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SHELL-MODEL CALCULATIONS ON ODD-PARITY LEVELS OF NUCLEI IN THE RANGE ^{33}S–^{41}Ca

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Abstract: The shell model is used to investigate quantitatively some odd-parity levels of nuclei in the range ^{33}S–^{41}Ca An inert ^{32}S core is assumed with a residual two-particle interaction of the outer nucleons. Only one nucleon is considered to be in the $1f_{\frac{7}{2}}$ shell, while the others are in the $1d_{\frac{3}{2}}$ shell. The interaction matrix elements are reduced to linear expressions in 14 parameters (i e 12 two-particle interactions and two binding energies to the core)

The numerical results are presented as they are obtained from fitting the 14 parameters to 60 nuclear level energies. From the values of these parameters the energies and spins of about 240 levels are derived. Though the root mean square deviation of the calculated energies from the experimental energies is about 0.5 MeV, most of the level orders are calculated correctly. In addition, the energies of some 0^+ levels are calculated from configurations containing two particles in the $1f_{\frac{7}{2}}$ shell. These level energies appear to have a larger spread around the experimental energies

An expansion of the matrix elements of the effective interaction of one nucleon in the $1d_{\frac{3}{2}}$ and one in the $1f_{\frac{7}{2}}$ shell in terms of interaction moments shows that the mean two-body interaction for the $T = 0$ case is attractive, with a significant quadrupole splitting; this agrees with a weak spin dependence of the effective nuclear forces. The mean two-body interaction for the $T = 1$ case is slightly repulsive, the octupole splitting being the most significant one

1. Introduction

In the region of the $1d_{\frac{3}{2}}$ shell nuclei only a few shell-model calculations have been performed of levels of odd parity. Amongst these are the well-known examples of Goldstein and Talmi [1]) and Pandya [2]), who successfully interpreted levels in ^{38}Cl and ^{40}K as *j-j* coupling configurations with one proton (or hole, respectively) in the $1d_{\frac{3}{2}}$ shell and one neutron in the $1f_{\frac{7}{2}}$ shell.

In the present paper, this *j-j* coupling calculation is extended to all possible configurations with an arbitrary number of nucleons in the $1d_{\frac{3}{2}}$ shell. All inner (closed) shells are considered as an inert ^{32}S core. The method of effective parameters, describing the residual two-particle interaction of the extra-core nucleons, is used. The energies of the configurations are expressed in 14 such parameters. These parameters were obtained from a least-squares fit to 60 nuclear levels, including even-parity levels belonging to configurations in the $1d_{\frac{3}{2}}$ shell only.

The neglect of excitations from the $2s_{\frac{1}{2}}$ and $1d_{\frac{5}{2}}$ shells and to the $2p_{\frac{3}{2}}$, $2p_{\frac{1}{2}}$ and $1f_{\frac{5}{2}}$ shells makes the description of the experimental situation somewhat sketchy. That the applied *j-j* coupling scheme works at all may be attributed to the considerable spin-orbit splitting in the 1d and 1f shells ($\approx$ 5 MeV) and the probably spherical form of the average nuclear field at the end of the $1d_{\frac{3}{2}}$ shell, which precludes strong orbital momentum mixtures in non-degenerate configurations.

The identification of experimental levels as $(1d_{\frac{3}{2}})^n\ 1f_{\frac{7}{2}}$ configurations is, for the time being, only possible for the very lowest odd-parity states which are, as appears from the calculations, for the major part states of lowest seniority, $v = 1$ or 2. These levels have $J^\pi = \frac{7}{2}^-, 2^-, 3^-, 4^-$ or 5^-. Some known 0^+ levels are interpreted as $(1d_{\frac{3}{2}})^n(1f_{\frac{7}{2}})^2$ configurations.

The wave functions are not so detailed that a successful calculation of magnetic moments, electromagnetic transitions etc., can be expected. However, the calculations presented might be useful for more complicated calculations where configuration mixing is taken into account. At the moment, there appears to be insufficient experimental material to perform the latter kind of calculations with the effective parameter method.

2. Energies of Nuclear Levels in Terms of the Effective Parameters

A detailed theoretical treatment of the decomposition of the interaction matrix elements in terms of two-particle matrix elements has been given e.g. in refs. [3,4]). Hence we only shall give a short summary of the assumptions made and the formulae to be used. A recent investigation of even-parity levels in the $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ shells by Glaudemans, Wiechers and Brussaard [5]) serves as an example and will be frequently quoted.

The nuclear Hamiltonian is replaced by a model Hamiltonian H^m:

$$H^m - H^0 - V^C = \sum_i T_i + \sum_i U_i + \rho \sum_i \boldsymbol{l}_i \cdot \boldsymbol{s}_i + \sum_{i<j} V_{ij}. \tag{1}$$

Here H^0 represents the internal interaction of the core, ^{32}S in this case. This interaction is considered to be constant with eigenvalues E^0 in the range of nuclei to be investigated. The Coulomb energy V^C of the outer nucleons (i.e. mutually as well as with the core), is derived from the experimental Coulomb energy differences of mirror nuclei [3,5]). In eq. (1), (i) T_i and U_i represent the kinetic and nuclear potential energies of the nucleons in the spherical field provided by the core; (ii) $\rho \sum_i \boldsymbol{l}_i \cdot \boldsymbol{s}_i$ denotes the spin-orbit coupling term; (iii) V_{ij} represents the (residual) nuclear interaction of the nucleons outside the core. This interaction is described as a two-body interaction, correlations between three or more nucleons being neglected. The isospin is considered to be a good quantum number.

The eigenvalues $E_b(nlj)$, of the single-particle eigenfunctions of the first three terms of the right-hand side of eq. (1) will be considered as unknown parameters. In fact they represent single-particle binding energies in the pure configurations n, l, j. The basis functions, to be denoted by χ, describing the (pure) many-particle configurations, are antisymmetric products of these single-particle wave functions, coupled to the appropriate total spin and isospin (and, if necessary, other quantum numbers). The model wave functions Ψ are linear combinations (mixed configurations) of these basis functions with the same spin and isospin (and possibly other quantum numbers).

For n nucleons in the $1d_{\frac{3}{2}}$ shell and m nucleons in the $1f_{\frac{7}{2}}$ shell the eigenvalue E^{m} of the model Hamiltonian is given by

$$E^{m}-E^{0}-E^{C} = nE_b(1d_{\frac{3}{2}})+mE_b(1f_{\frac{7}{2}})+\langle\psi|\sum_{1\leq i<j}^{n+m} V_{ij}|\psi\rangle_{JTe}\,. \tag{2}$$

The symbol e labels different solutions with the same n, m, J and T. The number m of nucleons in the $1f_{\frac{7}{2}}$ shell is limited to 0, 1 or 2 in this investigation. The last term on the right-hand side of eq. (2) can be evaluated as a linear function of 12 parameters, representing the interactions of two-body configurations. These parameters are, if both nucleons are in the $1d_{\frac{3}{2}}$ shell: $\langle 1d_{\frac{3}{2}}^2|V_{12}|1d_{\frac{3}{2}}^2\rangle_{JT}$ with JT = 01, 21, 10 or 30, and if the nucleons are in different shells: $\langle 1d_{\frac{3}{2}}1f_{\frac{7}{2}}|V_{12}|1d_{\frac{3}{2}}1f_{\frac{7}{2}}\rangle_{JT}$ with JT = 20, 30, 40, 50, 21, 31, 41 and 51.

In the case of one nucleon in the $1f_{\frac{7}{2}}$ shell the wave function becomes

$$\psi(1d_{\frac{3}{2}}^n 1f_{\frac{7}{2}})_{JTe} = \sum_a V_a^e \chi(1d_{\frac{3}{2}}^n(J_a T_a v_a)1f_{\frac{7}{2}})_{JT}\,,$$

where v_a denotes the seniority, which may be required as an additional label. The interaction energy of the state is given by

$$\langle\psi|\sum_{1\leq i<j}^{n+1} V_{ij}|\psi\rangle_{JTe} = \sum_{a,c} V_a^e V_c^e \langle 1d_{\frac{3}{2}}^n(J_a T_a v_a)1f_{\frac{7}{2}}|\sum_{1\leq i<j}^{n+1} V_{ij}|1d_{\frac{3}{2}}^n(J_c T_c v_c)1f_{\frac{7}{2}}\rangle_{JT}\,. \tag{3}$$

The coefficients V_a^e are obtained as eigenvectors from the diagonalization of the matrix at the right-hand side of eq. (3). The evaluation of the matrix elements (as shown in principle in the refs. [3,4]) is given by

$$\begin{aligned}
&\langle 1d_{\frac{3}{2}}^n(J_a T_a v_a)1f_{\frac{7}{2}}|\sum_{1\leq i<j}^{n+1} V_{ij}|1d_{\frac{3}{2}}^n(J_c T_c v_c)1f_{\frac{7}{2}}\rangle_{JT} \\
&= n\sum_{b,d}\langle 1d_{\frac{3}{2}}^n(J_a T_a v_a)|1d_{\frac{3}{2}}^{n-1}(J_b T_b v_b)\rangle\langle 1d_{\frac{3}{2}}^n(J_c T_c v_c)|1d_{\frac{3}{2}}^{n-1}(J_b T_b v_b)\rangle \\
&\times U(J_b\tfrac{3}{2}J\tfrac{7}{2};\,J_a J_d)U(T_b\tfrac{1}{2}T\tfrac{1}{2};\,T_a T_d)U(J_b\tfrac{3}{2}J\tfrac{7}{2};\,J_c J_d) \\
&\times U(T_b\tfrac{1}{2}T\tfrac{1}{2};\,T_c T_d)\langle 1d_{\frac{3}{2}}1f_{\frac{7}{2}}|V_{12}|1d_{\frac{3}{2}}1f_{\frac{7}{2}}\rangle_{J_d T_d} \\
&+\langle 1d_{\frac{3}{2}}^n|\sum_{1\leq i<j}^{n} V_{ij}|1d_{\frac{3}{2}}^n\rangle_{J_a T_a}\delta_{J_a J_c}\delta_{T_a T_c}\,.
\end{aligned} \tag{4}$$

The expansion of $\langle 1d_{\frac{3}{2}}^n|\sum_{1\leq i<j}^n V_{ij}|1d_{\frac{3}{2}}^n\rangle_{J_a T_a}$ (i.e. the case of $m = 0$) in terms of $\langle 1d_{\frac{3}{2}}^2|V_{12}|1d_{\frac{3}{2}}^2\rangle_{JT}$ has been evaluated by Edmonds and Flowers [6]).

The numerical values of the coefficients of fractional parentage $\langle 1d_{\frac{3}{2}}^n(J_a T_a v_a)|1d_{\frac{3}{2}}^{n-1}(J_b T_b v_b)\rangle$ can be found in ref. [6]). The U-coefficients are normalized Racah coefficients:

$$U(J_b\tfrac{3}{2}J\tfrac{7}{2};\,J_aJ_d) = \{(2J_a+1)(2J_d+1)\}^{\frac{1}{2}}W(J_b\tfrac{3}{2}J\tfrac{7}{2};\,J_aJ_d).$$

The addition of a second nucleon in the $1f_{\frac{7}{2}}$ shell gives rise to matrix elements which similarly can be reduced to

$$\begin{aligned}\langle 1d^n_{\frac{3}{2}}(J_aT_av_a)1f^2_{\frac{7}{2}}(J_bT_b)| &\sum_{1\leq i<j}^{n+2} V_{ij}|1d^n_{\frac{3}{2}}(J_cT_cv_c)1f^2_{\frac{7}{2}}(J_dT_d)\rangle_{JT} \\ &= 2\sum_{J_eT_e} U(\tfrac{7}{2}\tfrac{7}{2}JJ_a;\,J_bJ_e)U(\tfrac{7}{2}\tfrac{7}{2}JJ_c;\,J_dJ_e) \\ &\times U(\tfrac{1}{2}\tfrac{1}{2}TT_a;\,T_bT_e)U(\tfrac{1}{2}\tfrac{1}{2}TT_c;\,T_dT_e) \\ &\times \langle 1d_{\frac{3}{2}}(J_aT_av_a)1f_{\frac{7}{2}}|\sum_{1\leq i<j}^{n+1} V_{ij}|1d_{\frac{3}{2}}(J_cT_cv_c)1f_{\frac{7}{2}}\rangle_{J_eT_e} \\ &+\langle 1f^2_{\frac{7}{2}}|V_{12}|1f^2_{\frac{7}{2}}\rangle_{J_bT_b}\delta_{J_bJ_d}\delta_{T_bT_d}. \end{aligned} \tag{5}$$

3. The Calculations

The nuclear interaction of the extra-core nucleons, mutual and with the core, is represented by the right-hand side of eq. (2). The experimental values are given by the level energies with respect to the ^{32}S core, corrected for the Coulomb energy. The Coulomb energies E^{C} in the $1d_{\frac{3}{2}}$ shell region were calculated in ref. [5]) in two slightly different ways from binding energy differences between mirror nuclei. For a derivation of ground-state values of $E = E^{\mathrm{m}} - E^0 - E^{\mathrm{C}}$ used in the present investigation, we employed the values given in table 1 (second column) of ref. [5]), shifting the values by 47.06 MeV as a correction for the ^{32}S core. As the Coulomb energy is considered to be a constant for each nucleus, the values of E for the excited states are obtained similarly.

These experimental values of E should now be obtained as eigenvalues of eq. (2). This is achieved by an iterative least-squares procedure [5]), which also gives the coefficients V^e_a determining the wave functions. An initial set of the energy parameters $E_b(nlj)$ and $\langle n_1l_1j_1n_2l_2j_2|V_{12}|n_1l_1j_1n_2l_2j_2\rangle_{JT}$ is obtained in the following way. The wave functions of the experimentally known odd-parity states are approximated by taking basis functions of lowest seniority. In this approximation the energies of all levels are linear expressions in the energy parameters and can be fitted by a least-squares procedure. The parameters thus obtained now determine the matrix elements (3) in zeroth order. Subsequently the matrices (3) are diagonalized, and their eigenvectors are calculated. Substitution of these eigenvectors V^e_a into eq. (3) then gives, with eqs. (2) and (4), a set of simultaneous linear equations for the energy parameters that can be solved with a least-squares procedure. Three iterations in total are sufficient to reduce the variations in the parameters to the order of 0.01 MeV. The energies of all possible configurations are calculated from the last set of parameters.

All calculations, including the evaluation of the matrix elements, were performed on the X1 computer at the "Electronisch Rekencentrum der Rijksuniversiteit".

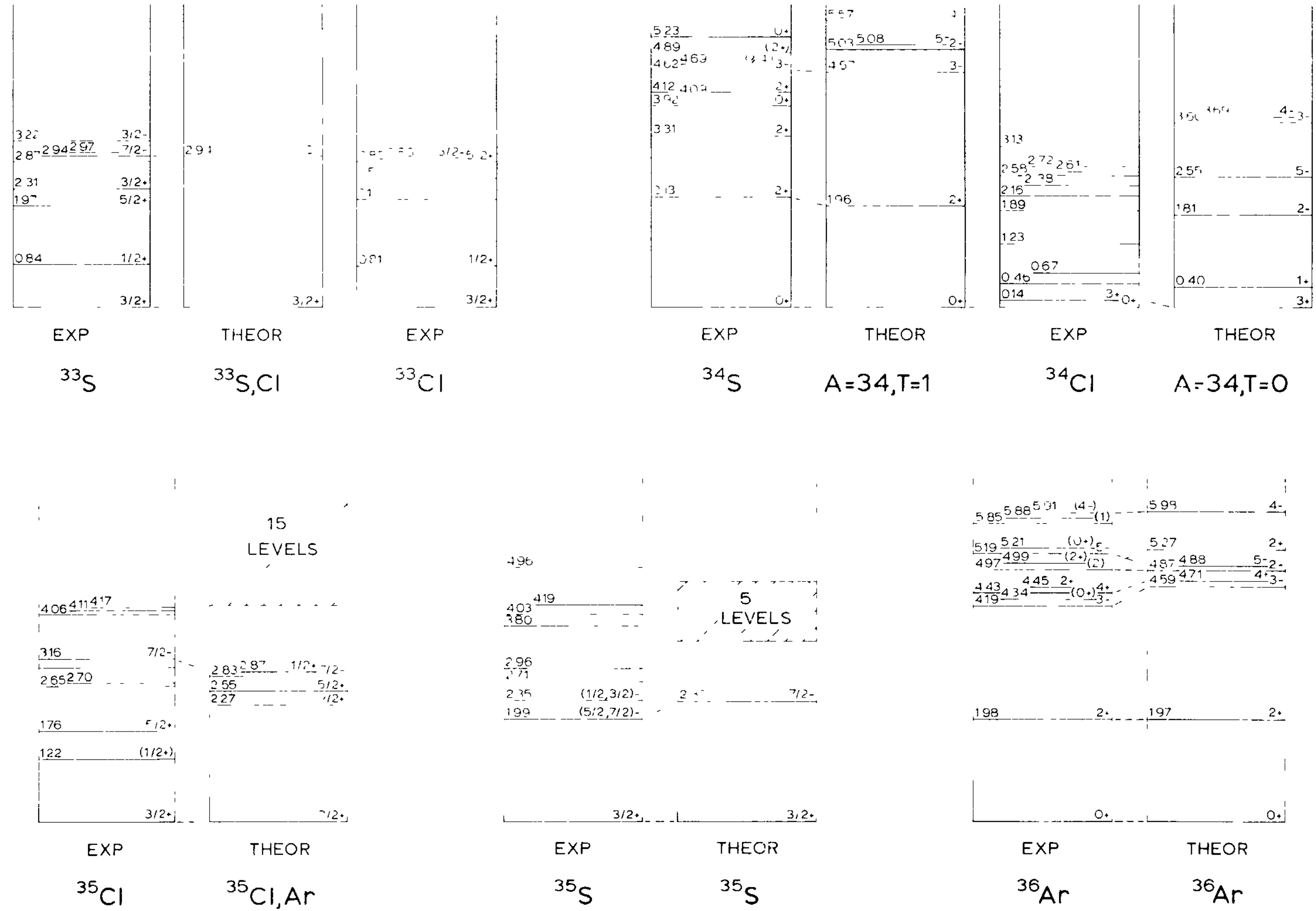
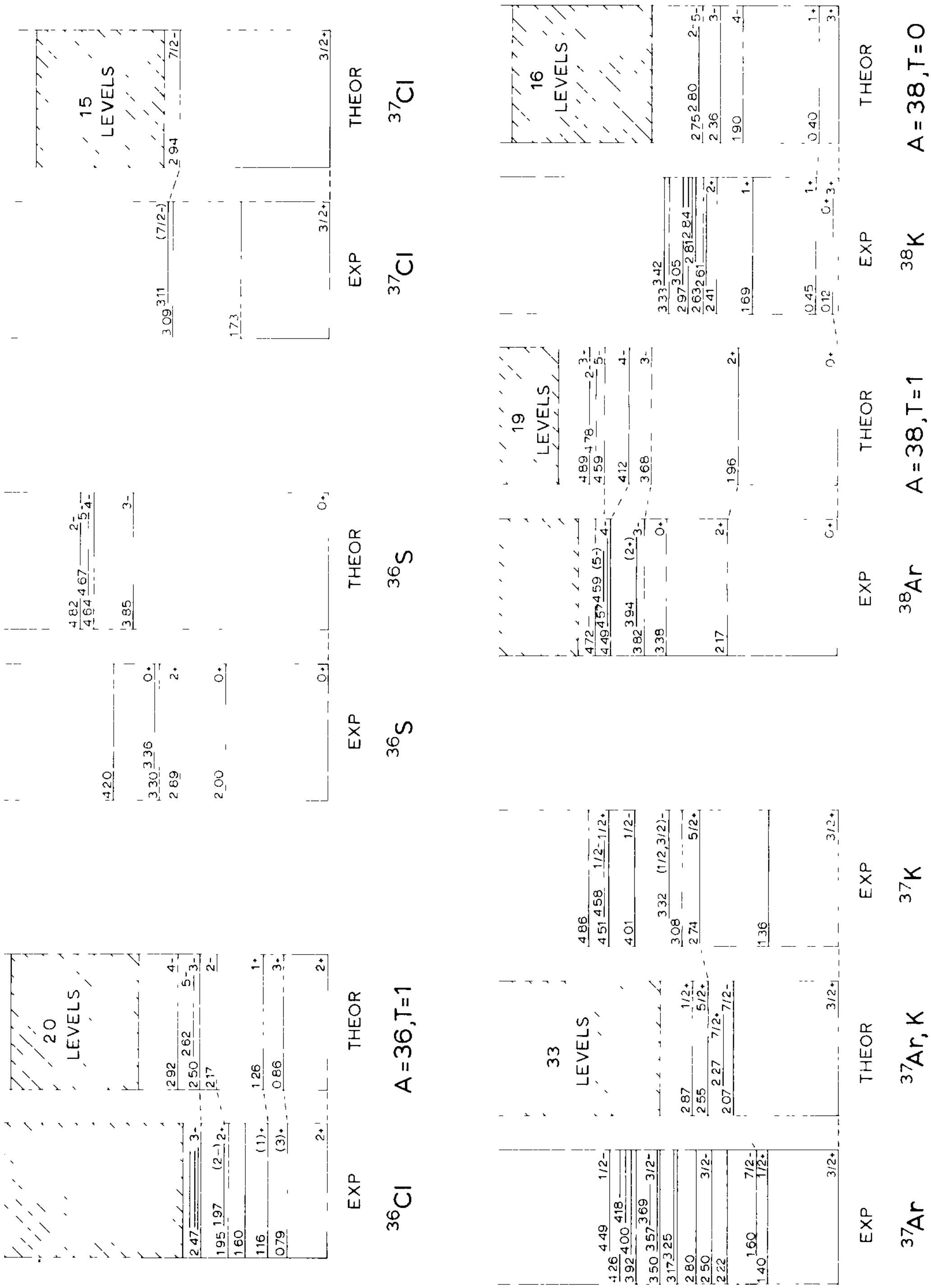


Fig 1. Level schemes of experimental and calculated levels. The excitation energies are given in MeV. The positions of the fitted levels and the corresponding experimental levels are interconnected by dashed lines. Of the odd-parity levels only the ones that predominantly consist of configurations of lowest seniority, are drawn. Hatched areas indicate high (theoretical and experimental) level densities.


EXP
THEOR
36Cl
A=36,T=1
20 LEVELS
36S
37Cl
15 LEVELS
37Ar
37Ar,K
33 LEVELS
37K
38Ar
A=38,T=1
19 LEVELS
38K
A=38,T=0
16 LEVELS

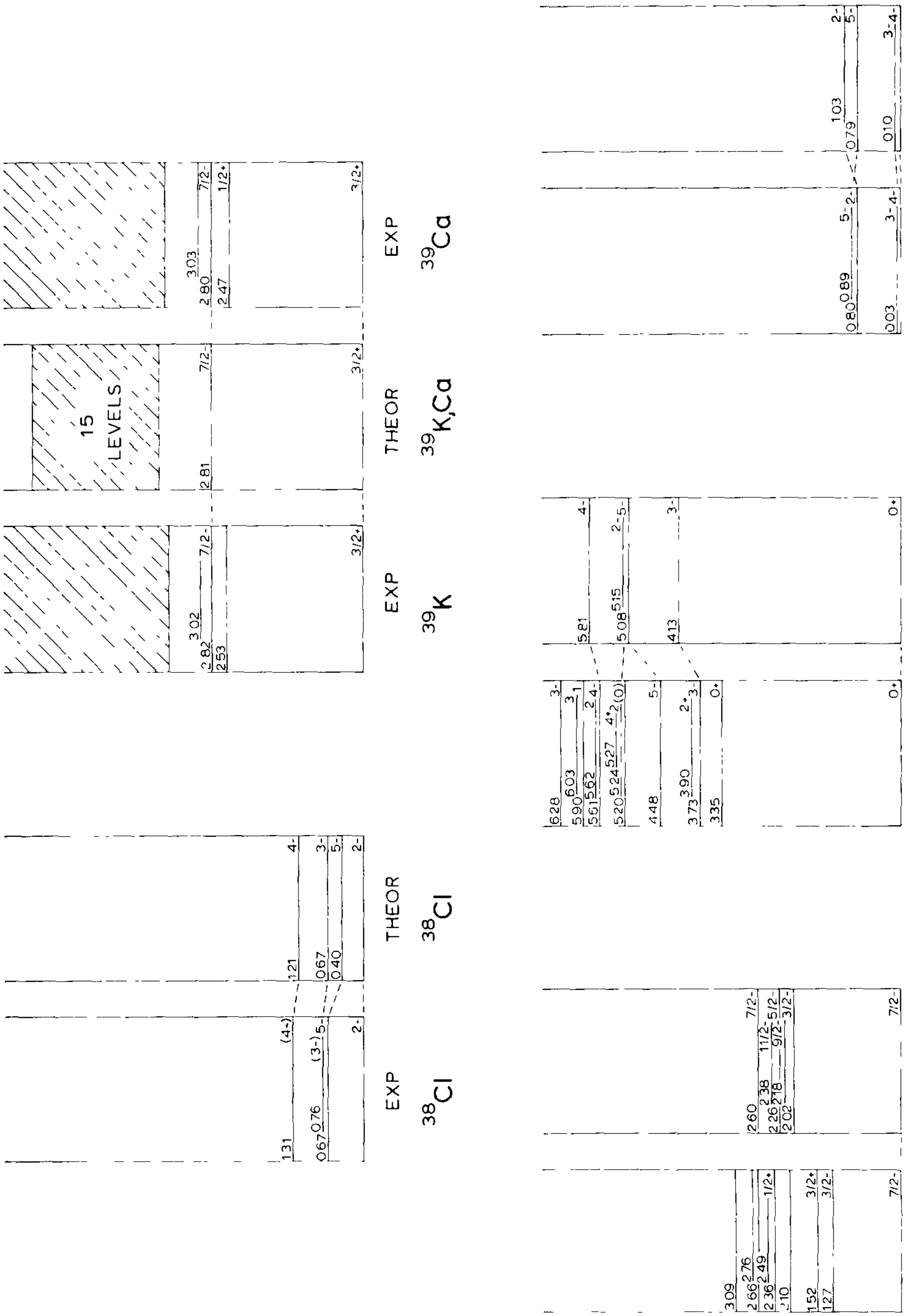


Fig 1 Continued, for figure caption see page 95.

TABLE 1

Experimental and calculated energies (in MeV)

Nucleus isospin	J^π	E^{exp}	E^{th}	E_x^{exp}	E_x^{th}	References to experiment
^{33}S, Cl	$\frac{3}{2}^+$	− 8 64	− 9.04	0	0	[7]
$T = \frac{1}{2}$	$\frac{7}{2}^-$			2.94	2 94	[7, 24]
^{34}Cl	1^+				0 40	
$T = 0$	3^+	−19.99	−20 59	0	0	[7]
	2^-				1.81	
	3^-				3.60	
	4^-				3.69	
	5^-				2 55	
^{34}S, Cl	0^+	−20 06	−19 78	0	0	[7]
$T = 1$	2^+			2.13	1 96	[7]
	2^-				5.03	
	3^-			4 62	4 57	[8]
	4^-				5 57	
	5^-				5 08	
^{35}Cl, Ar	$\frac{3}{2}^+$	−32.80	−32.77	0	0	[7]
$T = \frac{1}{2}$	$\frac{1}{2}^+$				2 87	[a]
	$\frac{5}{2}^+$				2 55	[a]
	$\frac{7}{2}^+$				2 27	
	$\frac{1}{2}^-$				4.83	
	$\frac{3}{2}^-$				4.27, 6.04	
	$\frac{5}{2}^-$				4.46, 5.73, 6.29	
	$\frac{7}{2}^-$			3 16	2.83, 5 27, 5 48, 6.02	[7, 20]
	$\frac{9}{2}^-$				4 76, 6.05, 6 69	
	$\frac{11}{2}^-$				5.31, 6.22	
	$\frac{13}{2}^-$				5.28	
^{35}S	$\frac{3}{2}^+$	−27.04	−27 33	0	0	[7]
$T = \frac{3}{2}$	$\frac{3}{2}^-$				3.49	
	$\frac{5}{2}^-$				4.55	
	$\frac{7}{2}^-$			(1.99)	2.32, 4.67	[7]
	$\frac{9}{2}^-$				3 89	
	$\frac{11}{2}^-$				4.62	
^{36}Ar	0^+	−48 04	−47.47	0	0	[7]
$T = 0$	2^+			1.98	1.97	[16]
	2^+			4.99	5 27	[16, b]
	4^+			4.43	4.71	[16]
	0^-				8.67	
	1^-				7.07, 8 39	
	2^-			4.97	4 87, 7 63, 8 57	[16]
	3^-			4 19	4 59, 7.32, 8 62, 8.80	[16]
	4^-			(5.91)	5 98, 7.28, 8.37, 8 84	[16]
	5^-			5.19	4 88, 7 39, 8 54	[16]
	6^-				8.28, 8.44	
	7^-				7 96	
^{36}Cl, Ar	1^+			(1 16)	1 26	[7]
$T = 1$	2^+	−41 37	−41.87	0	0	[7]
	3^+			(0.79)	0.86	[7]
	0^-				3 72	

TABLE 1 (continued)

Nucleus isospin	J^π	E^{exp}	E^{th}	E_x^{exp}	E_x^{th}	References to experiments.
	1^-				4 03, 5.42	
	2^-			1.97	2.17, 4 13, 5.21, 5.32	[9]
	3^-			2.58	2 50, 3.70, 4 64, 5.45, 6.37	[9,16,c]
	4^-			(2.90)	2 92, 3 68, 5 10, 5.58, 6 16	[16,c]
	5^-				2.62, 4.14, 5.40, 5 86	
	6^-				4 67, 6 19	
	7^-				5 25	
^{36}S	0^+	−36 93	−36.57	0	0	[7]
$T = 2$	2^-				4 82	
	3^-				3 85	
	4^-				4.64	
	5^-				4 67	
^{37}Ar, K	$\frac{3}{2}^+$	−56.84	−57.14	0	0	[7]
$T = \frac{1}{2}$	$\frac{1}{2}^+$				2 87	
	$\frac{5}{2}^+$			2 74	2.55	[23]
	$\frac{7}{2}^+$				2.27	
	$\frac{1}{2}^-$				5 71, 6.57	
	$\frac{3}{2}^-$				3.50, 5 32, 5 58, 6.88, 8.16	
	$\frac{5}{2}^-$				4 13, 4.78, 5.35, 6.91, 7.06, 7.70	
	$\frac{7}{2}^-$			1.60	2.07, 4 21, 4 83, 6.23, 6 54, 7 46, 7 96	[10,d]
	$\frac{9}{2}^-$				3.87, 4.48, 5.76, 6.38, 7.56, 8.19	
	$\frac{11}{2}^-$				4 18, 4.98, 6.45, 7 06, 8.15	
	$\frac{13}{2}^-$				5 51, 7 09	
	$\frac{15}{2}^-$				7.59	
^{37}Cl	$\frac{3}{2}^+$	−51.69	−51.69	0	0	[7]
$T = \frac{3}{2}$	$\frac{1}{2}^-$				3 76	
	$\frac{3}{2}^-$				3.24, 4.84	
	$\frac{5}{2}^-$				3 40, 4.97, 5.76	
	$\frac{7}{2}^-$			(3.11)	2 94, 4 22, 4.89, 5 54	[7,e]
	$\frac{9}{2}^-$				3 84, 4 78, 5.76	
	$\frac{11}{2}^-$				4.32, 5 34	
	$\frac{13}{2}^-$				4 00	
^{37}S	$\frac{7}{2}^-$	(−41 33)	−40 86	(0)	0	[7,f]
$T = \frac{5}{2}$						
^{38}K	1^+			0.45	0 40	[11,g]
$T = 0$	3^+	−68 87	−69.33	0	0	[7]
	0^-				4 21	
	1^-				3 69, 5 98	
	2^-			≧2 61	2 80, 3.70, 6.13	[11,g]
	3^-			,,	2 36, 3 92, 5.23, 5.79	,,
	4^-			,,	1.90, 4 06, 4.85, 6.41	,,
	5^-			,,	2.75, 4 14, 5.69	,,
	6^-				3.90, 5.12	
	7^-				5.78	
^{38}Ar	0^+	−68.68	−68.52	0	0	[7]
$T = 1$	2^+			2 17	1.96	[7]
	0^-				6 97	
	1^-				6.27, 7.21	

TABLE 1 (continued)

Nucleus isospin	J^π	$E^{\rm exp}$	$E^{\rm th}$	$E_x^{\rm exp}$	$E_x^{\rm th}$	References to experiments.
	2^-				4.78, 5 67, 6.79, 7.60	
	3^-			3.82	3.68, 4.89, 6.65, 7.42, 7.67	[7]
	4^-			4.49	4.12, 6.08, 6.53, 7.35, 7.83	[15]
	5^-			4.59	4.59	[15]
				5 67	5 51, 6 78, 7.57	[15]
	6^-				7 00, 7.40	
	7^-				7.17	
^{38}Cl	2^-	−57 98	−57.62	0	0	[7]
$T = 2$	3^-			(0.76)	0 67	[7]
	4^-			(1 31)	1 21	[7]
	5^-			0.67	0.40	[7]
^{39}K, Ca	$\frac{3}{2}^+$	−81 95	−82 15	0	0	[7]
$T = \frac{1}{2}$	$\frac{1}{2}^-$				5 95	
	$\frac{3}{2}^-$				3.78, 5.47	
	$\frac{5}{2}^-$				4 25, 4 81, 6 13	
	$\frac{7}{2}^-$			2.81	2 81, 4 24, 5.51, 6 11	[12, 13, 25]
	$\frac{9}{2}^-$				4.03, 4.79, 5.39	
	$\frac{11}{2}^-$				4 51, 5 80	
	$\frac{13}{2}^-$				5.55	
^{39}Ar	$\frac{3}{2}^-$				2.02	
$T = \frac{3}{2}$	$\frac{5}{2}^-$				2 26	
	$\frac{7}{2}^-$	−75.27	−75.13	0	0, 2.60	[7]
	$\frac{9}{2}^-$				2.18	
	$\frac{11}{2}^-$				2 38	
^{40}Ca	0^+	−97.58	−97.40	0	0	[7]
$T = 0$	2^-			(5.24)	5.15	[22, h]
	3^-			3.73	4.13	[21]
	4^-			5 61	5.81	[22]
	5^-			4.48	5 08	[21]
^{40}K	2^-			0 80	1 03	[7]
$T = 1$	3^-			0 03	0.10	[7]
	4^-	−89.93	−89.92	0	0	[7, 14, h]
	5^-			0.89	0 79	[7]
^{41}Ca	$\frac{7}{2}^-$	−105.94	−105.81	0	0	[7]
$T = \frac{1}{2}$						

[a]) The $\frac{1}{2}^+$ and $\frac{5}{2}^+$ levels in ^{35}Cl at 1 22 and 1.76 MeV excitation energies arise from excitation of one nucleon from the $2s_{\frac{1}{2}}$ shell to the $1d_{\frac{3}{2}}$ shell (see ref. [5])).

[b]) Experimentally, 2^+ levels are found in ^{36}Ar at 4.45 and 4 97 MeV excitation energies. The 4 45 MeV level presumably arises from excitation of one nucleon from the $2s_{\frac{1}{2}}$ shell into the $1d_{\frac{3}{2}}$ shell (see ref [5])).

[c]) Levels with spin and parity (2^-) and 3^- have been found in ^{36}Cl at 1.96 and 2.47 MeV by van Middelkoop and Spilling with the ^{35}Cl(n, γ)^{36}Cl reaction [9]). The 3^- level in ^{36}Ar has probably been observed as a $T = 1$ resonance in the ^{35}Cl(p, γ)^{36}Ar reaction at $E_x = 9.34$ MeV. The energies from both experiments have been averaged after correction for Coulomb energy and n-p mass difference. The $J = 4$, $T = 1$ resonance at $E_x = 9$ 57 MeV in the reaction ^{35}Cl(p, γ)^{36}Ar may have odd parity. The corresponding excitation energy in ^{36}Cl is 2 90 MeV.

[d]) The $\frac{7}{2}^-$ level in ^{37}Ar has been found in the ^{36}Ar(d, p)^{37}Ar reaction by Rosner and Schneid [10]) at

(see next page)

4. Results

4.1. LEVEL ENERGIES

The experimental and calculated level schemes are displayed in fig. 1. The position of the fitted levels and the corresponding experimental levels are interconnected by dashed lines. Of the calculated odd-parity levels in most cases only the ones that predominantly consist of configurations of the lowest seniority, are drawn. The calculated energies are presented in table 1. In this table are listed (i) the experimental values of $E = E^{m} - E^{0} - E^{C}$ for the ground states, (ii) the values calculated for the right-hand side of eq. (2) for the ground states, (iii) the experimental excitation energies E_x^{exp} for the fitted levels and (iv) all calculated excitation energies E_x^{th}. In the cases of unproven spins and/or parities, the experimental energies are given in parentheses. In the last column, references are given to experimental determinations of spins and parities of the fitted levels. The majority of the data has been taken from the review paper by Endt and van der Leun [7]).

Only the energies of odd-parity levels that predominantly consist of configurations of the lowest seniority were fitted to the experimental energies. Levels at higher excitation energies can not be identified so easily and are expected to contain stronger admixtures from configurations that are neglected in the present calculations.

The root mean square deviation of the 60 theoretical level energies with respect to the experimental energies amounts to 0.5 MeV. The excitation energies of levels of one specific isospin are described considerably better with a r.m.s. deviation of 0.3 MeV. In most cases the calculated level orders agree with experiment.

4.2. EFFECTIVE PARAMETERS

In table 2 the values of the parameters obtained are given, together with the estimated external errors. These errors are calculated from the final least-squares fit of the iteration process, and in such a way that with equal weight attributed to all levels, the r.m.s. deviation of 0.5 MeV is reproduced.

The values of the parameters obtained for the interaction in the $1d_{\frac{3}{2}}$ shell are slightly different from the values obtained by Glaudemans *et al.* [5]). The latter parameters can be found in the second column of table 2.

Expanding the matrix elements of the effective interaction of one nucleon in the

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1.60 MeV The spectroscopic factor is 0.82, which means that predominantly a single particle excitation is involved.

e) Allowed β-decay to the 3 105 MeV level in ^{37}Cl from the ^{37}S ground state has been observed

f) The spin and parity of the ground state of ^{37}S are not known The β-decay to the $\frac{3}{2}^+$ ground state of ^{37}Cl is forbidden; a $\frac{7}{2}^-$ assignment is therefore not unreasonable.

g) States with $J^\pi = 1^+$ have been found in ^{38}K at 0 45 and 1.69 MeV by Janecke [11]) in the ^{40}Ca(d, α)^{38}K reaction. Odd-parity levels should have excitation energies higher than 2.61 MeV, according to this experiment.

h) In ^{40}Ca, spin-2 levels are found at 5 24 and 5 62 MeV [22,26]). No parity determinations have been performed for these levels. The lowest $T = 1$ state in ^{40}Ca has been found in the β-decay of ^{40}Sc at 7.646 MeV by Anderson *et al.* [14])

$1d_{\frac{3}{2}}$ and one in the $1f_{\frac{7}{2}}$ shell in terms of interaction moments $T^{(r)}$ (see ref. [3])), one obtains

$$\langle j_1 j_2 | V_{12} | j_1 j_2 \rangle_J = \sum_r (-1)^{j_1+j_2+J} \begin{Bmatrix} j_1 & j_2 & J \\ j_2 & j_1 & r \end{Bmatrix} T^{(r)}.$$

The results are shown in the last column of table 2. For the $T = 0$ matrix elements the average interaction is attractive and only the quadrupole splitting is relevant. For the $T = 1$ matrix elements the average interaction is repulsive; the octupole splitting is the most significant one.

TABLE 2

Parameter values (in MeV)

	J	T	Present investigation	Glaudemans *et al.* [5]	Tensor expansion	
$E_b(1d_{\frac{3}{2}})$			-9.04 ± 0.06			
$E_b(1f_{\frac{7}{2}})$			$-6\ 10 \pm 0.18$			
$\langle 1d_{\frac{3}{2}}{}^2 \| V_{12} \| 1d_{\frac{3}{2}}{}^2 \rangle$	$J = 0$	$T = 1$	$-1.71 \pm 0\ 17$	$-2\ 28$		
	2	1	$+0.26 \pm 0\ 04$	$+0.16$		
	1	0	$-2\ 11 \pm 0\ 24$	$-0\ 92$		
	3	0	$-2\ 51 \pm 0\ 13$	-2.64		
$\langle 1d_{\frac{3}{2}} 1f_{\frac{7}{2}} \| V_{12} \| 1d_{\frac{3}{2}} 1f_{\frac{7}{2}} \rangle$	$J = 2$	$T = 0$	$-3\ 65 \pm 0\ 39$		$T = 0$	$T^{(0)} = -14.0 \pm 0\ 9$
	3	0	$-1\ 85 \pm 0\ 40$			$T^{(1)} = 0.0 \pm 1\ 6$
	4	0	$-1.77 \pm 0\ 30$			$T^{(2)} = -\ 8\ 7 \pm 2.0$
	5	0	$-2\ 90 \pm 0\ 28$			$T^{(3)} = +\ 2\ 6 \pm 2.6$
	2	1	$+0\ 38 \pm 0\ 26$		$T = 1$	$T^{(0)} = +\ 2\ 6 \pm 0.5$
	3	1	$-0\ 08 \pm 0\ 20$			$T^{(1)} = +\ 1\ 1 \pm 0.9$
	4	1	$+0\ 92 \pm 0\ 18$			$T^{(2)} = -\ 1\ 7 \pm 1\ 2$
	5	1	$+0\ 43 \pm 0.15$			$T^{(3)} = -\ 4.6 \pm 1\ 4$

5. Calculation of Some 0^+ Levels

The 0^+ levels of some nuclei are calculated with the assumptions that (i) two nucleons are excited to the $1f_{\frac{7}{2}}$ shell, and (ii) the nucleons in the $1d_{\frac{3}{2}}$ shell as well as in the $1f_{\frac{7}{2}}$ shell are coupled to spin zero. More complete *j-j* coupling calculations might give small corrections of the order of 0.1 to 0.5 MeV. The results of the calculated excitation energies are presented in table 3. The ground state of ^{42}Ca has been used to determine the two-body interaction of nucleons in the $1f_{\frac{7}{2}}$ shell: $\langle 1f_{\frac{7}{2}}^2 | V_{12} | 1f_{\frac{7}{2}}^2 \rangle_{01} = -3.00$ MeV.

The calculations are less reliable than the calculations on the odd-parity levels, as is obvious from the large deviations from the experimental values. In table 3 also the excitation energies of 0^+ levels from $(2s_{\frac{1}{2}})^{-n}(1d_{\frac{3}{2}})^m$ configurations are indicated; these were calculated by Glaudemans *et al.* [5]). Comparison shows that in most nuclei where 0^+ levels are known experimentally, the correct number of 0^+ levels is predicted.

TABLE 3

Calculated excitation energies of 0^+ levels (in MeV)

Nucleus	Isospin	E_x^{exp}	E_x^{th} ($1d_{\frac{3}{2}}^n 1f_{\frac{7}{2}}^2$) present calculation	E_x^{th} ($2s_{\frac{1}{2}}^{-n} d_{\frac{3}{2}}^m$) Glaudemans *et al.* [5]	References to experiment
^{34}S	1	3 92		3 78	[8]
^{34}S	1	(5 23)	4.61	5 12	[8]
^{34}S	1	5.86			[8]
^{36}Ar	0	(4 34)		3 99	[16]
^{36}Ar	0	(5 21)	5 58		[16]
^{36}Cl	1		2.86		
^{36}Cl	1			3.47	
^{36}S	2	2.00	3.38	4 28	[8]
^{36}S	2	3 36			[8]
^{38}Ar	1	3 38	3 62		[17]
^{38}Ar	1	(4.57)		6 67	[18]
^{38}Cl	2	(1.79)	0.71		[7]
^{40}Ca	0	3 35	4 58		[7]
^{40}K	1	$\geq$1 95	$-$0 08		[7]

6. Discussion

6.1. SPURIOUS STATES

In calculations starting from oscillator wave functions of a definite mathematical form, one customarily defines these functions with respect to a fixed point in space. As a consequence, a centre-of-mass motion is introduced and some of the states defined are spurious. This problem is less serious for the effective parameter method, where the single-particle wave functions are defined with respect to the centre of mass of the relatively heavy core (^{32}S in this case). As only a few nucleons are added to the core (up to nine), the centre of mass can be considered as almost at rest. Complete elimination of the centre-of-mass motion would be achieved if the wave functions were defined in the self-consistent field of each nucleus. For a small number of nucleons outside the core this would entail small variations in the parameters as a function of the number of nucleons. These variations have been neglected in the present investigation.

6.2. PREDICTIONS

One may have confidence in the predictions of the very lowest odd-parity states (with predominant seniority $v = 1$ or 2). The predicted spin sequences of these levels generally agree with the experimental ones. However, systematic deviations of the mean energies occur, which indicate the limitations of the model used. In the even, $T = 0$ nuclei ^{36}Ar and ^{40}Ca, the calculated energies are too high; in the odd nuclei ^{36}Cl, ^{38}Cl, ^{38}K and ^{40}K the calculated energies are too low. In the calculation of the 0^+ levels these features are exaggerated. States at higher excitation energies are expected to contain a strong admixture of configurations neglected in the present calculation. The calculation of excitation energies for these levels may be useful, however, for identification of predominant configurations. Generally, configurations with high isospin tend to be purer, which is manifested by the smaller errors in the parameters for isospin 1.

One readily sees from table 1, that for even nuclei the lowest observed and predicted odd-parity states have $J^\pi = 3^-$. This is in accordance with a general semi-empirical rule given by Talmi [19]), which states that the lowest odd-parity states in even nuclei have odd spins, and the lowest even-parity states in even nuclei have even spins. One also sees from table 1 that in the considered odd nuclei the lowest observed or predicted odd-parity states have even spins.

6.3. MATRIX ELEMENTS OF THE EFFECTIVE RESIDUAL INTERACTION

In spite of the observed differences between theoretical and experimental energies, one can try to take the derived two-body matrix elements seriously, to draw some conclusions about the nature of the responsible effective nuclear forces. The interaction matrix elements for one nucleon in the $1d_{\frac{3}{2}}$ shell and one in the $1f_{\frac{7}{2}}$ shell form a large enough set for analysis. The radial exchange integrals are expected to be small, as the two particles are in shells of opposite parities. For central forces the Wigner and Heisenberg part of the interaction consequently is mainly described by even interaction moments (see table 2). This seems to apply to the $T = 0$ matrix elements where the interaction moments $T^{(0)}$ and $T^{(2)}$ are large. Moreover, the ratio between the moments $T^{(2)}$ and $T^{(0)}$ can be obtained with forces of moderate range.

The odd moments should describe the spin exchange and tensor forces. The odd interaction moments are small for the $T = 0$ matrix elements, whereas the moments $T^{(1)}$ and $T^{(3)}$ have a different sign in the $T = 1$ case. An expansion of the scalar Bartlett and Majorana forces yields moments $T^{(1)}$ and $T^{(3)}$ of the same sign and of the same order of magnitude; the same should apply to the moments $T^{(0)}$ and $T^{(2)}$. Consequently, scalar forces are inadequate to describe the resulting interaction moments in this case.

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