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MEASUREMENTS OF (p, γ) RESONANCE STRENGTHS IN THE s-d SHELL

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Abstract: Resonance strengths of selected resonances in the $E_p = 0.3-2.1$ MeV region in the (p, γ) reactions on ^{23}Na, $^{24-26}$Mg, ^{27}Al, $^{28-30}$Si, ^{31}P, ^{32,34}S, ^{35,37}Cl, ^{39,41}K and ^{40}Ca are compared through relative yield measurements, using targets of many different chemical compounds, each containing at least two of the investigated isotopes. If in a (N, Z) diagram lines are drawn between isotopes connected in this way, one obtains several closed cycles, providing internal checks on the measured strength ratios. The final best values of the relative strengths are obtained by least-squares analysis. The $E_p = 621$ keV ^{30}Si(p, γ)^{31}P resonance of which the strength is known from a γ-ray resonant absorption experiment, was used to convert the relative into absolute strengths.

E NUCLEAR REACTIONS ^{23}Na, ^{24,25,26}Mg, ^{27}Al, ^{28,29,30}Si, ^{31}P, ^{32,34}S, ^{35,37}Cl, ^{39,41}K, ^{40}Ca(p, γ), $E = 0.3-2.1$ MeV; measured $\sigma(E_p)$. Deduced resonance strengths. Natural targets of different chemical compounds.

1. Introduction

The knowledge of (p, γ) resonance strengths is a prerequisite for the interpretation of most (p, γ) experiments. The resonance strength is defined here as $S = (2J+1)\Gamma_p\Gamma_\gamma/\Gamma$, where J is the resonance spin and Γ_p, Γ_γ and Γ are the partial widths for proton and γ-ray emission and the total width, respectively.

Absolute resonance strengths are notoriously difficult to measure. Cases are known in the literature where the values given by different authors differ by several orders of magnitude. Resonance strengths are usually derived from the steps in the thick target yield curve [1]). It requires a knowledge of the isotopic constitution of the target and its stopping power, of the proton charge deposited on the target during the measurement, and of the γ-ray detector efficiency. Most frequently, errors probably originate from insufficient knowledge of the target isotopic constitution. During evaporation *in vacuo* of the target material from a tantalum boat onto the target backing, the material may decompose, crystal water escape, tantalum evaporate along with the target material, etc. Contaminants may settle on the target either during evaporation or in the actual measurement. But also the measurement of the proton current may be

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in error if secondary electron emission has been insufficiently taken into account.

Relative measurements of resonance strengths are easier in many respects. Crystal water and contaminants evidently can cause no difficulties. The necessity for the knowledge of the stopping power drops out if one uses thin targets and measures the ratio of the areas under the two resonance peaks [1]). The detector solid angle is irrelevant. Other quantities, like secondary electron emission and the detector efficiency per unit solid angle, only enter "in second order" because of differences in proton energy and in γ-ray spectrum at the two resonances in question.

In the present paper such relative strength measurements are described in which use is made of targets of many different chemical compounds, covering the elements Na, Mg, Al, Si, P, S, Cl, K and Ca, all of natural isotopic constitution. As an example, by using a NaCl target one obtains the strength ratio of selected Na and Cl resonances. By using NaCl, KCl and K_2SO_4 targets, one can compute the Na/S strength ratio, which can be checked by a direct measurement on Na_2SO_4. Such closed cycles serve, above all, to make sure that the compounds in question have not decomposed during the preparation of the targets. As a consequence, the system of relations connecting the strengths is overdetermined, such that the strengths have to be obtained from a least-squares analysis which can be linearized by using as unknowns the logarithms of the strengths rather than the strengths themselves.

Of the selected resonances there is one of which the absolute strength is known from a completely independent measurement. This is the $E_p = 621$ keV $^{30}Si(p, \gamma)^{31}P$ resonance of which the absolute strength has been accurately determined in a γ-ray resonant absorption experiment [2]), a method which obviates the many difficulties mentioned in the beginning of this section. This resonance served to convert the relative strengths obtained as explained above, into absolute strengths. The experimental errors in the final results are of the order of 15 %.

2. Experimental Procedure

All elements in the $Z = 10$–20 region were selected for the present experiment, with the exception of the noble gases neon and argon. Maybe one could have considered the use of a gas cell filled with a mixture of a noble gas and e.g. H_2S, but this was not done. The strength ratios of selected resonances of different isotopes of one element can, of course, be found using any target containing the element in question. Elements were only used of natural isotopic constitution. This eliminates from the measurements the isotopes with a very small isotopic abundance (^{33}S and ^{36}S, ^{40}K, and the Ca isotopes with $A > 40$), but it has the advantage that the isotopic abundances are known with great precision.

Resonances were selected which were strong, sufficiently isolated in energy from other resonances, and which had a known γ-decay (see the references in sect. 3). All resonances were in the $E_p = 0.3$–2.1 MeV region, which could be covered with the Utrecht 850 keV Cockcroft-Walton generator and the 3 MeV Van de Graaff generator.

The gamma radiation was detected with a cylindrical 10 cm × 10 cm NaI scintillation crystal at a front-face-to-target distance of 40 mm. The counter was put at an angle of 55° to the proton beam to eliminate $P_2(\cos\theta)$ angular distribution effects. For all selected resonances it was known that $P_4(\cos\theta)$ terms in the angular distribution are small or absent, except for that in ^{40}Ca.

Most targets were prepared by evaporation *in vacuo* onto 0.3 mm tantalum backings. The chemical compounds which were used are listed in sect. 3. The main criterion for their selection was that the material should not decompose during evaporation. Always several targets of different thickness were prepared. About half of the targets could be regarded as "thick", the other half as "thin".

The strength ratios measured with targets of different thickness and prepared under different evaporation conditions did not show differences outside the combined experimental error. Such measurements serve as another check (next to the check, mentioned above) on the non-decomposition of the target material during evaporation.

For Na_2SiO_3, the evaporation procedure proved unsuitable. Of this compound targets were prepared by painting a thin layer of the powdered material, mixed with water, onto the target backing. Such targets are relatively thick such that only steps in the yield curve could be compared.

To avoid deterioration of the (water-cooled) targets under proton bombardment, the beam power was always kept below 3 W. A good check on target deterioration was to end a series of measurements on different resonances by repeating the measurement on the first resonance.

The proton charge on the target was measured with a calibrated current integrator. Secondary emission effects were prevented by the use of a suppressor ring at negative potential between the target and the beam-defining diaphragm.

For the thick targets, the strength ratio is evaluated from the measurements

$$S_1/S_2 = B_1/B_2,$$

where the quantities B (with index omitted) are given by

$$B = E(2I+1)\left(\frac{A}{A+1}\right)^2 \frac{\varepsilon}{f}\frac{N_\gamma}{K},$$

where E is the proton energy in the laboratory system,
I and A the spin and mass number of the initial nucleus, respectively,
$\varepsilon = (1/n)\,\mathrm{d}E/\mathrm{d}x$ the target stopping power per molecule,
n the number of target molecules per cm^3,
f the number of initial nuclei per target molecule,
K the partial γ-ray detection efficiency and
N_γ the step in the γ-ray yield curve at the resonance.

For a mono-energetic γ-ray, the partial detection efficiency K is defined as the probability that a γ-quantum emitted from the target gives rise to a pulse with the correct amplitude to be transmitted by the window of the differential discriminator. For a complex γ-ray spectrum, K is obtained by adding the partial detection efficiencies of all lines in the spectrum, multiplied with their relative intensities. The computation of K includes corrections for γ-ray absorption in the target backing and target holder, and in the aluminium cover of the scintillation crystal.

For thin targets an analogous formula holds, $S_1/S_2 = B'_1/B'_2$, with

$$B' = E^{\frac{1}{2}}(2I+1)\left(\frac{A}{A+1}\right)^2 \frac{1}{f}\frac{A_\gamma}{K},$$

in which A_γ is the area of the resonance peak, with the counting rate plotted as a function of the proton resonance fluxmeter frequency.

Of course, background has to be suitably subtracted. Sometimes, tails of preceding resonances have to be subtracted, for which purpose it is useful to measure the shape of an isolated resonance in the neighbourhood.

Corrections were applied for coincident and random summing effects [3]).

3. Results

In figs. 1 and 2, representative examples are shown of yield curves measured with a thin and a thick target, respectively.

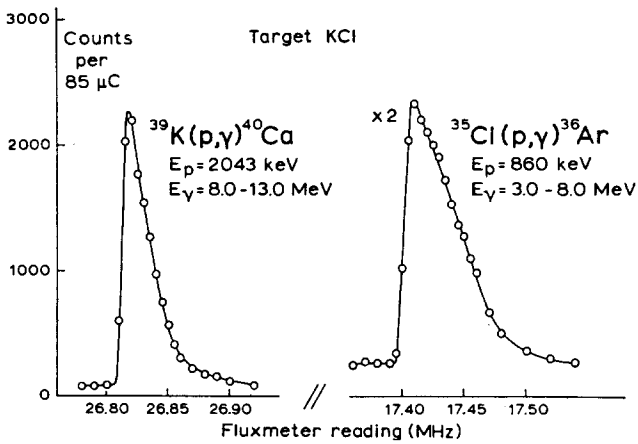

Fig. 1. Yield curves of the $E_p = 860$ keV ^{35}Cl(p, γ)^{36}Ar and the $E_p = 2043$ keV ^{39}K(p, γ)^{40}Ca resonances as measured with a thin target.

The measured ratios are listed in table 1. They are all averages of measurements with targets of different thickness. Estimated contributions for the insufficient knowledge of the ratio of the partial efficiencies and (for thick targets) of the stopping

power [35]) at the different resonant energies, were added quadratically to the statistical error in the average, resulting from counting statistics (the internal or external error was taken, whichever was the largest). Also errors resulting from background subtraction were taken into account.

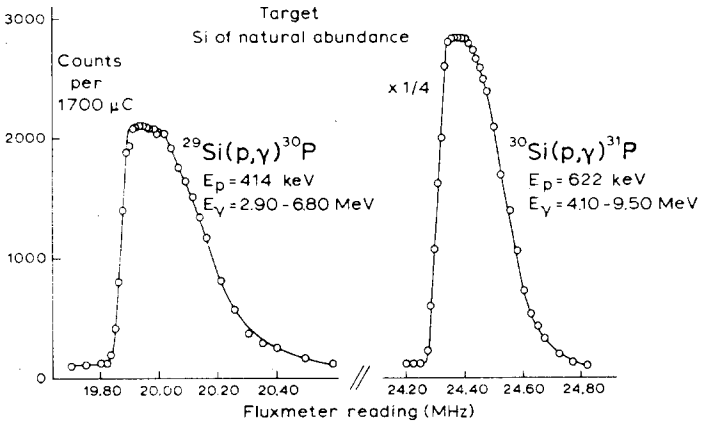

Fig. 2. Yield curves of the $E_p = 414$ keV ^{29}Si(p, γ)^{30}P and the $E_p = 621$ keV ^{30}Si(p, γ)^{31}P resonances as measured with a thick target.

TABLE 1

Measured (p, γ) strength ratios as obtained from measurements with different chemical compounds

1.	Na_2SiO_3	$S(^{23}Na)/S(^{29}Si)$	=	1.50 ± 0.15
2.	$Na_4P_2O_7$	$S(^{23}Na)/S(^{31}P)$	=	2.22 ± 0.22
3.	$Na_2S_2O_7$	$S(^{23}Na)/S(^{32}S)$	=	6.6 ± 0.7
4.	NaCl	$S(^{35}Cl)/S(^{23}Na)$	=	4.5 ± 0.5
5.	MgO	$S(^{26}Mg)/S(^{25}Mg)$	=	1.37 ± 0.14
	MgO	$S(^{26}Mg)/S(^{24}Mg)$	=	1.77 ± 0.18
6.	$MgSO_4$	$S(^{26}Mg)/S(^{32}S)$	=	6.6 ± 0.7
7.	$MgCl_2$	$S(^{35}Cl)/S(^{26}Mg)$	=	5.4 ± 0.5
8.	Al_2Cl_6	$S(^{27}Al)/S(^{35}Cl)$	=	1.08 ± 0.11
9.	Si	$S(^{30}Si)/S(^{28}Si)$	=	920 ±90
	Si	$S(^{30}Si)/S(^{29}Si)$	=	4.45 ± 0.45
10.	P_4S_6	$S(^{31}P)/S(^{32}S)$	=	3.80 ± 0.38
11.	$Ca_3(PO_4)_2$	$S(^{31}P)/S(^{40}Ca)$	=	2.16 ± 0.22
12.	ZnS	$S(^{34}S)/S(^{32}S)$	=	150 ±15
13.	K_2SO_4	$S(^{39}K)/S(^{32}S)$	=	230 ±50
	K_2SO_4	$S(^{39}K)/S(^{41}K)$	=	3.08 ± 0.31
14.	$CaSO_4$	$S(^{40}Ca)/S(^{32}S)$	=	2.02 ± 0.20
15.	$BaCl_2$ and $CoCl_2$ (averaged)	$S(^{35}Cl)/S(^{37}Cl)$	=	4.67 ± 0.47
16.	KCl	$S(^{39}K)/S(^{35}Cl)$	=	6.2 ± 0.6

Energies of the selected resonances are given in table 2.

The complicated system of links between different isotopes from table 1 is illustrated in fig. 3.

In addition to the measurements listed in table 1, the strength of the $E_p = 1966$ keV ^{26}Mg(p, γ)^{27}Al resonance was also determined, relative to the resonance at $E_p = 454$ keV in the same reaction. The result is $S(1966)/S(454) = 6.13 \pm 0.61$. The reason for this measurement will be explained in sect. 4.

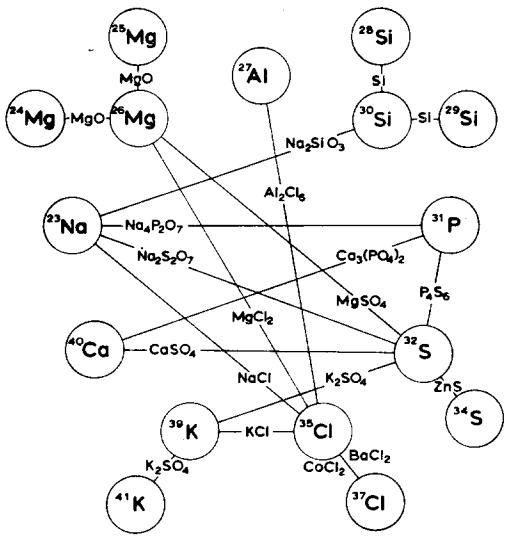

Fig. 3. Diagram showing the links between resonance strengths established by relative yield measurements using the chemical compounds indicated.

As seen in fig. 3, the system relating the strengths S_k in ^{23}Na, ^{24}Mg, ^{31}P, ^{32}S, ^{35}Cl, ^{39}K and ^{40}Ca is overdetermined. Because only strength ratios $\alpha_{kl} = S_k/S_l$ are measured, one may put one of the strengths, say that of ^{23}Na, equal to unity. One then is left with ten equations in six unknowns. The "best values" of the strengths are found by using linear least-squares analysis, minimizing the quantity

$$Q^2 = \sum_{k,l} W_{kl}(\ln S_k - \ln S_l - \ln \alpha_{kl})^2.$$

The $W_{kl} = (\alpha_{kl}/\delta\alpha_{kl})^2$ are the appropriate weights, with $\delta\alpha_{kl}$ indicating the error in α_{kl}, as given in table 1.

The normalized χ^2 values, defined as $Q^2_{\min}/(M-N)$, where $(M-N)$ is the number of free parameters, amounted to $\chi^2 = 1.07$. The fact that this value is nearly unity indicates that the errors in table 1 where neither under- nor overestimated, proving, in addition, that cycles really close nicely. From the best values in this primary system, the relative strengths of the secondary isotopes ^{24,25}Mg, ^{27}Al, $^{28-30}$Si, ^{34}S, ^{37}Cl and ^{41}K, are derived in a straightforward way.

The relative strengths thus obtained, converted into absolute strengths by normalizing on the value $(2J+1)\Gamma_p\Gamma_\gamma/\Gamma = 3.10\pm0.26$ eV for the $E_p = 621$ keV ^{30}Si(p, γ)^{31}P standard resonance, are listed in table 2. The yield for the standard resonance is found by combining the results from a γ-ray absorption experiment [2]), $\Gamma_{\gamma_0} = 1.52\pm0.12$ eV, $\Gamma = 40\pm7$ eV, with those given in ref. [4]), $J = \frac{1}{2}$, $\Gamma_{\gamma_0}/\Gamma_\gamma = 0.94\pm0.02$.

TABLE 2

Absolute strengths of sixteen selected (p, γ) resonances

Reaction	E_p (keV)	$(2J+1)\Gamma_p\Gamma_\gamma/\Gamma$ (eV) present work	other work	Refs. to γ decay	Refs. to angular distribution
^{23}Na(p, γ)^{24}Mg	512	1.05 ± 0.16	1.80 ± 0.36	[5, 6])	[6])
			0.45 ± 0.09	[7])	
^{24}Mg(p, γ)^{25}Al	823	0.52 ± 0.08	0.44 ± 0.08	[8])	[8])
^{25}Mg(p, γ)^{26}Al	435	0.67 ± 0.10	1.5 ± 0.7	[9])	[10])
			0.69 ± 0.10	[10])	
^{26}Mg(p, γ)^{27}Al	454 [a])	0.92 ± 0.14	2.8 ± 0.6	[11])	[34])
^{26}Mg(p, γ)^{27}Al	1966 [a])	5.7 ± 0.8	5.6 ± 1.8	[12])	[12])
^{27}Al(p, γ)^{28}Si	633	5.3 ± 0.8	5.4 ± 1.8	[13–15])	[13])
^{28}Si(p, γ)^{29}P	369	$(3.4\pm0.5)\times10^{-3}$	$(4.7\pm0.8)\times10^{-3}$	[16])	[16])
			$(4.8\pm0.6)\times10^{-3}$	[17])	
^{29}Si(p, γ)^{30}P	416	0.70 ± 0.10	0.23 ± 0.06	[18])	[18])
^{30}Si(p, γ)^{31}P	621	3.10 ± 0.26 [b])		[2, 4])	[4])
^{31}P(p, γ)^{32}S	642	0.52 ± 0.08	0.23	[19])	[19])
			0.36 ± 0.18	[20])	
^{32}S(p, γ)^{33}Cl	588	0.14 ± 0.02	0.10 ± 0.04	[21])	[21])
			0.06 ± 0.02	[22])	
^{34}S(p, γ)^{35}Cl	1214	21 ± 3	0.8 ± 0.1	[23])	[24])
			4.0 ± 1.2	[24])	
^{35}Cl(p, γ)^{36}Ar	860	4.9 ± 0.7	9.6	[25])	[27])
			$0.10^{+0.10}_{-0.05}$	[26])	
			5 ± 2	[27])	
^{37}Cl(p, γ)^{38}Ar	847	1.04 ± 0.15	$0.02^{+0.02}_{-0.01}$	[26])	[26])
^{39}K(p, γ)^{40}Ca	2043	31 ± 5	11.8 ± 1.7	[28])	[29])
			26 ± 10	[29])	
^{41}K(p, γ)^{42}Ca	1111	10.0 ± 1.5	8.5 ± 4.2	[30])	[30]) [d])
^{40}Ca(p, γ)^{41}Sc	1842	0.26 ± 0.04	$0.30^{+0.30}_{-0.15}$	[31]) [c])	[31])

[a]) All strength comparisons with other isotopes were done using the 454 keV resonance. For the 1966 keV resonance, see text.

[b]) This value is used as a standard for normalization of the relative strengths obtained in this paper.

[c]) Computed from the integrated cross section $\int\sigma dE = 0.3$ eV · b as given in ref. [31]).

[d]) From the spectrum given in ref. [30]), the γ_0 branching is deduced as 68 %.

The computer program for the least-squares analysis also gave the errors in the relative strengths. The errors in the absolute strengths, as listed in table 2, are then

found by adding to the relative error quadratically the error (8.4 %) in the strength of the standard resonance, and an estimated error (7 %) for insufficient knowledge of the γ-ray spectrum.

4. Discussion

It is seen in table 2 that in many cases the present results and those given in the literature agree to within the combined experimental errors. Notably, we are quite happy about the good agreement for ^{27}Al(p, γ)^{28}Si, because the measurements described in refs. [13, 14]) were done with great care, and because fresh Al targets are expected to be almost completely free of contaminants.

There is another group of cases where the agreement is to within a factor of 2 or 3. If the literature value is on the low side, the discrepancy might, e.g., have been caused by the target having been more oxidized than suspected by the experimenter.

Much more serious disagreement, up to a factor of 50, is found for the values reported in refs. [23, 24, 26]).

Strong support for the present system of strengths is obtained from a recent γ-ray resonant absorption experiment [12]) at the 1966 keV ^{26}Mg(p, γ)^{27}Al resonance. This resonance is known to be broad, $\Gamma_p^2/\Gamma = 21 \pm 5$ eV, from a ^{26}Mg(p, p)^{26}Mg elastic scattering experiment [32]). From these two measurements the resonance strengths can be computed as $S = 5.6 \pm 1.8$ eV, in beautiful agreement with the present result, $S = 5.7 \pm 0.8$ eV. The strengths given in ref. [36]) for all ^{26}Mg(p, γ)^{27}Al resonances in the $E_p = 0.3-3.0$ MeV region [11, 36]) were too large by a factor 3.0.

In principle, the $E_p = 774$ keV ^{27}Al(p, γ)^{28}Si and the $E_p = 2043$ keV ^{39}K(p, γ)^{40}Ca resonances, at which γ-ray absorption was also observed [33, 28]) would be suitable for similar checks. At these resonances, however, the additional necessary information on Γ_p or Γ is still lacking. The best way to obtain this information would be from a high-resolution proton elastic scattering experiment.

The present work has provided a set of resonances with known strength which can be used to obtain the strength of any other (p, γ) resonance on the same isotopes through relative measurements. Most of the resonances in table 2 are at rather low proton energies, $E_p < 1$ MeV, and it might be useful to establish a set of secondary standards in the $E_p = 1-3$ MeV region, a region more accessible to most Van de Graaff generators.

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