Emergence of a 4D World from Causal Quantum Gravity

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Causal Dynamical Triangulations in four dimensions provide a background-independent definition of the sum over geometries in nonperturbative quantum gravity, with a positive cosmological constant. We present evidence that a macroscopic four-dimensional world emerges from this theory dynamically.

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Introduction.—String theory has highlighted the fact that the notion of "dimension" in a quantum theory of all fundamental interactions has a very different status from that in any classical field theory or any perturbative quantum field theory on a fixed background, where the dimension of spacetime remains fixed throughout. This may be seen as a particular case of the more general truth, not always appreciated, that in any nonperturbative theory of quantum gravity dimension will become a dynamical quantity, along with other aspects of geometry. (By dimension we mean an effective dimension observed at macroscopic scales).

This Letter deals with an approach to quantum gravity in which the dynamical property of spacetime dimension is particularly transparent [1]. The approach is that of "nonperturbative quantum gravity from Lorentzian dynamical triangulations" or, for short, "Causal Dynamical Triangulations". In it, the sum over all spacetime geometries takes the form of a sum over four-geometries constructed from discrete building blocks [12]. Each collection of building blocks, glued together according to a set of rules, represents a geometry contributing to the path integral. To extract its physical geometric properties one has to perform the path integral, renormalize, and evaluate the expectation values of appropriate observables, including the effective (or Hausdorff) dimension d_h.

Note that the dynamical nature of "dimensionality" implies that the Hausdorff dimension of the quantum geometry is *not a priori* determined by the dimensionality at the cut-off scale a, which is simply the fixed dimensionality d of the building blocks of the regularized version of the theory. An example in point are the attempts to define theories of quantum geometry via "*Euclidean* Dynamical Triangulations", much-studied during the 1980s and "90s. In these models, if the dimension d is larger than 2, and if all geometries contribute to the path integral with equal weight, a geometry with no linear extension and $d_h = \infty$ is created with probability one. If instead—as is natural for a gravity-inspired theory—the Boltzmann weight of each geometry is taken to be the exponential of (minus) the Euclidean Einstein-Hilbert action, one finds for small values of the bare gravitational coupling constant a first-order phase transition to a phase of the opposite extreme, namely, one in which the quantum geometry satisfies $d_h = 2$. This is indicative of a different type of degeneracy, where typical (i.e. probability one) configurations are so-called branched polymers or trees (see [13–19] for details of the phase structure and geometric properties of the four-dimensional Euclidean theory). Unfortunately, this geometric degeneracy is also reflected in the expectation values of other geometric quantities, and makes Euclidean Dynamical Triangulations an unlikely candidate for a theory of four-dimensional quantum gravity.

The requirement that a background-independent quantum gravity theory possess the correct semiclassical limit, given by a macroscopically four-dimensional spacetime with microscopic quantum fluctuations, is highly nontrivial. The apparent failure of higher-dimensional Euclidean nonperturbative quantum gravity to do so (not limited to this particular approach [20]), together with a desire to formulate a quantum gravity with the correct Lorentzian signature and causal properties [21] were our main reasons for suggesting a radically different approach to the sampling of geometries in the path integral [22–25]. A further motivation was to construct a path integral more closely related to canonical formulations of quantum gravity. Starting from Lorentzian simplicial spacetimes with $d = 4$ we insist that only causally well-behaved geometries appear in the (regularized) path integral, as described in detail in [25]. A further crucial property of our explicit construction is that each configuration allows for a rotation to Euclidean signature, a necessary prerequisite for discussing the convergence properties of the sum over geometries, as well as for using Monte Carlo techniques.

In what follows we will report on the outcome of the first ever Monte Carlo simulations of four-dimensional causal dynamical triangulations. It differs radically from what was found in previous simulations of four-dimensional *Euclidean* dynamical triangulations. We will present strong evidence that the Lorentzian framework produces a quantum geometry which is both extended and effectively four-dimensional. This is to our knowledge the first example of a theory of quantum gravity that generates a quantum spacetime with such properties *dynamically.*

The emergent macroscopic 4d geometries.—All causal simplicial spacetimes contributing to the path integral are foliated by an integer-valued "proper-time" t, and each geometry can be obtained by gluing together four-simplices in a way that respects this foliation. Each four-simplex has timelike links of length-squared a_t^2 and spacelike links of length-squared a_s^2, with all of the latter being located in spatial slices of constant integer-t. These slices consist of purely spacelike tetrahedra, forming a three-dimensional piecewise flat manifold, whose topology we choose to be that of a three-sphere S^3.

Each spatial tetrahedron at time t is shared by two four-simplices [said to be of type (1,4) and (4,1)] whose fifth vertex lies in the neighboring slice of time $t-1$ and $t+1$, respectively. In addition we need four-simplices of type (2,3) and (3,2) which share one link and one triangle with two adjacent spatial slices [25]. Let us assume the link lengths are related by

$$a_t^2 = -\alpha a_s^2. \tag{1}$$

All choices $\alpha > 0$ correspond to Lorentzian and all choices $\alpha < -7/12$ to Euclidean signature, and a Euclideanization of geometry amounts to a suitable analytic continuation in α. Setting $\alpha = -1$ leads to a particularly simple expression for the (Euclidean) Einstein-Hilbert action of a given triangulation T (since all four-simplices are identical geometrically), namely,

$$S(T) = -k_0 N_0(T) + k_4 N_4(T), \tag{2}$$

with $N_i(T)$ denoting the number of i-dimensional simplices in T. In (2), k_0 is proportional to the inverse (bare) gravitational coupling constant, $k_0 \sim 1/G$, while k_4 is a linear combination of the cosmological and inverse gravitational coupling constants. The action (2) has been calculated from Regge's prescription for piecewise linear geometries. If we take $\alpha \neq -1$ the Euclidean four-simplices of type (1,4) and type (2,3) will be different and thus appear with different weights in the Einstein-Hilbert action [25]. For our present purposes it is convenient to use the equivalent parameterization

$$S(T) = -k_0 N_0(T) + k_4 N_4(T) + \Delta[2N_{14}(T) + N_{23}(T)], \tag{3}$$

where $N_{14}(T)$ and $N_{23}(T)$ denote the combined numbers in T of four-simplices of types (1,4) and (4,1) and of types (2,3) and (3,2), respectively. The explicit map between the parameter Δ in Eq. (3) and α can be readily worked out. For the simulations reported here we have used Δ's in the range 0.4–0.6.

The quantum-gravitational proper-time propagator is defined by

$$G_{k_0,k_4,\Delta}[T^{(3)}(0), T^{(3)}(t)] = \sum_{T_t} \frac{1}{C_{T_t}} e^{-S(T_t)}. \tag{4}$$

where the summation is over all four-dimensional triangulations T_t of topology $S^3 \times [0,1]$ and t proper-time steps, whose spatial boundary geometries at proper times 0 and t are $T^{(3)}(0)$ and $T^{(3)}(t)$. The order of the automorphism group of the triangulation T_t is denoted by C_{T_t}. The propagator can be related to the quantum Hamiltonian conjugate to t, and in turn to the transfer matrix of the (Euclidean) statistical theory [25].

While it may be difficult to find an explicit analytic expression for the full propagator (4) of the four-dimensional theory, Monte Carlo simulations are readily available, employing standard techniques from Euclidean dynamically triangulated quantum gravity [26]. Ideally one would like to keep the *renormalized* cosmological constant Λ fixed in the simulation [27]. The presence of the cosmological term $\Lambda \int \sqrt{g}$ in the action then implies that the four-volume V_4 fluctuates around $\langle V_4 \rangle \sim \Lambda^{-1}$. However, for simulation-technical reasons one fixes in-

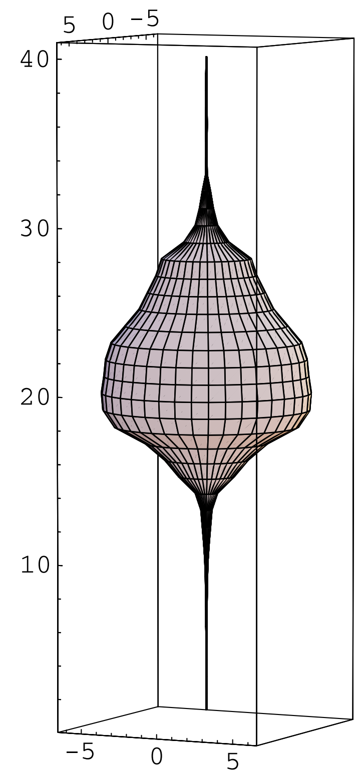

FIG. 1 (color online). Snapshot of a "typical universe" of volume 91.1 k from the Monte Carlo simulations. The total time extent (vertical direction) is $t = 40$ as indicated. The circumference at integer-t is the spatial three-volume $V_3(t) \times 0.02$ (in units where $a_s = 1$). The surface represents an interpolation between adjacent "spatial volumes". No attempt has been made to capture the actual 4d connectivity between neighboring spatial slices.

stead the average number $\langle N_4 \rangle$ of four-simplices, or $\langle V_4 \rangle$, from the outset [28], effectively working with a cosmological constant $\Lambda \sim \langle V_4 \rangle^{-1}$.

Figure 1 shows a snapshot of a configuration with $N_4 \approx 91100$ four-simplices as it appears in the Monte Carlo simulations. Important information is contained in how the volume V_3 of spatial slices and the time extent τ of the observed universe [29] behave in relation to the total spacetime volume V_4. To good approximation we find that the spatially extended parts of the spacetimes for various four-volumes V_4 can be mapped onto each other by rescaling the spatial volumes and the proper times according to

$$V_3 \to V_3/V_4^{3/4}, \qquad \tau \to \tau/V_4^{1/4}. \tag{5}$$

This is illustrated in Fig. 2, displaying the so-called volume-volume correlator

$$\langle V_3(0)V_3(\delta)\rangle = \frac{1}{t^2}\sum_{j=1}^{t} V_3(j)V_3(j+\delta) \tag{6}$$

measured for three different spacetime volumes V_4, using the rescaling (5), and where we have periodically identified $T_3(t) = T_3(0)$. The detailed statistical analysis leading to (5), (6) and the measurement of the spatial Hausdorff dimension below will be published elsewhere [30] (see also [23] for an analogous treatment in three dimensions).

Relation (5) strongly suggests that what we "observe" in the simulations are genuine four-dimensional universes whose spatial hypersurfaces are three-dimensional. To test this we have recorded the average geodesic distance $\langle r \rangle_{V_3}$ between points in the spatial three-volumes V_3. If the spatial geometry is indeed three-dimensional one expects a leading behavior

$$\langle r \rangle_{V_3} \sim V_3^{1/3}. \tag{7}$$

Figure3 shows the results of measuring $\langle r \rangle_{V_3}$ in the macroscopic spatial slices of our quantum universes [31]. It combines the data for universes with total $N_4 \approx 45.5$, 91.1, 184.4 and 371.2 k. The straight line in Fig. 3 has merely been drawn to guide the eye; it corresponds to the three-dimensional behavior (7). From these combined measurements, the evidence for *three*-dimensional slices is rather compelling. A best fit for the spatial Hausdorff dimension d_h in the relation $\langle r \rangle_{V_3} \sim V_3^{1/d_h}$ yields $d_h = 3.10 \pm 0.15$. As should be clear from the Introduction, reproducing this "classical" dimensionality dynamically from a fully nonperturbative formulation of quantum gravity constitutes a highly nontrivial result. However, it should be kept in mind that a property like $\langle r \rangle_{V_3} \sim V_3^{1/3}$ provides only a crude characterization of the spatial geometry, and by no means implies that the space "observed" in the computer simulations is a nice smooth 3d space at short distances. Further details about the simulations, measurements, fits, and the complete phase diagram of the model will be published shortly [30].

Discussion.—Causal dynamical triangulations are a framework for defining quantum gravity nonperturbatively as the continuum limit of a well-defined regularized sum over geometries. Interestingly, and in complete agreement with current observational data is the fact that the physical cosmological constant Λ in dynamical triangulations is necessarily positive [33].

Recall that the effective cosmological constant in our simulation is $1/\langle V_4 \rangle$. Since, according to (5), both the spatial three-volumes and the time extension of our quantum universe relate to Λ canonically (that is, in a way expected from their classical dimensionality), it is appropriate to call this universe macroscopic.

This leads us to conclude that we have observed the emergence of a four-dimensional macroscopic world with three-dimensional spatial geometries. Judging from the computer simulations, this dynamically generated quantum geometry acts as a background geometry around which small quantum fluctuations take place. Further

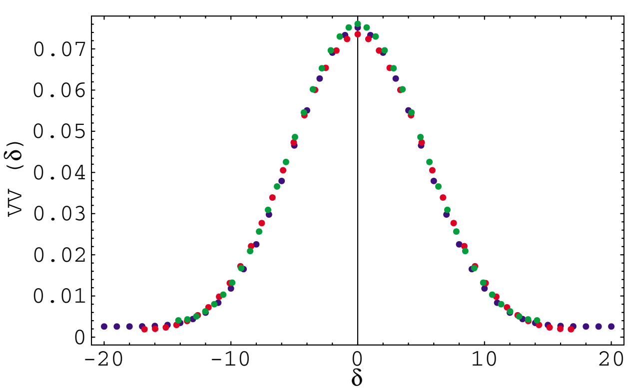

FIG. 2 (color online). Measurement of spatial volume-volume correlators for spacetimes with $N_4 \approx 45.5$, 91.1, and 184.4 k, and after performing the rescaling (5).

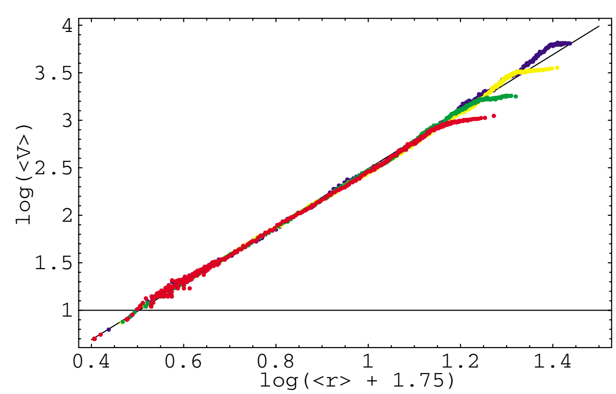

FIG. 3 (color online). The log-log plot of the average geodesic distance $\langle r \rangle_{V_3}$ versus V_3. It combines data from universes where $N_4 \approx 45.5$, 91.1, 184.4 and 371.2 k.

numerical studies will be needed to make this into a quantitative statement. The situation is really rather remarkable: we started out from an explicitly background-independent formalism and have rederived a particular stable background geometry. Obviously, questions about the nonrenormalizability of perturbative gravity will have to be readdressed in this new context (maybe along the lines outlined in [34]), with the benefit of having an explicit microscopic model of quantum spacetime and its quantum fluctuations. We hope future simulations and analytic studies of the model will teach us how to reconcile these aspects. The final picture may be that of an asymptotically safe theory in the sense of Weinberg [35], or turn out to require a completely new theoretical framework.

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