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RESONATOR Q MODULATION OF GAS LASERS WITH AN EXTERNAL MOVING MIRROR

Th. H. PEEK, P. T. BOLWIJN and C. Th. J. ALKEMADE

Physical Laboratory, State University Utrecht, The Netherlands

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The predicted central tuning dip in the modulated power output of gas lasers was observed by applying resonator Q modulation. The modulation was obtained by means of an external moving mirror. The signal shapes observed are explained for the quasistatic case.

Saturation and gain of single-mode gas lasers can be studied by analysis of the modulated power output [1-3]. For instance, we predicted in the a.c. power output a central tuning dip which reveals the saturation more clearly than the well-known Lamb-dip. The dip we observed previously by applying excitation density modulation [1]. Here we report our observation of the modulation dip in case of resonator Q modulation obtained by reflecting the laser beam back into the resonator by means of an external moving mirror [4].

If, for simplicity, we consider transmission losses only, the resonator quality Q_0 of a two-mirror laser is

$$Q_0 = (4\pi\nu L/c)\,(1-R_1 + 1-R_2)^{-1}. \qquad (1)$$

Here L is the distance between the laser mirrors M_1 and M_2, and R_1 and R_2 are their intensity reflectivities. The external mirror M_3 (reflectivity R_3) and the nearest laser mirror M_2 form a Fabry-Perot interferometer with length l. Its intensity reflectivity R can easily be derived [5-7]. To find the resonator quality Q with M_3 present we merely replace R_2 by R in eq. (1).

The steady-state laser intensity in single-mode operation is [8]

$$E^2 = (\alpha' - \pi\nu/Q)/\beta, \qquad (2)$$

where $\alpha \equiv \alpha' - \pi\nu/Q$ is the unsaturated net gain and β is the saturation parameter. Eq. (2) holds for low excitation level only. We find the change in laser intensity due to the presence of M_3 in the quasistatic case

$$E^2 - E_0^2 = \frac{c}{4L\beta}(1-R_2)\left[1 - \frac{1-R_3}{1+R_2R_3 + 2(R_2R_3)^{\frac{1}{2}}\cos\delta}\right], \qquad (3)$$

where $\delta = 4\pi l\nu/c$ is the external phase difference.

When the external moving mirror has constant velocity v along the beam direction, eq. (3) indicates that E^2 is modulated at the Doppler frequency $\omega_D = 4\pi\nu v/c$ and that, in general, the shape of the modulation is non-sinusoidal. Sinusoidal modulation is obtained if $2(R_2R_3)^{\frac{1}{2}} \ll 1 + R_2R_3$. The time-dependent part of E^2 can then be written

$$\Delta(E^2) \approx \frac{c}{4L\beta}\,\frac{(1-R_2)(1-R_3)}{(1+R_2R_3)^2}\,2(R_2R_3)^{\frac{1}{2}}\cos\omega_D t. \qquad (4)$$

The resonator quality can then be put into the form $Q \approx \bar{Q} + \Delta Q\cos\omega_D t$, with $(\Delta Q/\bar{Q}) \ll 1$ and $\bar{Q}$ slightly abobe Q_0. These conditions were needed in ref. 3 in the general case of resonator Q modulation.

We made experiments with a single-mode $1.15\,\mu$ He-Ne laser provided with plane internal mirrors [9] and with a hemispherical multimode $0.6328\,\mu$ He-Ne laser (OIP type 160-G). The experimental arrangement was as described in ref. 10; R_3 was varied from 0.01 up to 0.75 by using beam splitters in the external interferometer. The output was detected through mirror M_1.

At *low* modulation level we observed a dip in the a.c. power output, which is shown for the

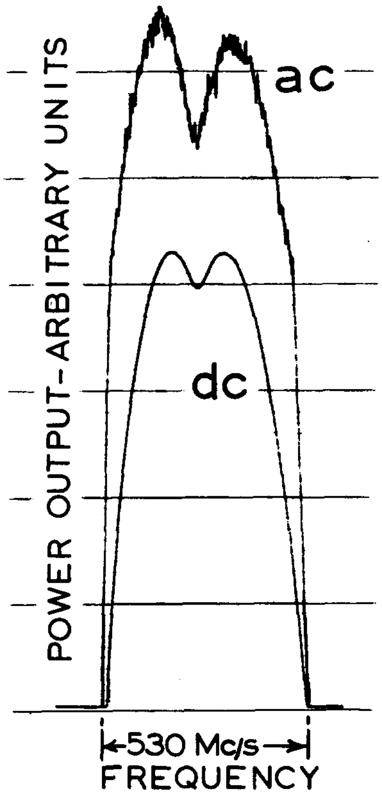

Fig. 1. The measured d.c. and a.c. power output as a function of the detuning of the interferometer, showing the Lamb-dip and the more pronounced modulation dip, respectively. Modulation frequency $\omega_D/2\pi = 1.8$ kc/s; total gas pressure 1.9 mm Hg, He-^{20}Ne ratio 5 : 1.

1.15μ laser in fig. 1 [cf. 1]. We also observed a modulation dip with the 0.6328μ laser, where we used a low excitation level to ensure single mode operation *.

At *high* modulation level the signals were non-sinusoidal. A typical example is shown in fig. 2a. The signals become more sinusoidal with decreasing modulation level (see fig. 2b). The signal shapes are in general agreement with our computer calculations of $E^2 - E_0{}^2$ for R_2 between 0.980 and 0.995 and for values of R_3 as used in the experiments. We note that the non-sinusoidal shape of the modulation is due to the non-sinusoidal modulation of Q rather than to the non-

* Another feature of external moving mirror modulation is that the number of axial modes can, in principle, be determined from the amplitude of the a.c. power output as a function of l [10,11 c.f. also 12]. We note that sinusoidal laser output modulation may also be treated as a case of two-beam interference, as shown in [10]. The relative contribution of each mode to the total modulation amplitude should be taken proportional to $\Delta(E^2)$ rather than to E^2 (see [11]).

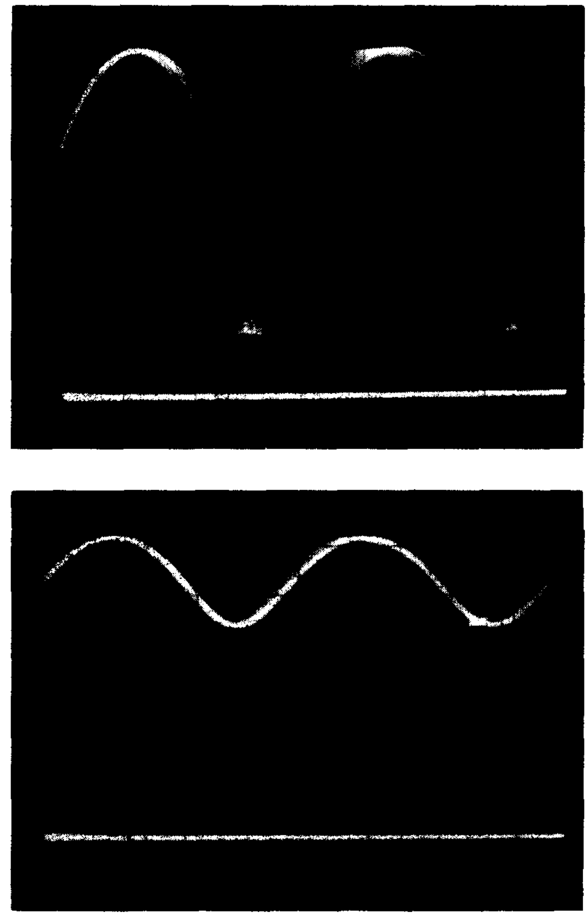

Fig. 2. Output modulation of the 0.6328μ laser for $R_3 \approx 0.50$ (a) and $R_3 \approx 0.10$ (b). Straight line indicates laser zero level. Modulation frequency $\omega_D/2\pi = 1$ kc/s. Phase instabilities shown are due to the non-uniform movement of the external mirror.

linear response of the laser. We did not observe any change in the shapes of the signals for frequencies up to 40 kc/s. Of course, at high ω_D such that $\omega_D \sim 2\alpha$ [cf. 1,3] the time-response of the laser should be taken into account. This may be also necessary for $\omega_D < 2\alpha$ since the rise- (and decay-) time of the modulation of Q can be short compared to the response-time of the laser. We checked that the intensity variation due to the laser frequency modulation which also occures when the resonator Q is periodically varied [cf. 8], could be disregarded.

Very recently, Hooper and Bekefi [13] have given an optical analysis of a system consisting of a laser resonator and an external mirror. They have used the well-known surface integral

[14] relating the field on one mirror to that one on the other mirror. The gain of the medium was introduced by starting from rate equations, thus excluding resonator effects. Their analysis involves a lengthy algebra. It can be shown, however, that for all practical cases the effect of M_3 can be derived directly from their intensity determining equation if $(R_2)^{\frac{1}{2}}$ is replaced by the amplitude reflectivity r of the external interferometer, similarly as was done in our simple analysis *. The general shape of the modulation signals turned out to be similar in their and our calculations, since it is closely related to the time-varying reflectivity of the external interferometer.

It might be of interest to consider external moving mirror modulation in terms of external signal injection [15].

* The quantity A in ref. 13 is equal to $(R_2)^{\frac{1}{2}}/r$, where r is assumed to be real at the laser oscillation frequency. This was not noticed by Hooper and Bekefi. The minus sign in the denominator of their eq. (18) has to be replaced by a plus sign (E.B.Hooper, private communication).

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NONLINEAR SCATTERING OF ELECTRONS BY LASER BEAM

P. T. CHANG
Systems Development Division, International Business Machines Corporation, Poughkeepsie, New York

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Direct observation of nonlinear scattering of electrons by a laser beam has been reported [1], but the probability of reflection appeared much higher than what was predicted by the Kapitza and Dirac formula [2]. Since it is well known that the intensity of a laser beam has a Gaussian distribution [3], it can be shown that energy conservation is preserved if this distribution taken explicitly into account in the calculation.

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