

HIGH RESOLUTION INVESTIGATION OF THE $^{26}\text{Mg}(p,p)^{26}\text{Mg}$ REACTION

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Synopsis

Differential cross sections for elastic scattering of protons on ^{26}Mg in the $E_p = 1\text{--}2$ MeV region, were measured with surface barrier counters at four angles. In this energy range 42 $^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$ resonances are known. Of 19 (p, γ) resonances the corresponding $^{26}\text{Mg}(p, p)^{26}\text{Mg}$ resonance was also observed; three (p, p) resonances were found which had no (p, γ) counterpart. The measured widths range from 6 keV down to 6 eV. The spins of 14 of these resonances and the proton orbital momenta of 20 resonances could be determined unambiguously. Partial widths for proton and γ -ray emission are computed from the observed widths and the (p, γ) yields. Upper limits are given for the widths of unobserved resonances.

1. *Introduction.* The investigation of elastic proton scattering on even-even nuclei is an ideal tool for the determination of resonance parities and partial proton widths. The results of this reaction in combination with those of the (p, γ) reaction on the same nucleus, lead to unique J^π determinations of the resonance levels. These determinations can be useful in the interpretation of $\gamma\text{--}\gamma$ angular correlation measurements for the determination of the J^π values of lower levels. The reduced proton widths, calculated from the partial proton widths, are important in the identification of analogue levels. From the partial proton width of a resonance together with its (p, γ) yield, the total width and the radiation width can be determined.

In a preceding article the results of an investigation of elastic proton scattering on ^{30}Si have been published¹⁾. This paper presents the results of an investigation of the $^{26}\text{Mg}(p, p)^{26}\text{Mg}$ reaction with the same experimental setup. Data from the $^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$ reaction^{2) 3)} were used. In the region $E_p = 1\text{--}2$ MeV, 42 resonances were observed in this reaction; the spins of several resonances were determined³⁾. Little information was obtained about the resonance parities.

Other $^{26}\text{Mg}(p, p)^{26}\text{Mg}$ work has been reported in refs. 4 and 5. The first investigation⁴⁾ was done with a considerably lower energy resolution than the second one⁵⁾; eight resonances were indicated in the energy region $E_p = 1.5\text{--}2$ MeV; only two of these were analysed, i.e. those at $E_p = 1883$

and 1900 keV. In ref. 5 about 15 resonances were listed in the energy region $E_p = 1\text{--}2$ MeV, of which nine resonances were analysed.

The high resolution in the present investigation is due to the small energy spread and good stability of the Utrecht 3MV HVEC Van de Graaff generator, and to the use of thin targets. A helpful factor is the knowledge of the (p, γ) resonance energies, with errors between 1.0 and 2.5 keV²). This makes it possible to concentrate the search for (p, p) resonances to narrow energy regions, which can be investigated in small energy steps of e.g. 0.1 keV. The simultaneous measurement of proton and γ -ray yields makes it possible to see whether or not an observed (p, p) resonance corresponds to a known (p, γ) resonance, to within 0.1 keV for narrow resonances.

The measurements lead to the determination of the parities of 20 and the spins of 14 of the $^{26}\text{Mg}(p, p)^{26}\text{Mg}$ resonances in the $E_p = 1\text{--}2$ MeV region.

Experimental details are given in section 2, the theoretical and computational analysis is presented in section 3, and the final results and conclusions are to be found in sections 4 and 5.

2. Experimental. Protons were magnetically deflected over 90° before hitting the ^{26}Mg target. The magnetic field was measured with a magnetic-resonance flux meter and this served as a measure of the particle energy. An absolute calibration of the incident proton energy could be obtained by comparison of the measured (p, γ) resonance energies with those reported in ref. 2. Targets were prepared by evaporation in vacuo of Mg after reduction of MgO enriched in ^{26}Mg (99.9%), from a tantalum boat onto a 10 $\mu\text{g}/\text{cm}^2$ carbon foil, obtained from the Yissum Research Dev. Cy., Israel. The target thickness was about 4 $\mu\text{g}/\text{cm}^2$, corresponding to an energy loss of 600 eV for 1.5 MeV protons. The targets can withstand a proton current of about 1 μA , but lower currents had to be used to avoid deformation of the pulse-height spectra by pile-up.

Elastically scattered protons were detected with four surface barrier

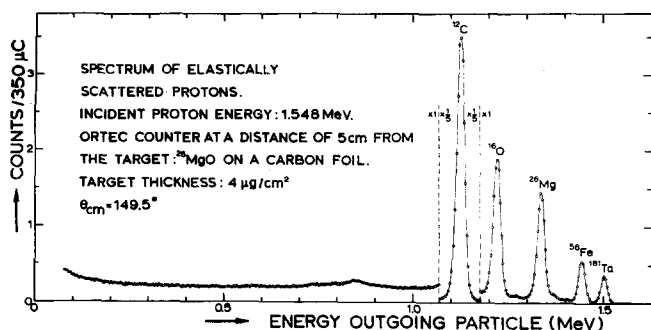


Fig. 1. Example of a pulse-height spectrum of elastically scattered protons.

counters at angles of 90.0° , 125.3° , 140.8° and 149.5° (c.m.) relative to the beam direction, and at a distance of 5 cm from the target. The four ORTEC detectors, each with a 25 mm^2 sensitive area, have an energy resolution of 20 keV. An example of a pulse-height spectrum is shown in fig. 1. The strong groups result from elastic scattering on ^{12}C , ^{16}O , ^{26}Mg , ^{56}Fe and ^{181}Ta . The iron and tantalum contaminations are due to the high temperature necessary to reduce the MgO during the preparation of the target.

A more extensive description of the experimental arrangement has been given in ref. 1.

3. *Analysis of the data.* For single resonances the method of analysis is nearly the same as mentioned in ref. 1. In comparison with the ^{30}Si case, the contribution from other isotopes than the one under investigation is smaller, because the enrichment of the target material is much higher.

The total proton yield outside a resonance can be described by the Rutherford contribution $CE_p^{-2}(\sin \theta_i/2)^{-4}$. The constant C only depends on the target thickness and the solid angle of the detector, if all yield curves are measured under the same circumstances. For each target, C has been determined by averaging over all the yield curves. The difference between this average Rutherford contribution and the measured nonresonant proton yield is considered as background; the background amounted to at most 10% of the nonresonant yield.

The method outlined above was extended for the case that two or more resonances interfere. This extension is based on the theory of Blatt and Biedenharn for the scattering cross section at a single resonance⁶). The contribution from hard-sphere scattering has been computed; it proved to be negligible. The scattering amplitude $f(\theta)$, related to the differential cross section by $d\sigma/d\Omega = |f(\theta)|^2$, can then be written, in the case of two interfering resonances, as

$$f(\theta) = f_C(\theta) + f_{R1}(\theta) + f_{R2}(\theta),$$

where $f_C(\theta)$ denotes the Rutherford scattering amplitude and f_{R1} and f_{R2} the resonance scattering terms. The notation is the same as in ref. 6.

The differential cross section can be written in four parts

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & [|f_C|^2] + [|f_{R1}|^2 + 2RP(f_C^* \cdot f_{R1})] + \\ & + [|f_{R2}|^2 + 2RP(f_C^* \cdot f_{R2})] + [2RP(f_{R1}^* \cdot f_{R2})]. \end{aligned}$$

Here RP stands for the real part of the expression in parentheses. The first term gives the potential scattering, the second and third term the pure resonance scattering and the interference between potential and resonance scattering for the first and second resonance, respectively, and the last term

the interference between the two resonance scattering amplitudes. For the case of one resonance only, the first two terms remain. The interference term for an even-even target nucleus can be written as

$$2RP(f_{R1}^* \cdot f_{R2}) = \frac{\lambda^2}{8} (2J_1 + 1)(2J_2 + 1) \cdot \frac{\Gamma_1 \cdot \Gamma_2}{\left[(E - E_1)^2 + \left(\frac{\Gamma_1}{2} \right)^2 \right]^{\frac{1}{2}} \left[(E - E_2)^2 + \left(\frac{\Gamma_2}{2} \right)^2 \right]^{\frac{1}{2}}} \times \cos(2\psi_{l_2} - 2\psi_{l_1} + 2\phi_{l_2} - 2\phi_{l_1} + \beta_2 - \beta_1) \cdot P_{l_1}(\cos \theta) \cdot P_{l_2}(\cos \theta).$$

In this expression the spins, the resonance energies, the level widths and the orbital momenta for the first and the second resonance are given by J_1 , E_1 , Γ_1 , l_1 and J_2 , E_2 , Γ_2 , l_2 , respectively; λ denotes the deBroglie wavelength, divided by 2π , of the incident particles; ψ_l , ϕ_l and β are the phase shifts as defined in ref. 6.

The dependence of the interference term on $P_{l_1}(\cos \theta) \cdot P_{l_2}(\cos \theta)$ facilitates

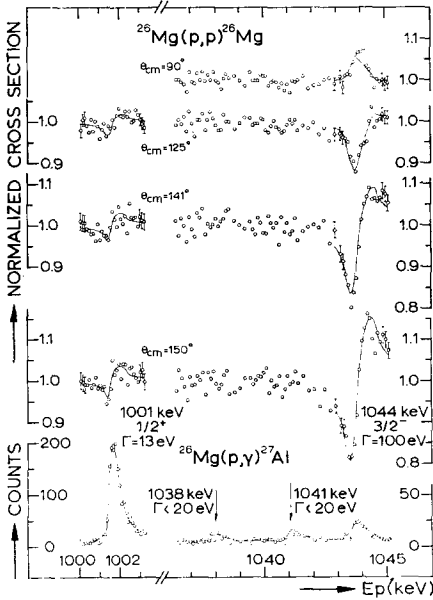


Fig. 2. Differential cross section for elastic proton scattering on ^{26}Mg with background subtracted, in units of the Rutherford contribution, at the $E_p = 1001$ and 1044 keV resonances. The drawn lines are the theoretical curves for the indicated J^π and Γ . The bottom curve gives the $^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$ yield.

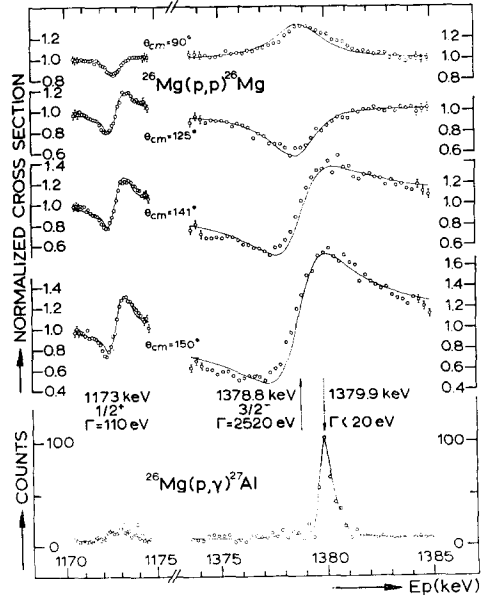


Fig. 3. The $E_p = 1173$ and 1378 keV resonances; for details, see caption of fig. 2.

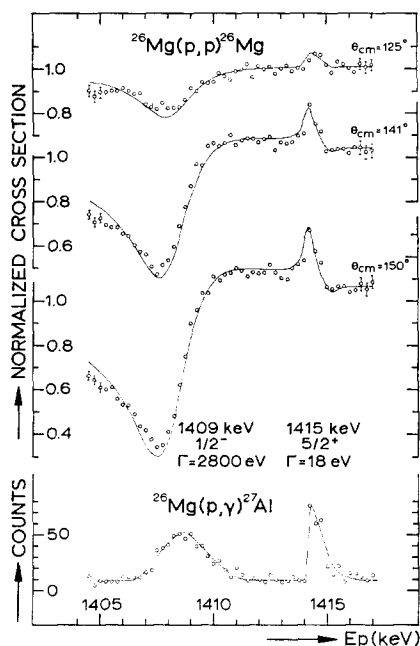


Fig. 4. The $E_p = 1409$ and 1415 keV resonances; for details, see caption of fig. 2.

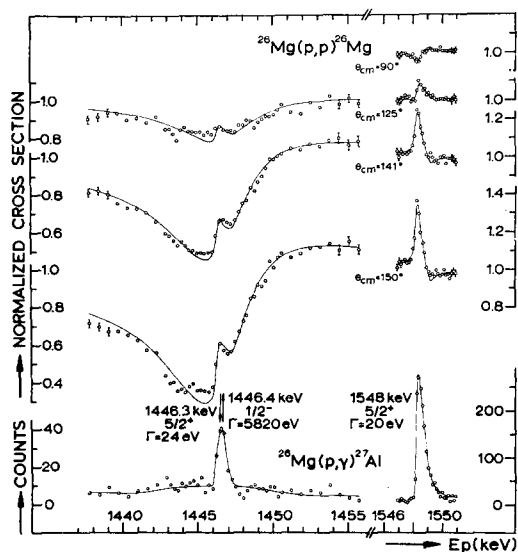


Fig. 5. The $E_p = 1446.3$, 1446.4 and 1548 keV resonances; for details, see caption of fig. 2.

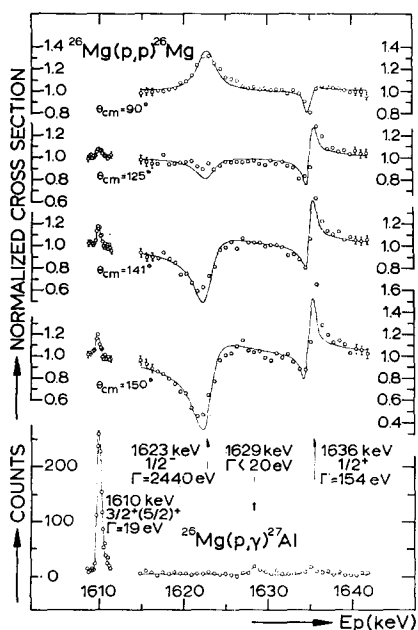


Fig. 6. The $E_p = 1610$, 1623 and 1636 keV resonances; for details, see caption of fig. 2.

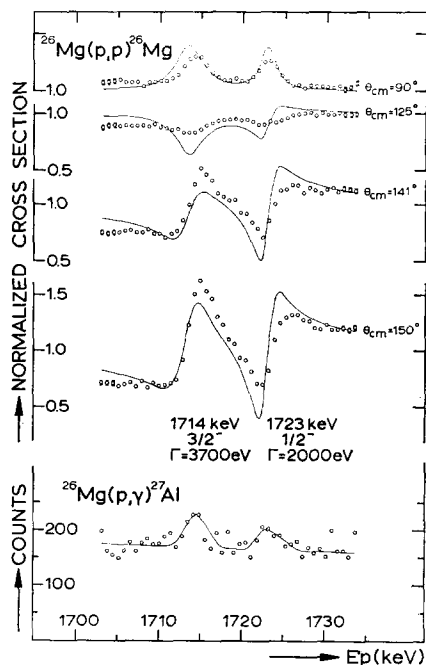


Fig. 7. The $E_p = 1714$ and 1723 keV resonances; for details, see caption of fig. 2.

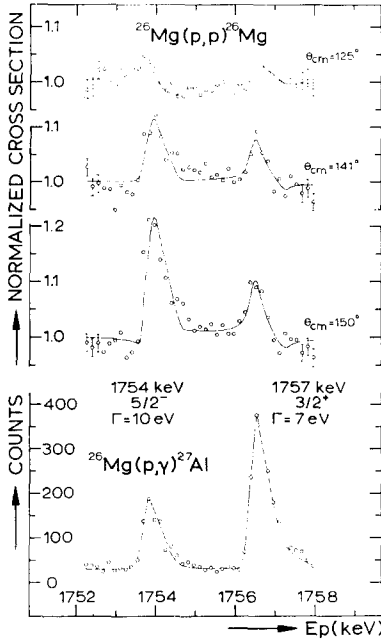


Fig. 8. The $E_p = 1754$ and 1757 keV resonances; for details, see caption of fig. 2.

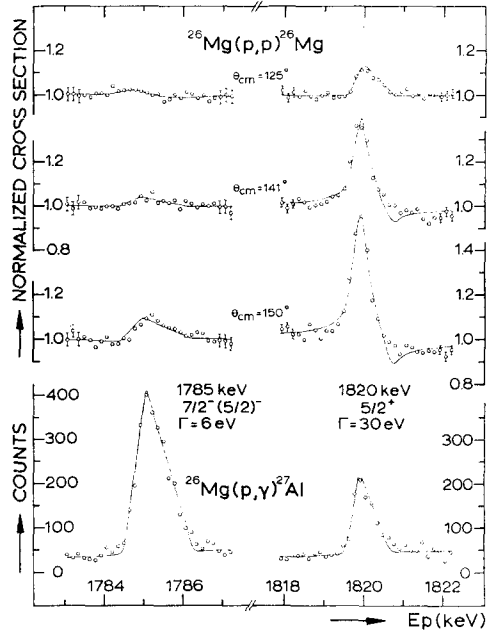


Fig. 9. The $E_p = 1785$ and 1820 keV resonances; for details, see caption of fig. 2.

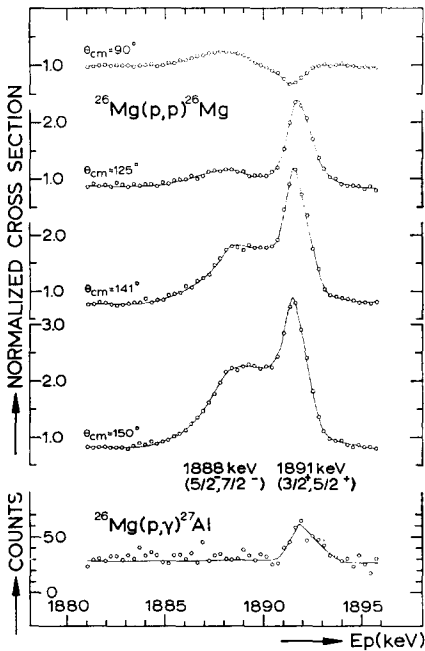


Fig. 10. The $E_p = 1888$ and 1891 keV resonances. Smooth lines have been drawn through the experimental data as a guidance for the eye.

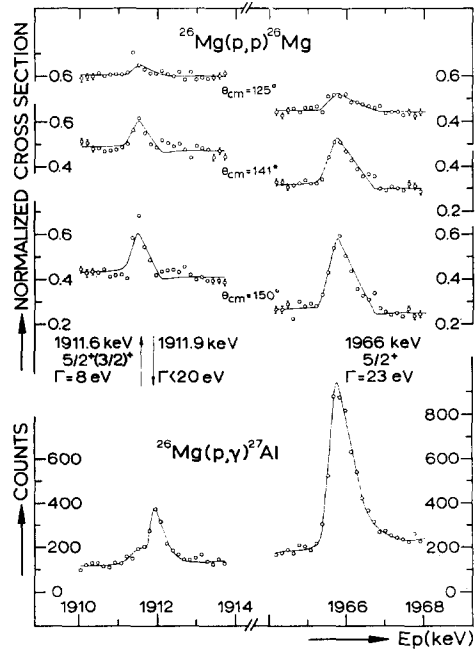


Fig. 11. The $E_p = 1912$ and 1966 keV resonances; for details, see caption of fig. 2.

the analysis, because at the angles θ_i which are zero's of the Legendre polynomial $P_{l_i}(\cos \theta_i)$ one only gets the sum of the two resonance contributions.

The extension to more than two interfering resonances can also easily be worked out.

The computed differential cross sections have to be compared to the experimental yield curves. With a computer a χ^2 value, χ_i^2 , is calculated for each angle θ_i , which is minimized by variation of the parameters E_0 , the resonance energy, Γ , a_i and b_i , where the constants a_i and b_i determine the background $B_i(E_p) = a_i E_p + b_i$.

A second program determines the minimum χ^2 for all contributing angles together, by variation of E_0 and Γ . The constants a_i and b_i are now known from the preceding fitting process.

The minimum values of χ^2 , obtained for the two possible J values, are compared to the 0.1% probability limit. The calculations were carried out in Utrecht and Amsterdam with EL-X8 computers and in Delft with a Telefunken TR4.

4. *Results.* The yield curves of protons elastically scattered from ^{26}Mg , measured at three or four angles, are shown in figs. 2-11. They are plotted in units of the Rutherford contribution; the background has been subtracted. The smooth curves are the results of the analysis described in the preceding section (except for fig. 10). The bottom curve gives the simultaneously measured γ -ray yield.

For 19 of the 42 (p, γ) resonances known from ref. 2, a corresponding (p, p) resonance has been found. Three resonances were observed, at $E_p = 1378.8, 1446.4$ and 1911.6 keV, which had no known (p, γ) counterpart.

At $E_p = 1378.8$ keV the proton yield curve shows a broad resonance ($\Gamma = 2.5$ keV, see fig. 3), which cannot be identified with the narrow $E_p = 1379.9$ keV (p, γ) resonance ($\Gamma \leq 0.02$ keV).

The $E_p = 1446$ keV (p, γ) resonance was reported as a possible doublet²⁾. This assumption is confirmed by the present experiment (see fig. 5). Elastic scattering resonances are observed at $E_p = 1446.3$ and 1446.4 keV; the measured widths show that the first resonance corresponds to the known (p, γ) resonance.

A small but significant difference in the resonance energies indicates that the $E_p = 1911.9$ keV (p, γ) resonance corresponds to another level than the (p, p) resonance at $E_p = 1911.6$ keV.

From the shapes of the yield curves of the elastically scattered protons, it is possible to determine immediately the spin and parity of a resonance level if $J = 1/2^+, 1/2^-$ or $3/2^-$. For higher orbital momenta a unique determination of the spin is in most cases impossible. At some resonances the 0.1% probability limit excludes one of the two possible J values. In three

cases a most probable spin could be indicated, namely for the resonances at $E_p = 1610, 1754$ and 1911.6 keV, where a factor of likelihood was calculated which exceeded 2. The other possibility for the resonance spin, however, could not be excluded in these cases. No conclusion could be drawn about the spins of the resonances at $E_p = 1757, 1785, 1888, 1891$ and 1966 keV.

In the analysis of the elastic scattering differential cross sections, the theoretical cross section must be folded into an instrumental resolution function¹⁾. This resolution function was approximated by a triangle. In some cases the assumed triangle shape of the resolution function is rather poor and causes some differences of the theoretical curve with the experimental data, especially in the high-energy tail; see e.g. the $E_p = 1754$ and 1820 keV resonances (figs. 8 and 9). However, a determination of Γ_p^2/Γ by a numerical folding of the theoretical cross section into the observed resolution function, proved that this value was equal, within the error, to the value obtained using the triangle shape.

A broad anomaly in the elastic proton yield at about 2 MeV has been observed and analysed in ref. 5 as a doublet with resonance energies at $E_p = 2025$ and 2050 keV and with widths $\Gamma = 40$ and 70 keV, respectively. This doublet interferes with resonances at $E_p = 1888, 1891, 1912$ and 1966 keV. The interference theory, as outlined in section 3, has been used in the analysis of these four resonances.

The doublet at $E_p = 1888$ and 1891 keV (fig. 10), however, could not be fitted with any spin combination using orbital momenta $l < 4$. Capture of $l \geq 4$ protons can be excluded since it would lead to reduced proton widths exceeding the Wigner limit. The shapes of the resonances at the four angles suggest $l = 3$ and 2 for the $E_p = 1888$ and 1891 keV resonances, respectively, but the fit is extremely poor. The conclusion must be that the structure around $E_p = 1.89$ MeV is caused by a triplet rather than by a doublet of levels.

5. Conclusions. The final conclusions on spins, parities and widths are presented in table I together with the data obtained from other (p, p) work⁵⁾ and from the (p, γ) work²⁾ ³⁾. The agreement of the nine analysed resonances of ref. 5 with the results of this experiment is very good; also the spins and parities determined at five (p, γ) resonances³⁾ agree very well with this work.

A comparison of the (p, γ) resonance strengths given in ref. 2 (as corrected in refs. 7 and 8) with the proton yields reported here leads to the conclusion that in most cases $\Gamma_\gamma \ll \Gamma_p$ for the observed (p, p) resonances in the $E_p = 1-2$ MeV region. The total widths thus nearly equal the proton widths and the (p, γ) resonance strengths determine the radiative widths of the resonance levels.

In table I only the values for the total width are presented; the partial radiation width and the transition strength for some resonance levels have been calculated and discussed in ref. 3.

The (p, γ) resonances which do not correspond to elastic proton scattering resonances, have a proton width $\Gamma_p \leq 40/(2J + 1)$ eV.

The reduced proton widths, calculated from the measured proton widths using $R = r_0(A^{\frac{1}{3}} + 1)$ with $r_0 = 1.20$ fm, are given in the fourth column of table I.

TABLE I

E_p (keV)	Results						
	This experiment			$^{26}\text{Mg}(p, p)^{26}\text{Mg}^5)$		$^{26}\text{Mg}(p, \gamma)^{27}\text{Al } ^2) ^3)$	
	J^π	$\Gamma(\text{eV})$	$\theta_p^2 \times 10^2$	J^π	$\Gamma(\text{eV})$	J^π	$\Gamma(\text{eV})$
1001	1/2 ⁺	13 \pm 11	0.02				
1044	3/2 ⁻	100 \pm 30	0.51				
1173	1/2 ⁺	110 \pm 50	0.09	1/2 ⁺	200		
1378.8	3/2 ⁻	2500 \pm 200	3.05	3/2 ⁻	2400		
1409	1/2 ⁻	2760 \pm 140	3.01	1/2 ⁻	2600	(1/2, 3/2)	3000 \pm 600
1415	5/2 ⁺	18 \pm 5	0.18				
1446.3	5/2 ⁺	24 \pm 8	0.21				(doublet?)
1446.4	1/2 ⁻	5820 \pm 100	5.65	1/2 ⁻	6000		
1548	5/2 ⁺	20 \pm 8	0.12	(5/2 ⁺)	10	5/2 ⁽⁺⁾	
1610	3/2 ⁺ (5/2 ⁺)	19 \pm 4 (12 \pm 6)	0.09 (0.06)			3/2 ⁽⁺⁾	
1623	1/2 ⁻	2440 \pm 120	1.42	1/2 ⁻	2000		4000 \pm 1000
1636	1/2 ⁺	150 \pm 60	0.03	1/2 ⁺	200		
1714	3/2 ⁻	3700 \pm 200	1.71	3/2 ⁻	3000		3400 \pm 800
1723	1/2 ⁻	2000 \pm 100	0.92	1/2 ⁻	2600		2600 \pm 700
1754	5/2 ⁻ (7/2 ⁻)	10 \pm 6 (8 \pm 5)	0.53 (0.42)				
1757	(3/2, 5/2) ⁺	7 \pm 5	0.02				
1785	(7/2, 5/2) ⁻	6 \pm ¹² ₆ 8 \pm ¹⁴ ₈	0.28 0.37			7/2	
1820	5/2 ⁺	30 \pm 7	0.08				
1888	(5/2 ⁻ , 7/2 ⁻)	—	—				4800 \pm 1200
1891	(3/2 ⁺ , 5/2 ⁺)	—	—				
1911.6	5/2 ⁺ (3/2 ⁺)	8 \pm 5 (10 \pm 7)	0.02				
1966	(3/2, 5/2) ⁺	23 \pm 5	0.04			5/2 ⁽⁺⁾	

In comparison with the preceding (p, p) experiment on ^{30}Si the resolution has been increased by a factor of 1.5. The narrowest resonance observed in the present experiment has a width of 6 eV; resonances with a smaller width could not be distinguished from background. The increase in the resolution is mainly due to the higher isotopic enrichment of the target (99.9% for ^{26}Mg as against 65% for ^{30}Si).

From the reduced proton widths, given in table I, one can conclude that none of the observed (p, p) resonances is an analogue state of a single-particle level in ^{27}Mg .

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