ISOTOPIC SPIN SELECTION RULES AND THE DECAY $K_L \to 2\pi$

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Recently, measurements have been performed on the decay of $K_L \to 2\pi^0$ [1], and it has been found that in all cases the final 2π state in $K_L \to 2\pi$ contains a rather large $I = 2$ admixture. Effectively there are two solutions [2], namely

1) The final state of two pions is dominantly $I=0$ with a 25% $I=2$ admixture.

2) The final state of two pions is practically a pure $I=2$ state.

Where possibility 1) is compatible with a good many proposed theories, notably with a theory of C-violating semi-strong interactions obeying $\Delta I = 0$ [3], or with a theory of C-violating e.m. interactions with $\Delta I = 0$ or 1 [4], the possibility 2) does not seem to correspond clearly to any proposed model. Logically, in an analysis of the properties of CP-violating interactions one must face possibility 2) also; it is the purpose of this paper to investigate in some detail the consequences of 2) [5]. Before doing so, however, we feel obliged to point out that in any case Weinberg's cancellation effect [6] will tend to suppress (how much is anybody's guess) an eventual $K_L \to 2\pi$ ($I=0$) transition with respect to $K_L \to 2\pi$ ($I=2$). For this reason, even if the solution 2) is experimentally established with some accuracy, one cannot take up assumption b) of the next section too confidently, but must rather see it as a possibility to be tested experimentally.

In order to derive some properties concerning the I-spin behaviour of the CP-violating interactions we make the following assumptions, consistent with experiment:

a) The final 2π state in $K_L \to 2\pi$ is pure $I=2$ *.

b) The absence of $K_L \to 2\pi$ ($I=0$) is not an accident but $K_L \to 2\pi$ ($I=0$) is forbidden *.

c) The CP-conserving non-leptonic strangeness changing interactions obey the rule $\Delta I = \frac{1}{2}$ **.

* We neglect effects of second or higher order in the CP-violating interaction.

** The conclusions of this paper are stable against a small $\Delta I = \frac{3}{2}$ or $\frac{5}{2}$ admixture.

Assumptions a), b) and c) imply that $\epsilon = 0$ [2]. This in turn implies that the CP-violating interaction must be such that the mass-matrix governing the composition of K_L and K_S in terms of K_0 and $\bar{K}_0$ is not affected *. Thus $K_S = K_1$, $K_L = K_2$, as if there were no CP-violation. We must now distinguish the various possibilities for the CP-violating interaction.

The CP-violating interaction obeys $\Delta S = 0$ (i.e. it belongs to the class of C-violating strong interactions). The mass-matrix is determined by second order weak transitions from a K-meson or anti-meson to a K-meson or anti-meson. From assumption c), and the fact that the K-meson belongs to an $I = \frac{1}{2}$ multiplet we conclude that the requirement of no C-violating effects in the mass-matrix leads to the rule $\Delta I \geq 3$ for the C-violating interaction. Allowing $K \to 2\pi$ ($I=2$) to first order in weak and C-violating interactions leads to the conclusion that at least part of the C-violating interaction must obey $\Delta I \leq 3$. Thus we conclude for the C-violating interaction:

No $\Delta I = 0, 1, 2$;

There must be a part with $\Delta I = 3$ †.

The immediate consequence of these selection rules is that no C-violating effects should exist in the decay $\eta \to 3\pi$, as only $\Delta I = 0$ or 2 C-violating interactions can contribute to such effects [3].

The C-violating interaction is of e.m. origin [4]. We now add the customary assumption:

d) The C-conserving e.m. interactions obey the rule $\Delta I = 0$ or 1.

The same reasoning as given above leads to the conclusion that C-violating effects due to virtual photons must obey $\Delta I \leq 3$, $\Delta I = 3$ being present. Now in processes of this kind we find two photon vertices present, one C-conserving obeying $\Delta I = 0$ or 1, and a C-violating vertex. The absence of $\Delta I = 0, 1, 2$ necessitates the rule:

† The author feels humble in the face of this high number.

The C-violating e.m. interaction obeys

$\Delta I \geqslant 4$, $\Delta I = 4$ being present.

As an immediate consequence we find:

No C-violating effects in $\eta \to 3\pi$;

Possibly large $\Delta I > 2$ effects in processes involving virtual photons (arising from the occurence of two C-violating e.m. vertices with $\Delta I = 4$). In particular possibly a large $\Delta I = 3$ contribution to $\eta \to 3\pi$;

No CP-violating effects in $K \to \pi\pi\gamma$.

The CP-violating interaction is weak and allows $\Delta S = 1$. The mass-matrix could now receive contributions of a CP-violating nature if the transition from a K or $\overline{K}$ to a K or $\overline{K}$ can proceed via one CP-violating weak interaction and one CP-conserving weak interaction. To avoid this we must impose $\Delta I \geqslant \frac{5}{2}$. Again, the necessity of having C-violating contributions to $K_L \to 2\pi$ ($I = 2$) necessitates $\Delta I = \frac{5}{2}$ to be present. Thus:

The CP-violating weak interaction obeys

$\Delta I \geqslant \frac{5}{2}$, $\Delta I = \frac{5}{2}$ being present.

For clarity it must be remarked that this CP-violating weak interaction is weaker by a factor of about 10^{-2} compared to the ordinary CP-conserving weak interactions. Therefore we must in this case not expect large effects in for instance $K \to \pi\pi\gamma$.

For non-leptonic, non-radiative weak processes all three cases produce equal predictions. In particular, no CP-violating effects in $\Lambda \to N\pi$, possibly small effects in $\Sigma \to N\pi$, possibly substantial effects in $K \to 3\pi$, as to be discussed below.

Experimental verification. If it turns out that assumptions a) and c) hold, i.e. if the phase between $K_L \to 2\pi^0$ and $K_S \to 2\pi^0$ differs by 180^o from the phase between $K_S \to \pi^+\pi^-$ and $K_L \to \pi^+\pi^-$ then it is very desirable, especially in view of the remarks in the beginning, to make a further test such as to be assured that assumption b) holds. To this purpose the decay $K \to 3\pi$, and especially interference experiments between $K_L \to 3\pi$ and $K_S \to 3\pi$ might be helpful.

The eventual effects of CP-violation in $K \to 3\pi$ decays have been studied extensively by Gaillard [7]. We just quote here the essential features. For definiteness we assume a C-violating semistrong interaction obeying $\Delta I = 3$, $\Delta S = 0$, with a coupling strength of about 10^{-2}. The three pions may be in a state of I-spin 0, 1, 2 or 3. The (totally anti-symmetric) $I = 0$ state is severely inhibited by angular momentum barriers and we will ignore it. The CP-conserving weak interactions obeying $\Delta I = \frac{1}{2}$ will then allow only $K_L \to 3\pi$ ($I = 1$) (remember $K_L = K_2$, $K_S = K_1$). The CP-violating transitions that may be obtained are $K_S \to 3\pi$ ($I = 3$) and $K_L \to 3\pi$ ($I = 2$). Interference may then be observed between $K_S \to 3\pi$ ($I = 3$) and $K_L \to 3\pi$ ($I = 1$, totally symmetrical). The ratio R_i of interference $K_S \to 3\pi^0$, $K_L \to 3\pi^0$ versus $K_S \to \pi^+\pi^-\pi^0$, $K_L \to \pi^+\pi^-\pi^0$ is then fixed:

$$|R_i| = \frac{1}{6}\left|\frac{\langle\pi^0\pi^0\pi^0|K_S\rangle\langle\pi^0\pi^0\pi^0|K_L\rangle}{\langle\pi^+\pi^-\pi^0|K_S\rangle\langle\pi^+\pi^-\pi^0|K_L\rangle}\right| = \left|\frac{(2)(-3)}{6}\right| = 1,$$

where the factor 6 arises from Bose statistics, the $3\pi^0$ phase space being 3! times smaller than the $\pi^+\pi^-\pi^0$ phase space.

If the C-violating interaction allows $\Delta I < 3$ also we can have $K_S \to 3\pi (I = 1)$ interfering with $K_L \to 3\pi$ ($I = 1$). This would in general give rise to an $|R_i|$ different from 1.

Leptonic decays. In case that the CP-violating interaction is semi-strong or e.m. statements can be made on CP-violation in leptonic decays. From $\Delta I \geqslant 3$ we conclude that no effects can show up in $K \to \pi\mu\nu$ or $\pi e\nu$ but some effects might be expected in for instance $\nu + N \to N^* + \mu$ (although they might be hard to detect). Also $K \to \pi\pi e\nu$ might be interesting in this respect, especially as the CP-violating effects are kinematically slightly favoured. In this decay the 2π system is, according to the rule $\Delta I = \frac{1}{2}$ for leptonic decays through ordinary weak interactions, in a state with $I = 0$ or 1. The C-violating interaction may give rise to the 2π ($I = 2$, S-wave) state, and this could for instance be observed by inspecting the π^+ distributions in $K_L \to \pi^+\pi^0 e^-\overline{\nu}$ with those of the π^- in $K_L \to \pi^-\pi^0 e^+\nu$.

Finally it must be remarked that no CP-violating effects should occur in $K_L, K_S \to \pi e\nu$. The reason is that for this decay the situation is exactly as without CP-violation: $K_L = K_2$, $K_S = K_1$ and there are no effects on the amplitudes $K_0, \overline{K}_0 \to \pi e\nu$.

Further speculations. A very important experiment on the question of CP-violation is the measurement of the neutron electric dipole moment (see the review of Prentki [3], end of section on theories of type 2). An electric dipole moment violates P and CP, and in case of C-violating semi-strong or e.m. interactions a parity violating but CP conserving weak interaction with $\Delta S = 0$ must be present also in order to generate such a moment. Lack of knowledge on the I-spin properties of non-leptonic, $\Delta S = 0$ weak interactions prevents us from making any definitive statement.

If, for definiteness we assume $\Delta I = 0$ for this weak interaction than, in the case of a C-violating $\Delta I = 3$ semi-strong, or an e.m. C-violating $\Delta I = 4$ interaction no moment should arise, to first order in those interactions. In this case we expect the neutron electric dipole moment to be of order 10^{-24}. It may be noted that C-violating semi-strong interactions causing the situation 1) of the first section would be effective for the electric dipole moment, assuming $\Delta I \leq 2$ for the weak interaction (giving about 10^{-22}). An e.m. C-violating interaction with $\Delta I \leq 3$ may give rise to a moment of 10^{-20} or 10^{-22}, depending on the I-spin selection rules for the e.m. and weak interactions.

With respect to a possible $\Delta I = 4$ C-violating e.m. interaction it may be noted that such an interaction would upset the Coleman-Glashow e.m. mass difference formula. As this formula works very well this must be considered as an argument against such an interaction.

Finally we wish to speculate for a moment on the possibility of a C-violating, $\Delta I = 3$ semi-strong interaction. As is well known some discrepancies exist in $\eta \to 3\pi$ such as that perhaps a $\Delta I = 3$, C-conserving interaction [8] * is necessary to explain the data. The coupling constant involved would be of order 10^{-2}, i.e. comparable to the coupling constant α expected for the e.m. transition $\eta \to 3\pi$ ($I = 1$) (see however ref. 7). If so it is tempting to argue that both the $\Delta I = 3$ C-violating and C-conserving semi-strong interactions are essentially two manifestations of some new fundamental interaction. In view of the fact that the experimental situation with respect to $\eta \to 3\pi$ is still very fluid, while also assumptions a) and b) are not yet established we do not go further into this.

* We wish to point out that knowledge of the Dalitz plot of $\eta \to 3\pi^0$ may serve to strengthen (or weaken) the arguments concerning $\Delta I = 3$ in $\eta \to 3\pi$. Many of the final state interaction models proposed in the literature to explain the low $\eta \to 3\pi^0$ rate produce quite amusing Dalitz plot distributions for $\eta \to 3\pi^0$, deviating substantially from the flat distribution expected if the final state interactions do not vary appreciably over the range of energy concerned. For an interesting attempt to explain a possible $\Delta I = 3$ admixture with the help of C-violating e.m. currents see ref.9.

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