained for the two possible l values, are compared to the 0.1% probability limit. In all cases but one (see below), this gave a unique determination of l_0.

The program proved to be too lengthy for the X1 computer of the Utrecht University. Instead, the calculations were carried out, by telex communication, with the Telefunken TR4 computer in Delft.

Against the analysis as outlined above, one might object that, for narrow resonances, the width Γ seems rather poorly determined. By replacing C, the multiplicative factor in front of the Rutherford contribution, and Γ by, say, $2C$ and $\Gamma/2$, the height of a resonance peak would seem to remain almost unchanged, which would seriously endanger the convergence of the fitting procedure. This is true to some extent at the angle ϑ_{l_0}, where no interference dip appears. But luckily, at the more backward angles, the peak to dip ratio appears to depend sensitively on Γ, such that the fitting process, taken from all angles together, converges very nicely.

A further proof for the correctness of the method of analysis is the fact that the background over Rutherford ratios obtained were quite reasonable. They varied from 0.55 (the minimum value, with only ^{28}Si and ^{29}Si contributing to the background), to 2.0. The highest values result from poor energy resolution (home-made counter, $\vartheta = 90°$, low E_p).

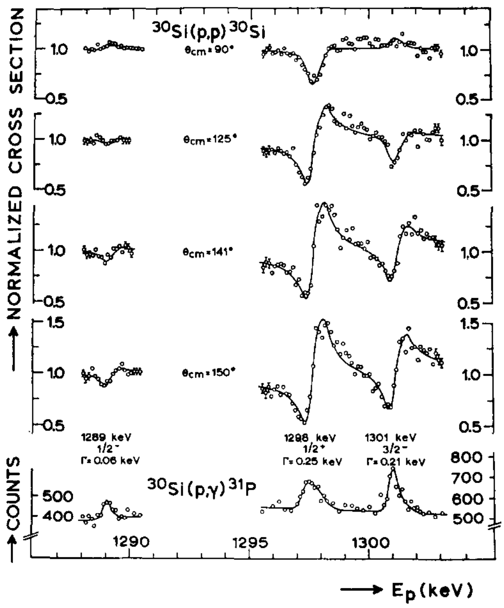

Fig. 2. Differential cross section at four angles for elastic proton scattering on ^{30}Si in units of the Rutherford contribution, with background subtracted, at the $E_p =$ = 1289, 1298 and 1301 keV resonances. The smooth curve is the theoretical curve for the indicated J^π and Γ. The bottom curve gives the γ-ray yield.

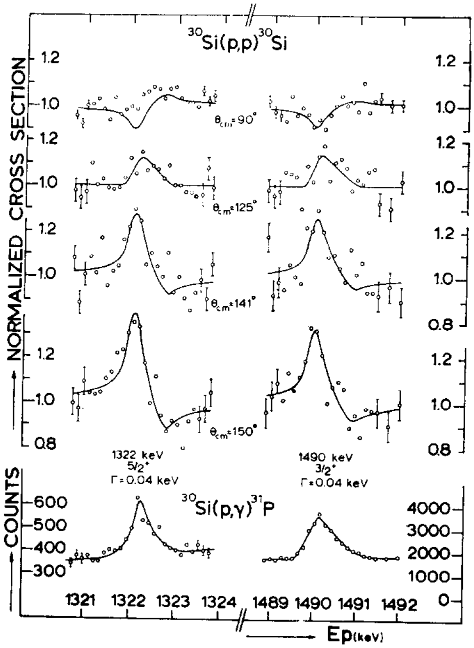

Fig. 3. The E_p = 1322 and 1490 keV resonances; for details, see caption of fig. 2.

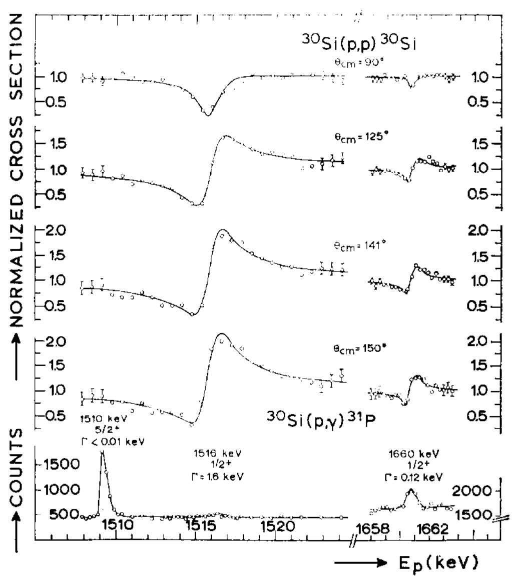

Fig. 4. The E_p = 1510, 1516 and 1660 keV resonances; for details, see caption of fig. 2.

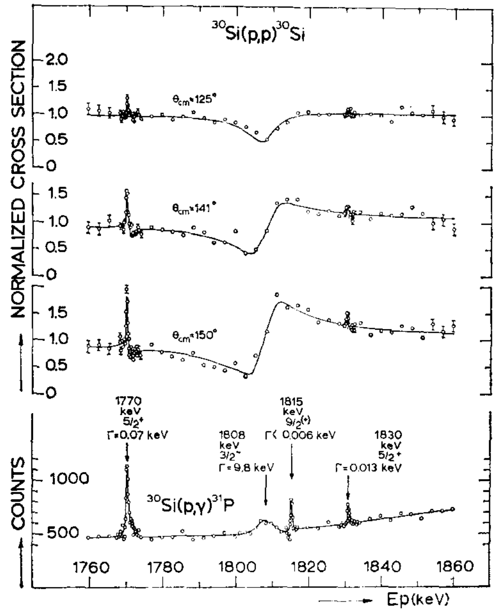

Fig. 5. The E_p = 1770, 1808, 1815 and 1830 keV resonances; for details, see caption of fig. 2.

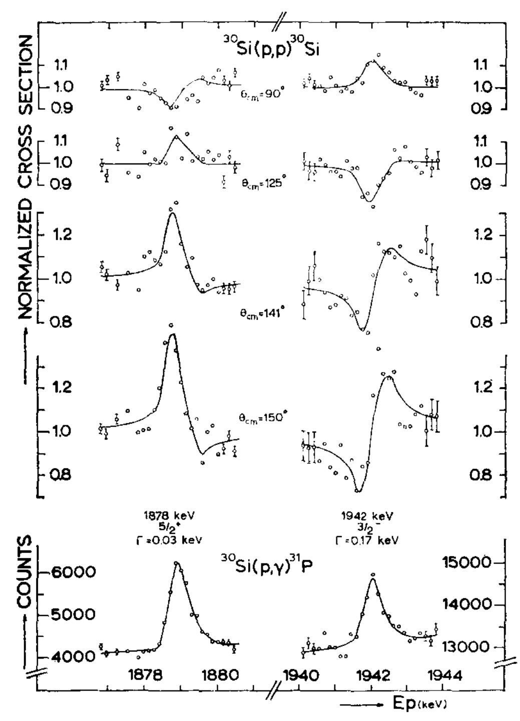

Fig. 6. The E_p = 1878 and 1942 keV resonances; for details, see caption of fig. 2.

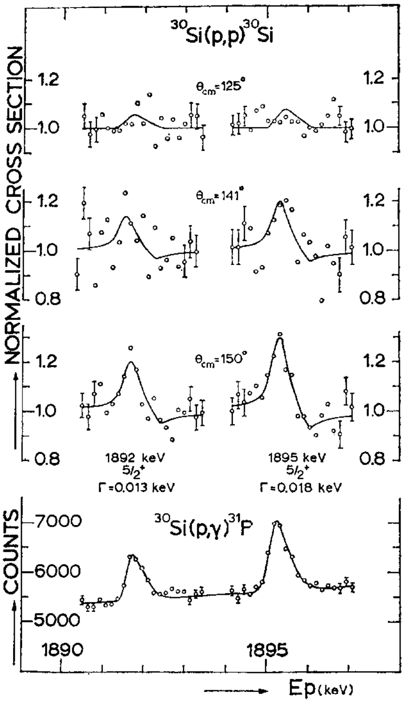

Fig. 7

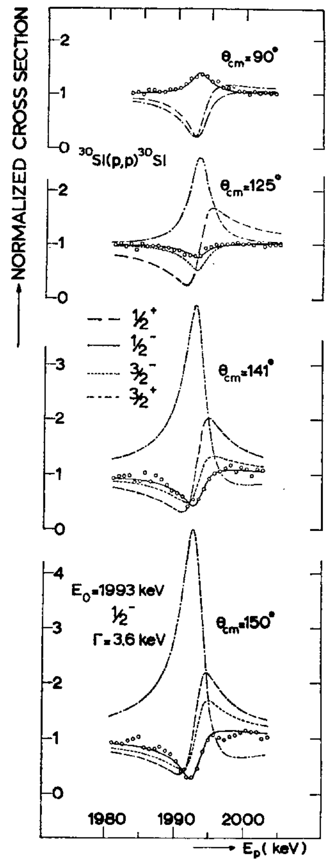

Fig. 8

Fig. 7. The E_p = 1892 and 1895 keV resonances; for details, see caption of fig. 2.

Fig. 8. Differential cross section at four angles for elastic proton scattering on ^{30}Si in units of the Rutherford contribution, with background subtracted, at the1993 keV resonance.

The theoretical curves have been drawn, not only for the correct assignment, $J^\pi =$ $= 1/2^-$, but also for $J^\pi = 1/2^+$, $3/2^-$ and $3/2^+$. See also text, explaining why $J^\pi =$ $= 5/2^+$ and *f* or *g* scattering can be neglected.

4. *Results.* The yields of protons scattered elastically from ^{30}Si, observed at four angles as a function of proton energy, are shown in figs. 2–8. They are plotted in units of the Rutherford contribution after subtraction of the background. The smooth curves are the results of the analysis as presented in section 3. The bottom curve gives, in arbitrary units, the γ-ray yield measured simultaneously. For better presentation, the zero is sometimes suppressed.

For some very narrow (p, γ) resonances, e.g. those at E_p = 1510 and 1815 keV, the corresponding (p, p) resonance structure is unobservably weak. All (p, p) resonances correspond to a known (p, γ) resonance. The

(p, γ) resonances at $E_p = 1516$ and 1993 keV hardly exceed the background in the present work, but they were clearly observed in the preceding (p, γ) experiments[1] [2]). This correspondence proves that all (p, p) resonances observed here can be assigned to the ^{30}Si(p, p)^{30}Si reaction. The ^{28}Si (32%) in the target only shows one, very broad, resonance in the $E_p = 1$–2 region, at $E_p = 1652$ keV with $\Gamma = 53$ keV[9]), which was seen but could be treated as a slow variation of the background. Resonances in ^{29}Si (3%) can be neglected as unobservably weak.

The resonances at $E_p = 1298$ and 1301 keV (fig. 2) are so close that they had to be analysed together. This presents some particular problems which may not have been solved quite succesfully, because for the accepted parities the combined χ^2 value remains just above the 0.1% probability limit. The fit, however, for any other parity combination is so very much worse, that also for these levels the parity determination may be considered as unambiguous.

In fig. 8, showing the $E_p = 1993$ keV resonance, theoretical curves have also been drawn for J^π values, other than the $J^\pi = 1/2^-$ which could be assigned from this experiment. Curves for f and g capture have not been drawn but would show high peaks at the most backward angles, contrary to experiment. Nor has the $J^\pi = 5/2^+$ case been drawn, which is almost undistinguishable from $J^\pi = 3/2^+$. As a conclusion, one can say that different l values can easily be distinguished, but that only for p capture the two different J values, $J^\pi = 1/2^-$ and $3/2^-$, yield a sufficiently different resonance structure.

TABLE I

Results from ^{30}Si(p, p)^{30}Si					
E_p	Results from this experiment			Results from ^{30}Si(p, γ)^{31}P [1])	
(keV)	J^π [b])	Γ (eV)	$\vartheta_p^2 \times 10^2$	J^π	Γ(keV)
1289	$1/2^-$	60 ± 50	0.13	$1/2^+$	[a])
1298	$1/2^+$	250 ± 70	0.19	$1/2^+$	0.7 ± 0.4
1301	$3/2^-$	210 ± 110	0.45	$3/2^+$	[a])
1322	$5/2^+$	40 ± 20	0.49	5/2	—
1490	$3/2^+$	40 ± 30	0.28	3/2	—
1516	$1/2^+$	1600 ± 400	0.6	1/2	1.5 ± 0.4
1660	$1/2^+$	120 ± 50	0.03	1/2	0.11 ± 0.05
1770	$5/2^+$	70 ± 30	0.18	$5/2^{(+)}$	0.10 ± 0.05
1808	$3/2^-$	9800 ± 600	4.3	$3/2^{(-)}$	9.0 ± 1.0
1830	$5/2^+$	13 ± 8	0.03	5/2	—
1878	$5/2^+$	31 ± 17	0.06	5/2	—
1892	$5/2^+$	13 ± 10	0.02	$5/2^{(+)}$	—
1895	$5/2^+$	18 ± 11	0.03	5/2	—
1942	$3/2^-$	170 ± 100	0.06	3/2	0.12 ± 0.05
1993	$1/2^-$	3580 ± 120	1.0	1/2	3.1 ± 0.4

[a]) In the (p, γ) work, these resonances were also seen to have a non-negligible width, but the width was not determined.

[b]) The spins given in this column are from ref. 1.

The final results for parities and widths are presented in table I. They are compared to those obtained from the (p, γ) work[1][2]. The widths agree within the experimental error. The parities also agree, but for the resonances at $E_p = 1289$ and 1301 keV. The cause of these wrong assignments from the (p, γ) work was already discussed in ref. 2.

The J^π values and widths obtained by Val'ter *et al*[3]) at $E_p = 1292$, 1514 and 1812 keV agree with the values found here for the resonances at $E_p = 1298$, 1516 and 1808 keV. They also observed (p, p) resonances at $E_p = 1030$, 1325, 1660, 1879 and 1996 keV, of which the latter four evidently correspond to those in table I at $E_p = 1322$, 1660, 1878 and 1993 keV. A search for the 1030 keV resonance, to which no known (p, γ) resonance corresponds, gave a negative result. An upper limit for the width is $60/(2J + 1)$ eV. The same upper limit can be taken to hold for the twenty-one (p, γ) resonances in the $E_p = 1$–2 MeV region, where no corresponding (p, p) resonance could be observed.

From the total widths, which closely approximate the proton widths, one can derive the reduced proton widths. These are given in the last column of table I, as computed[6]) using $R = r_0(A^{\frac{1}{3}} + 1)$ with $r_0 = 1.45$ fm. The purest single particle character is apparently found for the p-wave resonances at $E_p = 1808$ and 1993 keV.

For the resonances of table I, one can also compute the γ-ray widths from the (p, γ) yields. These values, and, for some resonances, the observed $E1$–$M2$ mixtures, were already discussed in ref. 1.

5. *Conclusions*. The narrowest resonances observed in the present (p, p) work have a width of 13 eV. Although this is quite an improvement over preceding $^{30}Si(p, p)^{30}Si$ work, it is felt that further improvements are still possible. Counter solid angles can certainly be taken somewhat larger, and one could hope for target material with an enrichment close to 100%, and for a better evaporation technique eliminating the Ta contamination on the target. Taken together, this might push down the lower limit of observability by a factor of 3 or 4. As all twenty-one resonances, observed in (p, γ) but not in (p, p), have a width between about 0.1 and 30 eV, such an improvement would greatly increase the number of observable resonances.

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REFERENCES

1) Van Rinsvelt, H. A. and Smith, P. B., Physica **30** (1964) 59.
2) Van Rinsvelt, H. A., thesis Utrecht (1965);
Van Rinsvelt, H. A. and Endt, P. M., Physica, to be published.
3) Val'ter, A. K., Popov, A. I. and Storizhko, V. E., Zh. eksp. teor. Fiz. **43** (1962) 2038; JETP **16** (1963) 1439.
4) Smulders, P. J. M., thesis Utrecht (1965).
5) Dearnaley, G., Rev. sci. Instrum. **31** (1960) 197.
6) Kuperus, J., Glaudemans, P. W. M. and Endt, P. M., Physica **29** (1963) 1281.
7) Blatt, J. M. and Biedenharn, S. C., Rev. mod. Phys. **24** (1952) 258.
8) Smith, P. B., Physica **24** (1958) 1085.
9) Endt, P. M. and Van der Leun, C., Nuclear Phys. **34** (1962) 1.