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A STUDY OF THE SU(3)* LIMIT OF IBM-2

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Abstract: In this paper the SU(3)* limit of IBM-2 is studied. It is shown that the complete dynamical symmetry group of IBM-2 contains a U(6) subgroup (called U(6)*) of which SU(3)* is a subgroup. The explicit form of the U(6) Casimir operator is given, and the complete reduction from U(6)* to SU(3)* is tabulated. The SU(3)* limit is applied to the nucleus ^{192}Os. After introduction of appropriate symmetry breaking terms the description is as good as that obtained through a phenomenological fit.

1. Introduction

The interacting boson model for one kind of bosons (IBM-1) [1-3]) has U(6) as a dynamical symmetry group, i.e. any IBM-1 hamiltonian can be expressed in terms of the generators of U(6) only. In the references mentioned above it has been shown that there are only three subgroups of U(6) which contain the angular-momentum subgroup SO(3). These are the subgroups U(5), SU(3) and O(6).

In the IBM-2 model it is taken into account that both protons and neutrons occur in nuclei. Hence proton and neutron bosons are introduced. This allows for a richer subgroup structure. There exist two SU(3) subgroups: one, which resembles the IBM-1 SU(3) limit, is referred to as SU(3); the other, which has completely different properties, is named SU(3)* [ref. [4])].

In this paper attention will be focused on the group SU(3)*. Through a semi-classical approach [5]) it can be shown that in this limit the nucleus has a triaxial ground-state shape, generated by proton and neutron distributions of spheroidal shape with an angle $\frac{1}{2}\pi$ between the two axes of axial symmetry. The question whether triaxial nuclei exist is still open. A study of the SU(3)* limit may shed some light on the occurrence of triaxiallity, and is therefore also interesting from this point of view.

In ref. [4]) a simplified SU(3)* classification has been applied to the nucleus ^{104}Ru. In this paper we will consider a more general hamiltonian, which will be applied to the nucleus ^{192}Os. Since transition strengths are often more sensitive to the hamiltonian used than excitation spectra are, a detailed comparison of B(E2) strengths will be made.

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The occurrence of "scissors mode" excitations is a topic of much experimental interest after their first identification [6]). The structure of these excitations in the SU(3)* limit will also be discussed.

2. Properties of the SU(3) limit of IBM-1

In this section a few properties of the SU(3) limit of IBM-1 will be tabulated for later use. The eight generators of the SU(3) subgroup of U(6) are given by

$$Q_m = s^\dagger \tilde{d}_m + d^\dagger_m s + \chi[d^\dagger \tilde{d}]^{(2)}_m ,$$
$$L_m = \sqrt{10}[d^\dagger \tilde{d}]^{(1)}_m , \tag{1}$$

with either sign of $\chi = \pm\frac{1}{2}\sqrt{7}$.

The operators L_m generate the angular-momentum subalgebra SO(3). The most general hamiltonian with SU(3) symmetry consists of the Casimir invariants of SU(3) and the subgroup SO(3)

$$H = A(\tfrac{2}{9}Q \cdot Q - \tfrac{1}{12}L \cdot L) + BL \cdot L . \tag{2}$$

The spectrum of this hamiltonian can be presented in analytical form as

$$E(\lambda, \mu, L) = \tfrac{1}{9}A(\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu)) + BL(L+1) . \tag{3}$$

Here the allowed SU(3) representations are given by

$$(\lambda, \mu) = (2N - 6i - 4j, 2j) \tag{4}$$

for all integers i and j such that $\lambda \geq 0$ and $\mu \geq 0$, as follows from the reduction of the irrep $[N]$ of U(6) to SU(3), where each boson carries two oscillator quanta [2]). In a more compact form this branching rule will be written as

$$[N] = \sum_{\lambda\mu} a_{\lambda\mu}(\lambda, \mu) , \tag{5}$$

where $a_{\lambda\mu}$ is an integer which counts the multiplicity of irreps, and consequently is zero when such an irrep does not occur in the reduction. The two possible realizations of the quadrupole operator $Q(\chi = \pm\frac{1}{2}\sqrt{7})$ can be related by means of the transformation

$$d^\dagger_m \to -d^\dagger_m ,$$
$$\tilde{d}_m \to -\tilde{d}_m ,$$
$$Q_m(\chi) \to -Q_m(-\chi) . \tag{6}$$

3. IBM-2 in the SU(3) limits

For two kinds of bosons, i.e. in IBM-2, one can consider the group chains

$$\begin{array}{ccccc} U^{\pi}(6) & \times U^{\nu}(6) & \supset U^{\pi\nu}(6) & & \\ \cup & \cup & \cup & & \\ SU^{\pi}(3) & \times SU^{\nu}(3) & \supset SU^{\pi\nu}(3) & \supset SO^{\pi\nu}(3)\,. & \end{array} \tag{7}$$

Since the reductions in eq. (7) are reductions of lie groups, and since due to angular momentum conservation the angular momentum SO(3) subgroup should remain a subgroup, the infinitesimal generators of $SU^{\pi\nu}(3)$ must have the structure

$$Q_m^{\pi\nu} = x_Q Q_m^{\pi} + y_Q Q_m^{\nu}\,,$$
$$L_m = L_m^{\pi} + L_m^{\nu}\,. \tag{8}$$

The further requirement that $Q^{\pi\nu}$ and L again obey SU(3) commutation relations results in the restriction

$$x_Q^2 = y_Q^2 = 1\,. \tag{9}$$

From eq. (6) it is readily verified that the transformation $\chi^{\nu} \to -\chi^{\nu}$ and a change of phase factor in all operators $\tilde{d}_m$ and $d_m^{\dagger}$ changes Q_m^{ν} into $-Q_m^{\nu}$, but still leaves the SU(3) symmetry intact. This operation transforms $Q_+^{\pi\nu} = Q^{\pi} + Q^{\nu}$ (with $\chi_{\pi} = \chi_{\nu}$) into $Q_-^{\pi\nu} = Q^{\pi} - Q^{\nu}$ (with $\chi_{\pi} = -\chi_{\nu}$). An SU(3) algebra where the quadrupole generators can be transformed by eq. (6) into

$$Q^{\pi\nu} = Q^{\pi} + Q^{\nu}\,, \qquad \chi_{\pi} = \chi_{\nu} \tag{10}$$

will be called and $SU^{\pi\nu}(3)$ algebra. An SU(3) algebra of which the generators can be transformed into

$$Q^{\pi\nu} = Q^{\pi} - Q^{\nu}\,, \qquad \chi_{\pi} = \chi_{\nu} \tag{11}$$

will be referred to as an $SU^{\pi\nu}(3)^*$ algebra.

4. The SU(3)* limit

In this section details of the SU(3)* group will be discussed. For the SU(3) group the reader is referred to the literature [e.g. ref. [9])]. The transformation from $SU^{\pi\nu}(3)$ to $SU^{\pi\nu}(3)^*$ corresponds to application of the transformation

$$Q_m \to -Q_m\,, \qquad L_m \to L_m \tag{12}$$

in the neutron SU(3) algebra.

The transformation (12) is a restricted form of a transformation on all generators of U(6) that leaves the commutation rules invariant

$$s^{\dagger}s \to -s^{\dagger}s\,, \qquad [d^{\dagger}\tilde{d}]_m^{(L)} \to (-1)^{L+1}[d^{\dagger}\tilde{d}]_m^{(L)}\,,$$
$$s^{\dagger}\tilde{d}_m \to -d_m^{\dagger}s\,, \qquad d_m^{\dagger}s \to -s^{\dagger}\tilde{d}_m\,. \tag{13}$$

These operators generate a different realization of the group U(6) that will be denoted by $\overline{\mathrm{U}(6)}$. The covariant symmetric irrep $[N, 0^5]$ of U(6) is now transformed into the contravariant symmetric irrep $[0^5, -N]$ of U(6) [see ref. [10])] for the definition of these generalized Young tableaux). This can be verified as follows:

(i) The linear Casimir operator $n = s^\dagger s + d^\dagger \cdot \tilde{d}$ transforms to $-n$.

(ii) A symmetric irrep has at most one nonzero row.

(iii) The linear Casimir invariant is the sum of the length of all the rows in the Young tableau.

(iv) The rows in a Young tableau are ordered such that their length always decreases as the row number increases.

(v) Thus all rows but the last one must have zero length, and the last row must have length $-N$.

The group generated by the sums of corresponding generators of $\mathrm{U}^\pi(6)$ and $\overline{\mathrm{U}^\nu(6)}$, denoted by $\mathrm{U}^{\pi\nu}(6)^*$, has $\mathrm{SU}^{\pi\nu}(3)^*$ as a subgroup. For the labelling according to the new U(6) group one needs to calculate the product of a covariant and a contravariant transformation, as can be seen from the outer product [10])

$$[N_\pi, 0^5] \times [0^5, -N_\nu] = ([N_\pi, 0^5] \times [N_\nu, 0]) \times [-N_\nu^6]$$
$$= \sum_{i=0}^{\min(N_\pi, N_\nu)} [N_\pi - i, 0^4, i - N_\nu]\,. \tag{14}$$

In order to return to the ordinary Young tableaux with rows of positive length, one can introduce an intermediate $\overline{\mathrm{SU}(6)}$ group where one can make use of the identity of irreps $[x_1, x_2, \ldots, x_6] = [x_1 + a, x_2 + a, \ldots, x_6 + a]$, which allows equation of contravariant irrep $[0^5, -N]$ with the covariant irrep $[N^5, 0]$. (This is possible since the relation between the transformation of an element of a contravariant irrep and an element of the corresponding covariant irrep is a multiplication with a nonzero power of the determinant of that transformation only [10]).)

We are now in a position to write down the labelling according to all the subgroups, which is given in table 1. The reduction of $\overline{\mathrm{U}^\nu(6)}$ to $\overline{\mathrm{SU}^\nu(3)}$ for the $[0^5, -N_\nu]$ irrep

TABLE 1

The reduction form $\mathrm{U}^\pi(6) \times \mathrm{U}^\nu(6)$ to $\mathrm{SU}^{\pi\nu}(3)^*$ and the labelling of the irreps at each stage of the reduction

Groups					Irreps				
$\mathrm{U}^\pi(6)$	×	$\overline{\mathrm{U}^\nu(6)}$	⊃	$\mathrm{U}^{\pi\nu}(6)^*$	$[N_\pi, 0^5]$	×	$[0^5, -N_\nu]$	=	$\sum_{i=0}^{\min(N_\pi,N_\nu)} [N_\pi - i, 0^4, i - N_\nu]$
∪		∪		∪	‖		‖		‖
$\mathrm{SU}^\pi(6)$	×	$\overline{\mathrm{SU}^\nu(6)}$	⊃	$\mathrm{SU}^{\pi\nu}(6)^*$	$[N_\pi, 0^4]$	×	$[N_\nu^5]$	=	$\sum_{i=0}^{\min(N_\pi,N_\nu)} [N_\pi + N_\nu, (N_\nu - i)^4]$
∪		∪		∪	‖		‖		‖
$\mathrm{SU}^\pi(3)$	×	$\overline{\mathrm{SU}^\nu(3)}$	⊃	$\mathrm{SU}^{\pi\nu}(3)^*$	$\sum_{\lambda_1,\mu_1} (\lambda_1, \mu_1)$	×	$\sum_{\lambda_2,\mu_2} (\lambda_2, \mu_2)$	=	$\sum_{\lambda,\mu} a_{\mu\lambda}(\lambda, \mu)$

of $\overline{U^{\nu}(6)}$ can most easily be found by the use of the analogy with the corresponding $U^{\nu}(6)$ irreps, since the multiplicities at each step are identical. The steps in the group reduction of $U^{\nu}(6)$ are

$$\begin{array}{ccccc} U^{\nu}(6) \supset & & U^{\nu}(3) & \supset & SU^{\nu}(3) \\ | & & | & & | \\ [N_{\nu}] & & = \sum_{\lambda_1,\lambda_2,\lambda_3} [\lambda_1, \lambda_2, \lambda_3] & = & \sum_{\lambda_1,\lambda_2,\lambda_3} (\lambda_1-\lambda_2, \lambda_2-\lambda_3) = \sum_{\lambda\mu} a_{\lambda\mu}(\lambda, \mu) . \end{array} \tag{15}$$

Application of analogous steps for $\overline{U^{\nu}(6)}$ shows

$$\begin{array}{ccccc} \overline{U^{\nu}(6)} & \supset & \overline{U^{\nu}(3)} & \supset & \overline{Su^{\nu}(3)} \\ | & & | & & | \\ [0^5, -N_{\nu}] & = \sum_{\lambda_1,\lambda_2,\lambda_3} & [-\lambda_3, -\lambda_2, -\lambda_1] & = \sum_{\lambda_1,\lambda_2,\lambda_3} & (\lambda_2-\lambda_3, \lambda_1-\lambda_2) = \sum_{\lambda\mu} a_{\lambda\mu}(\mu, \lambda) . \end{array} \tag{16}$$

One should note that $a_{\lambda\mu}$ is the same in two formulae, but the coefficients λ and μ are interchanged. The eigenvalues of the Casimir operator C_2 acting on an irrep $[N_{\pi}-i, 0^4, i-N_{\nu}]$ can be expressed in the following form:

$$\begin{aligned} \langle C_2(U^{\pi\nu}(6)^*)\rangle &= (N_{\pi}-i)(N_{\pi}-i+5)+(N_{\nu}-i)(N_{\nu}-i+5) \\ &= \tfrac{1}{2}(N_{\pi}-N_{\nu})^2+\tfrac{5}{2}(N_{\pi}+N_{\nu})+\tfrac{1}{2}\Delta N(\Delta N+5) , \end{aligned} \tag{17}$$

where we introduce the symmetry quantum number

$$\Delta N = (N_{\pi}-N_{\nu}-2i) \; (= N_{\pi}+N_{\nu}, N_{\pi}+N_{\nu}-2, \dots, |N_{\pi}-N_{\nu}|)$$

which we prefer to use in the following instead of i.

The Casimir operator C_2 cannot be realized by a part dependent on N_{π}, N_{ν} and the square of a spin-like quantity as is the case for the $U^{\pi\nu}(6)$ group [8]). Addition of this Casimir operator to an SU(3)* invariant hamiltonian results in a shift of the energy of states with different values of ΔN, and so removes part of the degeneracy in the SU(3)* Casimir operator (see appendix). The explicit form of the operator C_2 is readily derived

$$C_2(U^{\pi\nu}(6)^*) = C_2(U^{\pi}(6)) + C_2(U^{\nu}(6)) - 2(s_{\pi}^{\dagger}s_{\nu}^{\dagger} + d_{\pi}^{\dagger}\cdot d_{\nu}^{\dagger})(s_{\pi}s_{\nu} + \tilde{d}_{\pi}\cdot\tilde{d}_{\nu}) . \tag{18}$$

A hamiltonian of the form

$$H = \tfrac{1}{2}\kappa_{\pi\nu}C_2(SU^{\pi\nu}(3)^*) + \xi C_2(U^{\pi\nu}(6)^*) + a_{\pi\nu}L\cdot L \tag{19}$$

has the spectrum

$$\begin{aligned} E(N_{\pi}, N_{\nu}, \Delta N, \lambda, \mu, L) = {} & \tfrac{1}{4}\kappa_{\pi\nu}(\lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu)) \\ & + \xi(\tfrac{1}{2}(N_{\pi}-N_{\nu})^2+\tfrac{5}{2}N+\tfrac{1}{2}\Delta N(\Delta N+5)) \\ & + a_{\pi\nu}L(L+1) . \end{aligned} \tag{20}$$

The tensor decomposition of the irreps of $U^{\pi\nu}(6)^*$ into $SU^{\pi\nu}(3)^*$ irreps is given in the appendix. This can be used to determine the allowed $SU^{\pi\nu}(3)^*$ irreps and the symmetry quantum number ΔN.

5. Discussion of spectra

In order to gain some more physical insight we give a few examples of SU(3) spectra. The total number of bosons is assumed to be six and an equal number of

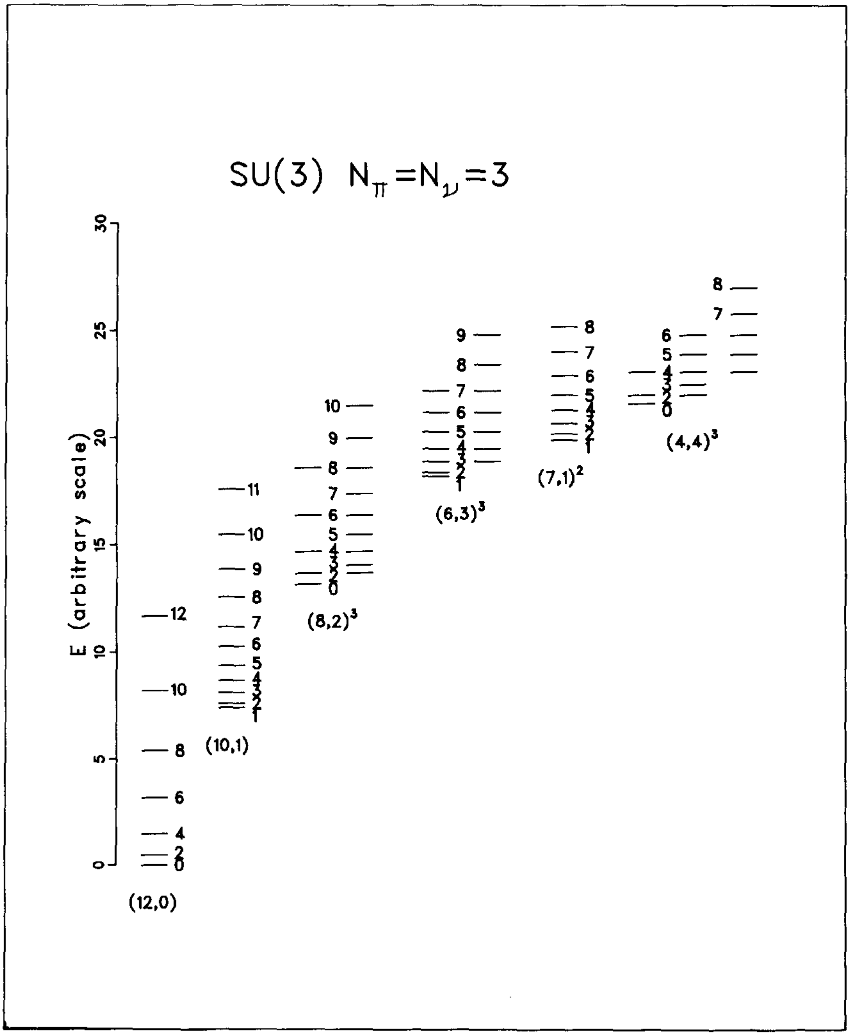

Fig. 1. Low energy part of the SU(3) spectrum for $N_\pi = N_\nu = 3$, eq. (21). The value of κ is chosen to be negative. If several irreps are degenerate only one is plotted.

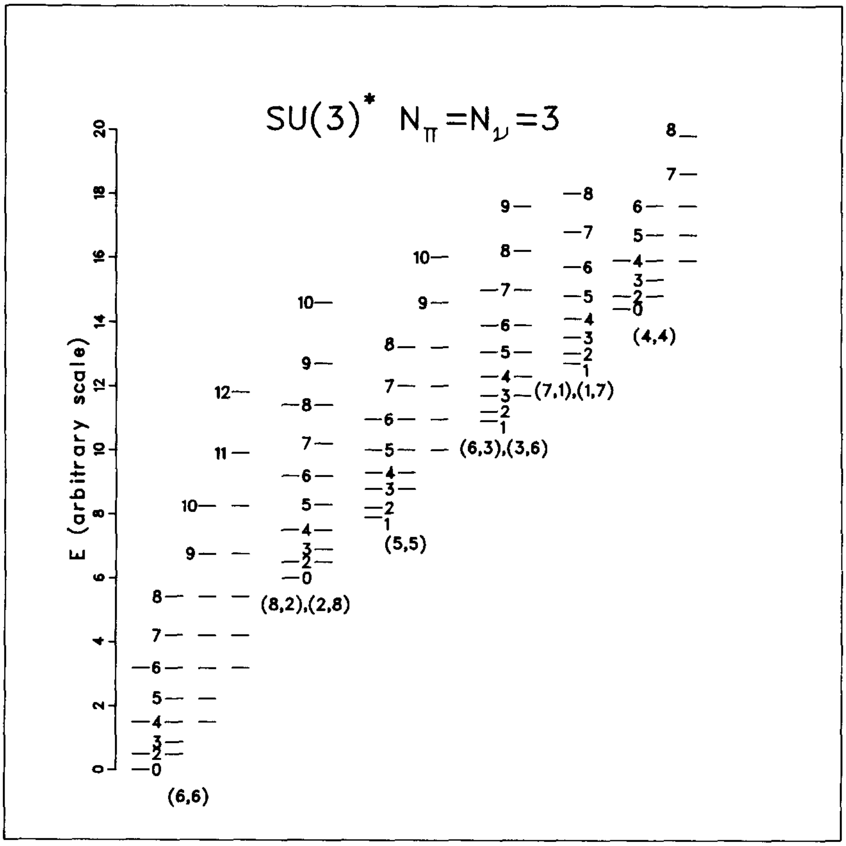

Fig. 2. Low energy part of the SU(3)* spectrum for $N_\pi = N_\nu = 3$, eq. (21). The value of κ is chosen to be negative. If several irreps are degenerate only one is plotted.

proton and neutron bosons is used in order to see the maximum effect of the proton-neutron degree of freedom. Consider the following family of hamiltonians

$$H = \tfrac{1}{2}\kappa(Q^\pi \pm Q^\nu)\cdot(Q^\pi \pm Q^\nu)\,, \qquad (\chi^\pi = \chi^\nu)\,. \tag{21}$$

These hamiltonians possess $SU^{\pi\nu}(3)$ symmetry ($SU^{\pi\nu}(3)^*$ symmetry) for the plus (minus) sign in eq. (21). In figs. 1 and 2 the low-energy spectra of SU(3) and SU(3)* are plotted.

The SU(3) spectrum has a nondegenerate ground-state band. If one does not use the Casimir operator of $U^{\pi\nu}(6)$ to push this band up in energy, one finds the $(2N-2, 1)$ mixed-symmetry band at very low energy. The band head is the scissors mode state as identified, e.g., in ^{156}Gd [ref. [6]].

The SU(3)* spectrum shows a very large degeneracy, even for the lowest (6, 6) irrep, due to the large value of μ. The effect of the majorana operator of U(6)*, eq. (18), would be to push several irreps with $\Delta N = 4$, 2, or 0, (e.g., one of the (4, 4) irreps) relative to the (6, 6) irrep.

Dieperink and Talmi [11]) have identified the SU(3)* spectrum with that of a triaxial rotor. Application of the standard formulae of the triaxial rotor [12]) shows that the SU(3)* triaxial rotor corresponds to the peculiar case where the three moments of inertia are equal. In the geometric model this implies that a rotation about any axis gives the same physical state, which results in the fact that all rotations are "forbidden". Thus one can conclude that the excitations in the SU(3)* limit cannot be described in the standard geometric model.

6. The scissors mode

The $(2N-2, 1)$ band of SU(3) corresponds in a geometrical picture to an oscillation of a deformed neutron shape against a deformed proton shape, while the centres-of-mass of both shapes are fixed [13]). These deformed shapes can each be related to the $(2N, 0)$ ground-state band of IBM-1 in the SU(3) limit. The coupling of two such bands by the SU(3) hamiltonian (21) gives the irreps $(N = N_\pi + N_\nu)$

$$(\lambda, \mu) = (2N-2i, i)\,, \qquad F = \tfrac{1}{2}N - i\,, \qquad i = 0, 1, \ldots, \tfrac{1}{2}N\,. \tag{22}$$

The difference in excitation energies of the band heads in SU(3) for $N \to \infty$, $i \ll N$ is given by $(2n+K)\omega_1$ with $K = i, i-2, \ldots, 1$ or 0, $n = \frac{1}{2}(i-K)$ and $\omega_1 = -\frac{3}{2}\kappa N$. This is the spectrum of a two-dimensional harmonic oscillator.

Similar excitations can also be identified in the SU(3)* limit, where the coupling of two axial rotors, i.e. SU(3) ground-state bands, via the SU(3)* hamiltonian results in the SU(3)* irreps

$$\begin{aligned}(\lambda, \mu) &= (2N_1 - i, 2N_2 - i)\,,\\ \Delta N &= N_1 + N_2 - 2i\,, \qquad i = 0, 1, \ldots, \tfrac{1}{2}(N_1 + N_2)\end{aligned} \tag{23}$$

with $N_1(N_2)$ the larger (smaller) of N_π and N_ν.

For $N_\pi \to \infty$, $N_\nu \to \infty$, $i \ll N_\pi$, $i \ll N_\nu$ the excitation energy of the band heads is given by $n\omega_2$ with $n = i$ and $\omega_2 = -\frac{3}{4}\kappa N$. In this case one thus finds a one-dimensional harmonic oscillator spectrum.

7. Broken SU(3)* symmetry in ^{192}Os

The classification according to SU(3)* will be illustrated to the nucleus ^{192}Os. Bijker *et al.* [14]) have made IBM-2 fits to several nuclei in the osmium region, and concluded that these nuclei can be described as transitional between the O(6) and SU(3) limits of IBM-2. The hamiltonian obtained in these fits contains a $Q^\pi \cdot Q^\nu$ interaction with $\chi_\pi \approx -\chi_\nu \approx -\frac{1}{2}\sqrt{7}$ for $A = 190$–194. We have seen that this can be one of the components of an SU(3)* interaction.

For this reason it is worth while to apply an SU(3)* classification to these nuclei. The nucleus ^{192}Os $(N_\pi = 5,\ N_\nu = 3)$ is chosen as an example, since this nucleus has been studied experimentally with many different probes, and as a result much is known about this nucleus [ref. [15,16])]. Especially the work on the low-spin states observed in the reaction $(n, n\gamma)$ [ref. [15])] makes identification of the various band structures found in IBM-2 possible.

Consider the SU(3)* hamiltonian

$$H = \tfrac{1}{2}\kappa C_2(\mathrm{SU}(3)^*) + \kappa_L L \cdot L + \xi C_2(\mathrm{U}(6)^*)\,. \quad (24)$$

For the nucleus ^{192}Os the parameter values

$$\kappa = -0.03356\ \mathrm{MeV}\,,$$
$$\kappa_L = 0.029\ \mathrm{MeV}\,,$$
$$\xi = -1.061\ \mathrm{MeV}\,, \quad (25)$$

give the correct moment of inertia for the $K=0$ ground state band and the correct excitation energies of the 0^+ bands. In fig. 3a all states below 2.5 MeV have been plotted. In fig. 3b the experimental counterparts of all states in fig. 3a are drawn, as far as these could be identified.

It is clear that the SU(3)* symmetry, if present, is broken. The states which are degenerate in SU(3)* show a rather large splitting, e.g., the 2_1^+ and the 2_2^+ states show a splitting of about 270 keV. The second 0^+ state is identified as a state of non-maximal symmetry quantum number, $\Delta N = 6$. This gives a possible explanation for the anomaly of the B(E2) ratios $B(\mathrm{E2};\, 0_i^+ \to 2_1^+)/B(\mathrm{E2};\, 0_i^+ \to 2_2^+)$ presented in ref. [15]). Experimentally it is found that this ratio has the value 0.019, >40, 1.2 for the first, second and third excited 0^+ state, respectively. In the IBM-2 fit [14]) the value of this ratio for the second 0^+ state is not reproduced. Since in the current description the symmetry of this state is different from that of the other two 0^+ states, the difference in value for this ratio is not surprising.

From the discussion given above it is clear that SU(3)* provides an excellent means to label the spectrum of ^{192}Os. This may indicate that a hamiltonian with broken SU(3)* symmetry can describe the nucleus ^{192}Os. *A priori* it is not entirely

TABLE 2

Values of the parameters in the hamiltonian of eq. (26)

Parameter	Value of parameter (MeV)	
	SU(3)*	Fit
ε_d	0.0	0.6351
κ	−0.03856	−0.05165
κ_L	0.00383	−0.00895
ξ	0.085	0.03466

The column labelled SU(3)* contains the values for the unbroken SU(3)* symmetry, the column labelled "fit" contains the values for the fit to the energy spectrum of ^{192}Os.

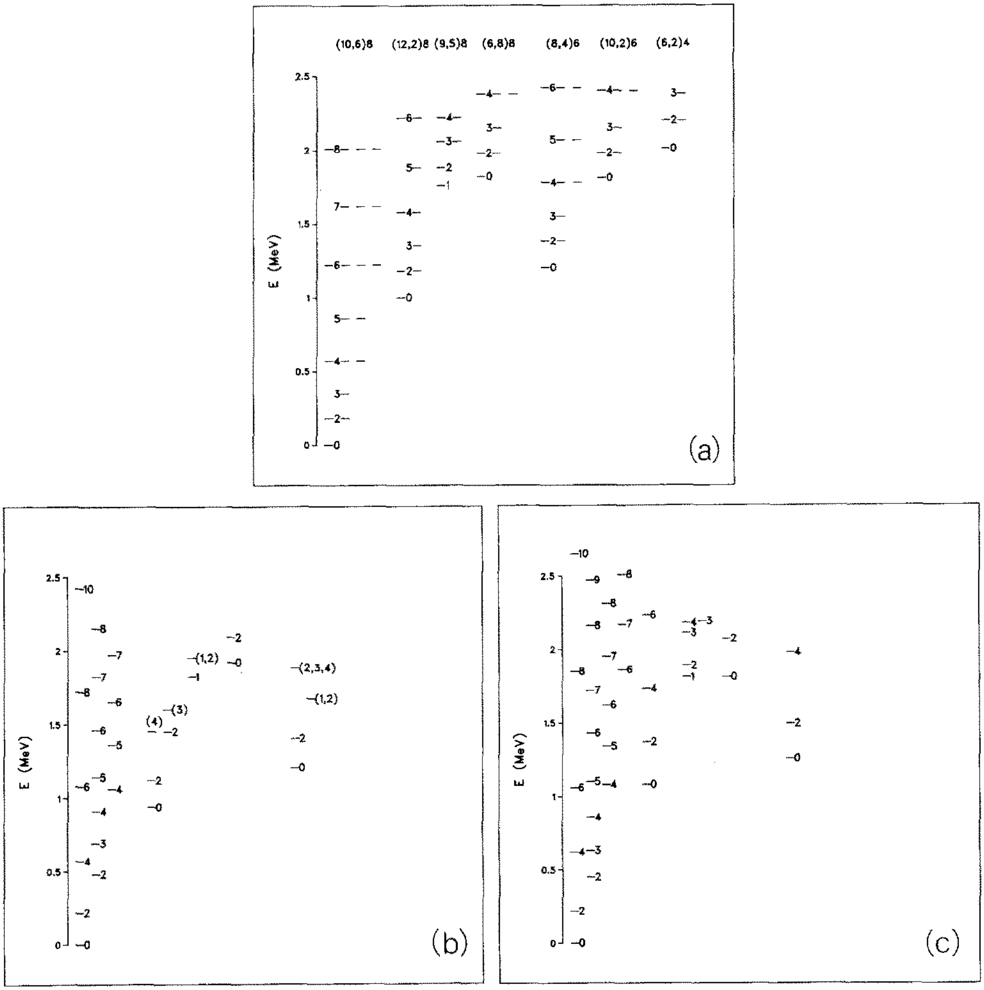

Fig. 3. (a) Theoretical spectrum obtained from the hamiltonian eq. (24) with the parameters eq. (25). The numbers labelling the columns are the SU(3) quantum numbers and the symmetry quantum number i.e. $(\lambda, \mu)\Delta N$. Next to each state the value of the spin is indicated. All states plotted have positive parity. (b) Experimental counterparts to the theoretical levels in fig. 3a. (c) Broken-SU(3)* fit to the experimental spectrum.

clear what symmetry-breaking terms should be introduced. One choice would be an SU(3) invariant term, which is present for the neutron-deficient osmium isotopes which have an SU(3)-like spectrum. Introduction of such a term also requires the introduction of the majorana operator of $U^{\pi\nu}(6)$ to push the first 1^+ state up in energy, since this will be calculated too low otherwise. This gives two extra parameters which are not very well determined in a fit to the energy levels of ^{192}Os alone.

Another choice is a U(5)-invariant term, which is dominant near the closed shell where the spectra are vibrational. For this reason we introduce a one-body d-boson term in the hamiltonian

$$H = \varepsilon_d(n_{d\pi}+n_{d\nu})+\kappa(Q_\pi+Q_\nu)\cdot(Q_\pi+Q_\nu)$$
$$+\kappa_L L\cdot L+\xi(s_\pi^\dagger s_\nu^\dagger - d_\pi^\dagger\cdot d_\nu^\dagger)(s_\pi s_\nu-\tilde{d}_\pi\cdot\tilde{d}_\nu)\,. \quad (26)$$

with $\chi_\pi=-\chi_\nu=-\frac{1}{2}\sqrt{7}$.

TABLE 3a

Comparison of values of B(E2) ratios

B(E2) ratio	Exp.	Bijker	Present work	
			I	II
$0_2^+\to 2_1^+/0_2^+\to 2_2^+$	0.019	0.048	0.22	1.54
$0_3^+\to 2_1^+/0_3^+\to 2_2^+$	>40	0.30	0.078	0.14
$0_4^+\to 2_1^+/0_4^+\to 2_2^+$	0.73	2.43	1.99	2.81
$2_2^+\to 0_1^+/2_2^+\to 2_1^+$	0.104	0.095	0.204	0.170
$3_1^+\to 2_1^+/3_1^+\to 2_2^+$	0.081	0.085	0.096	0.073
$4_2^+\to 2_1^+/4_2^+\to 2_2^+$	0.004	0.003	0.011	0.003
$4_2^+\to 3_1^+/4_2^+\to 2_2^+$	0.839	0.291	0.791	0.924
$4_2^+\to 4_1^+/4_2^+\to 2_2^+$	1.05	0.59	0.34	0.28
$5_1^+\to 4_1^+/5_1^+\to 3_1^+$	0.076	0.052	0.044	0.031

The second column contains the experimental data from refs. [15,16]), the third column the IBM-2 results from ref. [14]), the last two columns contain the results obtained in the present work. For both columns the equality $e_\pi=e_\nu$ is assumed. The column labelled I corresponds to $\chi_\pi^e=-\chi_\nu^e=-\frac{1}{2}\sqrt{7}$, and the column labelled II to $\chi_\pi^e=-1.3$, $\chi_\nu^e=0.95$.

TABLE 3b

Comparison of absolute B(E2) values

B(E2) value ($e^2\cdot b^2$)	Exp.	Bijker	Present work	
			I	II
$2_1^+\to 0_1^+$	0.418 (4)	0.46	0.373	0.412
$2_2^+\to 2_1^+$	0.36 (3)	0.31	0.144	0.152
$2_2^+\to 0_1^+$	0.040 (2)	0.029	0.026	0.026
$4_1^+\to 2_1^+$	0.54 (5)	0.81	0.52	0.58
$4_2^+\to 2_2^+$	0.17 (3)	0.40	0.16	0.17
$4_2^+\to 4_1^+$	0.37 (18)	0.241	0.052	0.058
$6_1^+\to 4_1^+$	0.87 (17)	0.71	0.56	0.57
$Q(e\cdot b)$	Exp.	Bijker	Present work	
			I	II
2_1^+	−0.96 (1)	−1.089	−0.850	−0.940

See caption of table 3a for meaning of columns. A boson charge of 0.19 $e\cdot b$ was assumed in the present results. In recalculating the values of ref. [14]) a boson charge of 0.17 $e\cdot b$ was assumed.

The minus sign in the factor multiplying ξ, which was a plus sign in eq. (18), is due to the different realization of the groups $SU^{\pi\nu}(3)^*$ and $U^{\pi\nu}(6)^*$ used here ($\chi_\pi = -\chi_\nu$).

The hamiltonian (26) contains four parameters, which were determined in a fit to the energy levels of ^{192}Os (table 2). The energy spectrum obtained for these parameter values is sketched in fig. 3c. In order to judge the quality of this description one should consider also the values of observables. For this reason we have compared several B(E2) values and ratios of B(E2) values with the experimental values.

The most general E2 operator

$$O = \varepsilon_\pi(s^\dagger_\pi \tilde{d}_\pi + d^\dagger_\pi s_\pi + \chi^e_\pi[d^\dagger_\pi \tilde{d}_\pi]^{(2)}) + \varepsilon_\nu(s^\dagger_\nu \tilde{d}_\nu + d^\dagger_\nu s_\nu + \chi^e_\nu[d^\dagger_\nu \tilde{d}_\nu]^{(2)}) \qquad (27)$$

contains four parameters. These parameters were not all fitted to the experimental data, but equality of charges, $e_\pi = e_\nu$, was used; only two choices for the χ's were applied: $\chi^e_\pi = -\chi^e_\nu = -\frac{1}{2}\sqrt{7}$ as in the hamiltonian eq. (26) and $\chi^e_\pi = -1.3$, $\chi^e_\nu = 0.95$ as used in ref. [14]). The results of the present work are also compared to the values obtained by means of the parameter values of Bijker [14]). As can be seen from table 3 a, b the present values have approximately the same quality as those from the IBM fit. It should be noted that most of the calculated values are not very sensitive to the parameters in the E2 operator, with the exception of the B(E2) ratios for the transition form the excited 0^+ states to the first and second 2^+ states. The latter are sensitive to a change in parameter value due to a strong cancellation between proton and neutron contributions. It is clear that the calculated values for the transition strengths within a band are closer to the experimental values than in the case of interband transitions.

The results obtained in this section show that the broken SU(3)* symmetry gives a reasonable description, where many experimental values of observables are reproduced. Some discrepancies found indicate that the present form of the hamiltonian is not completely satisfactory, e.g., most transition strengths from a $K = 2$ to a $K = 0$ state are underestimated.

8. Conclusions

The SU(3)* limit of IBM-2, which describes triaxial nuclei, presents a good labelling scheme for the spectrum of ^{192}Os. The addition of a symmetry breaking part consisting of single-boson energies only, gives a good description, and also for E2 transition strengths.

The SU(3)* limit contains also scissors mode states. The complete spectrum of these states is that of a one-dimensional harmonic oscillator.

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Appendix

In this appendix a tabulation of the branching ratios for the group reduction $U^{\pi\nu}(6)^* \supset SU^{\pi\nu}(3)^*$ is given, for $U^{\pi\nu}(6)^*$ irreps as found in IBM-2.

The product of a proton and a neutron U(6) irrep decomposes as (eq. (14))

$$\begin{array}{ccc} U^{\pi}(6)\times U^{\nu}(6) \supset & & U^{\pi\nu}(6) \\ | \quad\quad | & & | \\ [N_\pi] \times [N_\nu] = & \sum_{i=0}^{\min(N_\pi,N_\nu)} & [N_\pi - i, 0^4, i-N_\nu]\,. \end{array} \quad (A.1)$$

From eq. (A.1) the following identity is readily verified

$$[x, 0^4, -y] = [x]\times\overline{[y]} - \begin{cases} [x-1]\times\overline{[y-1]} & (x, y>0) \\ 0 & (x=0 \text{ or } y=0)\,. \end{cases} \quad (A.2)$$

The SU(3) decomposition of the symmetric irreps is known:

$$\begin{array}{cc} U^{\pi}(6) \supset & SU^{\pi}(3) \\ | & | \\ [N_\pi] = & \sum_{i,j} (2N_\pi - 6i - 4j, 2j)\,, \end{array} \quad (A.3)$$

$$\begin{array}{cc} \overline{U^{\nu}(6)} \supset & \overline{SU^{\nu}(3)} \\ | & | \\ [N_\nu] = & \sum_{k,l} (2l, 2N_\nu - 6k - 4l)\,. \end{array} \quad (A.4)$$

One thus needs to multiply the SU(3) contents of the symmetric irreps in eq. (A.2) and subtract the results of the two products to obtain the SU(3) contents of the irrep on the left-hand side of eq. (A.2).

The tabulation is restricted to those $U^{\pi\nu}(6)^*$ irreps for which $N_\pi + N_\nu < 9$. A more complete table can be obtained from the authors [17]).

$$[1] = (2,0)$$

$$[2] = (4,0) + (0,2)$$

$$[1, 0^4, -1] = (2,2) + (1,1) + (0,0)$$

$$[3] = (6,0) + (2,2) + (0,0)$$

$$[2, 0^4, -1] = (4,2) + (0,4) + (3,1) + (1,2) + 2(2,0)$$

$$[4] = (8,0) + (4,2) + (0,4) + (2,0)$$

$$[3, 0^4, -1] = (6,2) + (5,1) + (2,4) + (3,2) + 2(4,0) + (1,3) + (2,1) + 2(0,2)$$

 $(2,2)+2(1,1)+2(0,0)$

$$[2,0^4,-2]=(4,4)+(6,0)+(0,6)+(3,3)+(4,1)+(1,4)+4$$

$$[5]=(10,0)+(6,2)+(2,4)+(4,0)+(0,2) \qquad (3,3)+(4,1)$$

$$[4,0^4,-1]=(8,2)+(7,1)+(4,4)+(5,2)+2(6,0)+(0,6)+$$

$$+(1,4)+3(2,2)+(1,1)+(0,0) \qquad 1,5)+4(4,2)$$

$$[3,0^4,-2]=(6,4)+(8,0)+(2,6)+(5,3)+(6,1)+(3,4)+($$

$$+2(2,3)+3(0,4)+3(3,1)+2(1,2)+4(2,0) \qquad (0,0)$$

$$[6]=(12,0)+(8,2)+(4,4)+(0,6)+(6,0)+(2,2)+ \text{-}(5,3)+(6,1)$$

$$[5,0^4,-1]=(10,2)+(9,1)+(6,4)+(7,2)+2(8,0)+(2,6)+) + (1,2)+2(2,0)$$

$$+(3,4)+(1,5)+3(4,2)+(2,3)+2(0,4)+(3,1(0,8)+4(6,2)$$

$$[4,0^4,-2]=(8,4)+(10,0)+(7,3)+(4,6)+(8,1)+(5,4)+, 2)+5(4,0)+3(1,3)$$

$$+(3,5)+(1,6)+2(4,3)+3(5,1)+5(2,4)+3(3$$

$$+2(2,1)+4(0,2) \qquad ,1)+(1,7)$$

$$[3,0^4,-3]=(6,6)+(8,2)+(2,8)+(5,5)+(6,3)+(3,6)+(7(3,3)+3(4,1)$$

+ [illegible](4, 4) + [illegible](5, 2) + [illegible] (5, 0)

$$[7]=(14,0)+(10,2)+(6,4)+(8,0)+(2,6)+(4,2)+(0,4)+(2,0)$$

$$[6,0^4,-1]=(12,2)+(11,1)+(8,4)+(9,2)+2(10,0)+(7,3)+(4,6)+(8,1)$$

$$+(5,4)+(0,8)+3(6,2)+(3,5)+(1,6)+(4,3)+(5,1)+3(2,4)$$

$$+(3,2)+2(4,0)+(1,3)+(2,1)+2(0,2)$$

$$[5,0^4,-2]=(10,4)+(12,0)+(9,3)+(10,1)+(6,6)+(7,4)+4(8,2)+(2,8)$$

$$+(5,5)+2(6,3)+(3,6)+3(7,1)+(1,7)+5(4,4)+3(5,2)+2(2,5)$$

$$+5(6,0)+3(0,6)+4(3,3)+3(4,1)+3(1,4)+7(2,2)+2(1,1)$$

$$+2(0,0)$$

$$[4,0^4,-3]=(8,6)+(10,2)+(4,8)+(7,5)+(8,3)+(0,10)+(5,6)+(9,1)$$

$$+(3,7)+4(6,4)+(1,8)+2(7,2)+3(8,0)+2(4,5)+5(2,6)+5(5,3)$$

$$+3(6,1)+5(3,4)+4(1,5)+10(4,2)+(5,0)+5(2,3)+6(0,4)$$

$$+6(3,1)+4(1,2)+6(2,0)$$

$$[8]=(16,0)+(12,2)+(8,4)+(10,0)+(4,6)+(0,8)+(6,2)+(2,4)+(4,0)$$

$$+(0,2)$$

$$[7, 0^4, -1] = (14, 2) + (13, 1) + (10, 4) + (11, 2) + 2(12, 0) + (9, 3) + (10, 1) + (6, 6) + (7, 4) + 3(8, 2) + (2, 8) + (5, 5) + (6, 3) + (3, 6) + (1, 7) + (7, 1) + 3(4, 4) + (2, 5) + (5, 2) + 2(6, 0) + 2(0, 6) + (3, 3) + (4, 1) + (1, 4) + 3(2, 2) + (1, 1) + (0, 0)$$

$$[6, 0^4, -2] = (12, 4) + (14, 0) + (11, 3) + (12, 1) + (8, 6) + (9, 4) + 4(10, 2) + (4, 8) + (7, 5) + 2(8, 3) + (0, 10) + (5, 6) + 3(9, 1) + (3, 7) + 5(6, 4) + (1, 8) + 3(7, 2) + 5(8, 0) + 2(4, 5) + 5(2, 6) + 4(5, 3) + 3(6, 1) + 4(3, 4) + 3(1, 5) + 8(4, 2) + 3(2, 3) + 5(0, 4) + 3(3, 1) + 2(1, 2) + 4(2, 0)$$

$$[5, 0^4, -3] = (10, 6) + (12, 2) + (9, 5) + (6, 8) + (10, 3) + (11, 1) + (7, 6) + (2, 10) + 4(8, 4) + (5, 7) + 2(9, 2) + 3(10, 0) + (3, 8) + 2(6, 5) + (1, 9) + 5(7, 3) + 5(4, 6) + 3(8, 1) + 2(2, 7) + 5(5, 4) + 3(0, 8) + 10(6, 2) + 6(3, 5) + (7, 0) + 4(1, 6) + 7(4, 3) + 7(5, 1) + 11(2, 4) + (0, 5) + 7(3, 2) + 8(4, 0) + 6(1, 3) + 4(2, 1) + 6(0, 2)$$

$$[4, 0^4, -4] = (8, 8) + (10, 4) + (4, 10) + (7, 7) + (12, 0) + (0, 12) + (5, 8) + (8, 5) + (3, 9) + (9, 3) + 4(6, 6) + (10, 1) + (1, 10) + 2(7, 4) + 2(4, 7) + 5(8, 2) + 5(2, 8) + 5(5, 5) + 5(6, 3) + 5(3, 6) + 4(7, 1) + 4(1, 7) + 13(4, 4) + 6(5, 2) + 6(2, 5) + 7(6, 0) + 7(0, 6) + 11(3, 3) + 7(4, 1) + 7(1, 4) + 14(2, 2) + (3, 0) + (0, 3) + 5(1, 1) + 4(0, 0) .$$

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