PHYSICAL BASIS OF BALLISTOCARDIOGRAPHY. IV

The Relative Movement of Subject and Ballistocardiograph

H. C. Burger, D.Sc., A. Noordergraaf, M.Sc., J. J. M. Korsten, M.Sc., and P. Ullersma, B.Sc.

Utrecht, Netherlands

INTRODUCTION

To gather information about human and animal circulatory problems several types of ballistocardiographs are in use. By the periodic contraction of the heart, mass displaces within the body (only the longitudinal axis is attended). Assuming that the binding between the subject's body and the ballistocardiograph (bcg; the ballistocardiogram will be abbreviated as BCG) was infinitely strong and the subject himself or herself was a rigid body, it appeared from a preceding paper[1] that the following quantity concerning the common center of gravity of subject and bcg is measured if the *displacement* of the bcg is recorded:

A. The displacement of the common center of gravity of subject and bcg, using a *low-frequency* bcg, according to Gordon,[2] Henderson,[3] and Burger,[4]

B. The velocity of the center of gravity using a *middle-frequency* bcg, according to Nickerson,[5] and

C. The acceleration of the center of gravity, using a *high-frequency* bcg, according to Starr.[6]

The requirements that these bcg's have to meet follow from considerations discussed in one of the above-mentioned papers.[1] Moreover, it has been shown in that paper what are the mutual relations between the records procured by these types of bcg's, and in which way all three quantities, displacement, velocity, and acceleration, concerning the common center of gravity of subject and bcg, were to be found by any of the three above-mentioned types of bcg's. The last mentioned idea has been worked out and applied already for the low-frequency bcg by Elsbach[7] and, but partly, for the Dock type by Smith and Bryan.[8]

The force acting on the body follows from the recorded acceleration by multiplication of the moving mass of subject and bcg and its acceleration at every moment during a heart cycle.

However, in reality the binding between the subject's body and the bcg is not infinitely strong. Because of the tissue layer between the skeleton of the subject and the bcg there will be a difference in movement of subject and bcg.

From the Department of Medical Physics, Physical Laboratory of the University, Utrecht.
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To this phenomenon attention was drawn for the first time, as far as we know, in our paper concerning the physical basis of ballistocardiography,[4] in which this difference in movement, the "relative movement," was shown experimentally for the low-frequency bcg.

In the present paper the influence of this relative movement of subject and bcg will be investigated for the three mentioned types of bcg's and for the Dock type (direct-body type). Some results of this investigation were communicated in the discussion by the present authors of a paper by Nickerson.[9,10] The movement of the heart with respect to the skeleton will not be discussed in this paper.

THEORY

In its simplest form the problem of the relative movement of subject and bcg is a special form of a more general problem: that of two coupled harmonic oscillators.

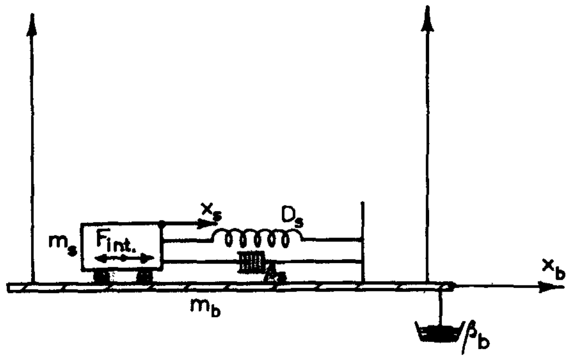


Fig. 1.—Schematical representation of the ballistocardiographic system in which the relative movement is taken into account. The subject with mass m_s is coupled to the bcg with mass m_b by a directive force and a frictional force. The whole system is coupled to the surroundings in an analogous way.

The problem in the form we have to face it can schematically be represented as in Fig. 1, as was done in a preceding paper.[9] The subject with mass m_s is represented by a cart that is coupled to the bcg with mass m_b by a directive force and a frictional force. The system of subject and bcg is coupled to the surroundings in the same way.

The internal force F_{int} equals $m_s\ddot{x}_c$, x_c, $\dot{x}_c$, and $\ddot{x}_c$, respectively, being the displacement, the velocity, and the acceleration of the center of gravity, calculated with respect to the skeleton and positive to the left in Fig. 1 (footward). The force exerted on the body equals this force but has an opposite direction (action equals reaction). The displacement, x_s, the velocity, $\dot{x}_s$, and the acceleration, $\ddot{x}_s$, of the subject are chosen positive to the right in Fig. 1 (headward). The same holds good for the respective quantities x_b, $\dot{x}_b$, and $\ddot{x}_b$ of the bcg. The frictional force acting between subject and bcg is assumed proportional to the difference in velocity of subject and bcg and acts in a direction opposite to this difference. So the frictional force exerted on the subject by the bcg can be written in the form $-\beta_s(\dot{x}_s - \dot{x}_b)$ and that exerted on the bcg by the subject $\beta_s(\dot{x}_s - \dot{x}_b)$. In an analogous way the directive force acting on the body can be written in the form $-D_s\ (x_s - x_b)$; that exerted on the bcg by the body has again the opposite sign. D_s is the proportionality factor between the difference in displacement and the force. The total force acting on the subject equals the product of the mass m_s of the subject and its acceleration $\ddot{x}_s$, so:

$$m_s\ddot{x}_c - \beta_s\ (\dot{x}_s - \dot{x}_b) - D_s\ (x_s - x_b) = m_s\ddot{x}_s \tag{1a}$$

In the same way the differential equation for the movement of the bcg can be found. Besides the two forces $\beta_s\ (\dot{x}_s - \dot{x}_b)$ and $D_s\ (x_s - x_b)$ two other forces act on the bcg. They are the frictional force $-\beta_b\dot{x}_b$, caused by the applied damping of subject and bcg with respect to the surroundings and $-D_bx_b$, caused by the directive force acting between the bcg and the surroundings. The sum of these forces equals, once again, the product of the mass m_b of the bcg and its acceleration $\ddot{x}_b$, so:

$$-\beta_b\dot{x}_b - D_bx_b + \beta_s\ (\dot{x}_s - \dot{x}_b) + D_s\ (x_s - x_b) = m_b\ddot{x}_b \tag{2a}$$

In this way two simultaneous differential equations describing the movement of subject and bcg are found. They read as follows, written in a more usual form, to be found from the equations (1a) and (2a) by interchanging some terms:

$$m_s\ddot{x}_s + \beta_s\ (\dot{x}_s - \dot{x}_b) + D_s\ (x_s - x_b) = m_s\ddot{x}_c \tag{1}$$

$$m_b\ddot{x}_b + \beta_b\dot{x}_b + D_bx_b + \beta_s\ (\dot{x}_b - \dot{x}_s) + D_s\ (x_b - x_s) = 0 \tag{2}$$

To find the solution of these differential equations, we cannot entirely do without mathematics.

The center of gravity of the subject is assumed to move periodically and, therefore, its movement can be developed in a Fourier series. An arbitrary term of such a series can be written in the usual exponential form

$$x_c = |\,x_c\,|\ e^{j\omega t} \tag{3a}$$

in which x_c is the displacement and $|x_c|$ is the amplitude of this displacement of the center of gravity, $\omega = 2\pi\nu$, with ν the frequency, t the time, and j the imaginary unit.

The solution of the differential equations (1) and (2) (valid after a time until the movement of the bcg is stationary) is:

$$x_s = |\,x_s\,|\ e^{j(\omega t + \varphi_s)} \tag{3b}$$

$$x_b = |\,x_b\,|\ e^{j(\omega t + \varphi_b)} \tag{3c}$$

in which $|x_s|$ and $|x_b|$ represent the amplitudes of the displacement of subject and bcg, respectively. φ_s and φ_b represent the phase shift (time-lag) between the mass-movement within the subject and the movement of subject and bcg, respectively.

Substituting the formulas (3a), (3b), and (3c) in the differential equations (1) and (2), the amplitude distortion and the phase shift (see below) can be derived.

After some calculation we get:

a. for the amplitude of the subject:

$$|\,x_s\,| = m_s\,\omega^2 M_s\,|\,x_c\,| \tag{4}$$

b. for the amplitude of the bcg:

$$|x_b| = m_s\,\omega^2 M_b\,|x_c| \tag{5}$$

in which the abbreviations M_s and M_b stand for:

$$M_s = \left[\frac{(-m_b\omega^2+D_s+D_b)^2+\omega^2(\beta_s+\beta_b)^2}{\{m_s m_b\omega^4-\omega^2(m_sD_b+m_bD_s+m_sD_s+\beta_b\beta_s)+D_sD_b\}^2+\{-\omega^3(m_s\beta_s+m_s\beta_b+m_b\beta_s)+\omega(\beta_sD_b+\beta_bD_s)\}^2}\right]^{\frac{1}{2}} \tag{4a}$$

$$M_b = \left[\frac{D_s^2+\omega^2\beta_s^2}{\{m_s m_b\omega^4-\omega^2(m_sD_b+m_bD_s+m_sD_s+\beta_b\beta_s)+D_sD_b\}^2+\{-\omega^3(m_s\beta_s+m_s\beta_b+m_b\beta_s)+\omega(\beta_sD_b+\beta_bD_s)\}^2}\right]^{\frac{1}{2}} \tag{5a}$$

c. for the ratio between the amplitude of the relative movement and the amplitude of the movement of the subject:

$$\left|\frac{x_s - x_b}{x_s}\right| = \left[\frac{(m_b\omega^2 - D_b)^2 + \beta_b^2\omega^2}{(-m_b\omega^2 + D_s + D_b)^2 + \omega^2(\beta_s + \beta_b)^2}\right]^{\frac{1}{2}} \tag{6}$$

From this general solution follow the amplitude characteristics of the different types of bcg's.

A. As is referred to in the introduction, the displacement of the *low-frequency* bcg, if applied correctly, represents the displacement of the center of gravity.[1] The way in which the amplitude of this displacement of the center of gravity is distorted if the displacement of the subject is recorded is described by

$$|x_s| = m_s\omega^2 M_s\,|x_c| \tag{4}$$

and by

$$|x_b| = m_s\omega^2 M_b\,|x_c| \tag{5}$$

if the displacement of the bcg is recorded.

B. The displacement of the *middle-frequency* bcg represents, with a certain approximation, the velocity of the center of gravity.[1] The way in which the amplitude of this velocity is distorted can be deduced from formula (4) after a simple calculation presented earlier.[1] In the case where the displacement of the subject is recorded it follows that:

$$|x_s| = m_s\omega M_s\,|\dot{x}_c|\,, \tag{7}$$

and in the case that the displacement of the bcg is recorded:

$$|x_b| = m_s\omega M_b\,|\dot{x}_c|\,. \tag{8}$$

C1. The displacement of the *high-frequency* bcg according to Starr represents, with a certain approximation, the acceleration of the center of gravity.[1] The way in which the amplitude of this acceleration is distorted is to be found from formula (4). In the case that the displacement of the subject is recorded it follows that:

$$|x_s| = m_s\,M_s|\ddot{x}_c|\,, \tag{9}$$

and in the case that the displacement of the bcg is recorded:

$$|x_b| = m_s\,M_b|\ddot{x}_c|\,. \tag{10}$$

C2. Although the bcg, according to Dock,[11] does not fit this scheme, it can be discussed here. This bcg has a table that is assumed infinitely strongly bound to the surroundings. So the natural frequency of the (loaded) bcg is very high with respect to the frequency of the heart. Therefore, the Dock type bcg has to be considered as a *high-frequency* bcg. As there is only one degree of freedom in movement (the amplitude of the bcg equals zero), the distortion can be found by direct calculation.[9] The same result can be found from the formulas (9) and (10), holding for this type of bcg, by substituting the infinitely great value of D_b or m_b in formula (4a). This substitution must be made because of the fact that the binding of the bcg is infinitely great. So when the displacement of the subject is recorded, it follows that:

$$|x_s| = m_s \; M_s'|\ddot{x}_c| \;, \tag{11}$$

in which

$$M_s' = \left[\frac{1}{(-m_s\omega^2 + D_s)^2 + \omega^2\beta_s^2}\right]^{\frac{1}{2}} \tag{12}$$

From the same substitution in formula (5a) it follows that $M_b' = 0$, so the amplitude of the bcg $|x_b|$ equals zero too, as can be seen without calculation.

In all cases, except that of the low-frequency bcg, the ratio between the amplitude of the relative movement and the amplitude of the movement of the subject $|x_s - x_b|/|x_s|$ is so great that it is superfluous to calculate this relation in those cases beside the calculation of the amplitude characteristics for $|x_s|$ and $|x_b|$.

In the case of the high-frequency bcg, according to Dock, the ratio $|x_s - x_b|/|x_s| = 1$, since $x_b = 0$.

The time-lag between the phenomena occurring within the body and the movement of the body is indicated by the phase shift φ_s in formula (3b), that between the occurrences within the body and the movement of the bcg by the phase shift φ_b in formula (3c).

The phase shifts φ_s and φ_b can, just as the amplitudes, be calculated from the differential equations (1) and (2) by substituting the formulas (3a), (3b), and (3c). From this calculation it follows, if the displacement of the subject is recorded, that:

$$tg\ \varphi_s = \frac{uv - tw}{tv + uw} \;, \tag{13}$$

if the displacement of the bcg is recorded:

$$tg\ \varphi_b = \frac{u'v - t'w}{t'v + u'w} \;, \tag{14}$$

in which

$$t = -m_b\omega^2 + D_s + D_b \qquad u = \omega(\beta_s + \beta_b)$$

$$t' = D_s \qquad u' = \omega\beta_s$$

$$v = m_s m_b\omega^4 - \omega^2(m_sD_b + m_bD_s + m_sD_s + \beta_b\beta_s) + D_bD_s$$

$$w = -\omega^3(m_s\beta_b + m_b\beta_s + m_s\beta_s) + \omega(\beta_sD_b + \beta_bD_s) \;.$$

The value of the phase shifts φ_s^* and φ_b^*, if the velocity or the acceleration of subject or bcg is recorded, can be found from φ_s and φ_b with the relations derived earlier.[1]

Ballistocardiographs that have to give a reliable representation of the quantities one is interested in ought to have amplitude and phase characteristics meeting certain requirements. We have chosen for these requirements:

1. The amplitude characteristic must be flat within 10 per cent in the frequency range from 1 to 30 c/s.
2. The phase characteristic must show a phase shift less than 20 degrees (0.06 period) in the same frequency range.

We have chosen the frequency range up to 30 c/s, because in the acceleration records made in our group, we noticed details corresponding to frequencies up to that amount.

In some cases the first requirement is not met. Because of this fact it is not necessary to investigate the fulfilment of the second requirement in those cases.

With the above-derived formulas (4), (5), and (7) to (11) for the amplitude characteristics and with (13) and (14) for the phase characteristics alone, it is not possible to calculate amplitude and phase characteristics. Two of the constants that need to be known (m_s, m_b, β_s, β_b, D_s, and D_b), namely β_s and D_s, cannot be deduced from the experimental circumstances. We therefore measured them separately on several subjects.

EXPERIMENTS

The measurements of β_s and D_s were done with the subject lying on a bcg of the low-frequency type, but clamped rigidly, and next lying on a rigid underlayer (Dock type). The subject was always in a supine position.

Our low-frequency bcg consists of a rectangular frame in which a piece of canvas has been stretched. The frame is suspended on four wires of about 3 meters length. A further description is to be found in a previous paper.[4] To measure β_s and D_s the bcg was fixed to the surroundings. On the foot end of the bcg a footplate is attached to the frame. The subject lying on the bcg can exert a force F on this footplate by pressing his feet toward it. This force F exerted on the footplate causes a spring to be impressed. F can be measured from the impression of this spring. The force exerted on the footplate also acts on the feet.

By a tap on the shoulders the subject, lying on the bcg, is given a deflection from its zero position. Then the subject oscillates about the equilibrium position with decreasing amplitude. This oscillation is recorded in the following manner. A light screen is attached to one of the shins. When the subject is in zero position, the screen's edge is in about the middle of a lightbeam incident on a phototube. When the subject is oscillating, alternatively more and less light is transmitted to the phototube. Between the terminals of the resistance in the phototube circuit a voltage arises proportional to the amount of incident light. The variations of voltage can be recorded with the aid of an electrocardiograph or cathode-ray oscillograph. This phototube method of recording, described earlier by us,[4] has not been changed in principle, but in practical arrangement only. In the light beam two small mirrors are placed, so that the light beam changes direction twice over ninety degrees. In this way a considerable diminution of the volume of the box containing the light source, the lenses, the phototube (and the added cathode-follower) was made possible. The present dimensions are 30 by 13 by 8 cm.^{3}, so that the box is easily displaceable.

The oscillating movement of the body after giving a tap on the shoulders is recorded on two places of the body.

1. A screen of about 60 Gm. was attached to a shin in such a way that the screen pointed through a hole in the canvas to the floor. By placing the phototube-box on the floor under the shin the movement of the shin was recorded.

2. A screen of about 20 Gm. was attached to the head in such a way that the screen pointed in the direction of the axis of the body. By fixing the phototube-box to the frame of the bcg the movement of the head was recorded.

As the screens were very light and very firmly attached to the body, the natural frequencies in the different directions in which oscillation was possible were not lower than 70 c/s. Moreover, the damping of these oscillations was about critical. Because of these reasons, together with the consideration that we are only interested in frequencies less than 30 c/s, the screens will follow the movement of the leg and the head so exactly that the difference in movement between leg and corresponding screen and between head and corresponding screen is negligible.

Measurements were done on leg and head to find a possible important difference in movement of leg and head (i. e., in the respective β_s and D_s) caused by the fact that head, spine, and leg are more or less movable with respect to each other.

These two places were chosen since they are convenient for recording the BCG. As will be shown below the binding between head, spine, and legs is fairly strong for the longitudinal movement of the body. Then, recording the movement of a screen attached to a leg is preferable to that of a screen attached to the head because the subject's head is apt to make a rolling movement. To prevent the latter movement, our subjects were invited to lay their heads in a _/-shaped quilted wooden frame of 10 cm. length, measured along the subject's axis.

Using this kind of screen with a natural frequency comparatively high with respect to the frequencies one is interested in, it is no longer necessary to make further investigations into the influence of the screen on the movement of the body, or to calculate the distortion of the body movement caused by the relative movement of the transducer (screen), as is done by Smith and Rosenbaum[12] and Smith, Rosenbaum, and Ostrich.[13] In their case the natural frequency is not high enough and the mass of the transducer not small enough to prevent this complication.

The recordings of the free vibration of the subject on the fixed bcg were done under the following circumstances:

1. Without the use of a footplate ($F = 0$ Kg.), while the canvas in the frame of the bcg was not stretched.
2. With the use of a footplate exerting a force on the feet of 25.10^6 dynes ($F = 25$ Kg.). Canvas as under 1.
3. $F = 0$ Kg., while the canvas in the frame was stretched.
4. $F = 25$ Kg., canvas as under 2.

Moreover, measurements on the free vibration were done on a rigid underlayer (a plank fixed on a concrete floor) to procure data on the Dock type. The previously mentioned values of F were used. The shins of the subjects lay across a piece of wood with a height of about 10 cm., according to Dock's indication.

After a few experiments we stopped using shoulder clamps combined with a footplate to make the binding between body and bcg as large as possible, as is done by von Wittern.[14] Only the footplate was used for this purpose. In our opinion, the shoulder clamps were too troublesome to the subject for routine use.

It is not possible to improve the fixation of the bcg to the subject by increasing the force F, exerted by the footplate on the feet above the value of 25.10^6 dynes (25 Kg.). With larger F the subject will glide over the canvas or the plank. On the other hand, not every arbitrarily chosen spring that can exert a force of 25 Kg. is useful for this purpose. For a better coupling of subject and bcg an increase of the directive force per centimeter D_s is required of the subject's tissue layer between the skeleton and the bcg, and an increase of the damping coefficient β_s. One should prefer to choose the directive force per centimeter D_f of the footplate large with respect to D_s. However, this is not possible without making use of shoulder clamps. Therefore a spring must be chosen that impresses a few centimeters to exert a force of 25 Kg., to prevent a little gliding movement of the subject from causing the loss of contact between feet and footplate. But a spring with D_f large with respect to D_s, impressed a few centimeters, exerts a force on the feet much greater than 25 Kg. and causes gliding. So a spring must be chosen with the property that D_f is smaller than D_s.

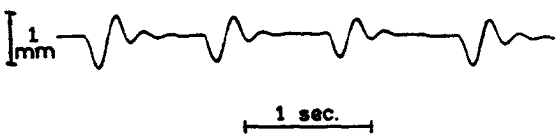


Fig. 2.—Damped vibrations of the subject lying on a rigid underlayer, caused by taps on the shoulders.

A tap on the shoulders generating the free vibration must be so great that the displacement caused by the heart action has an amplitude that is small with respect to the amplitude of the free vibration. On the other hand, the amplitude of the vibration must be small enough to provide proportionality. As a compromise we have chosen a beginning amplitude of about 0.5 mm. Fig. 2 represents a record of four successive taps, the subject lying on a rigid underlayer.

CALCULATIONS AND RESULTS

The records of the free vibrations must be conceived as a superposition of the vibrations of two coupled systems: the head on one hand and trunk and legs on the other. So, the record of the movement of a shin is a superposition of two vibrations with different amplitudes and different frequencies. In general, the movement of the head will be another superposition of the same frequencies.

The alinear properties of the tissue layer between skeleton and bcg may be a second reason why the ratio of the amplitudes in two consecutive inversion points and the vibration time are not constant during one vibration.

As the coupling between the above-mentioned systems is great and the amplitude of the vibration is small we assumed the vibration to be that of one degree of freedom. The values of β_s and D_s can then be calculated by making use of the following formulas[4]:

$$\beta_s = \frac{4\ m_s\ ln\ x_1/x_2}{T} \qquad (15)$$

$$D_s = \frac{16\ \pi^2\ m_s^2 + \beta_s^2\ T^2}{4\ m_s\ T^2} \qquad 16)$$

T is the time of oscillation of the free vibration and x_1/x_2 is the ratio of the amplitudes in two consecutive inversion points (Fig. 2). From our records we calculated mean values of β_s and D_s during a vibration caused by one tap and during a few oscillations caused by several taps.

The values of β_s and D_s were measured on subjects of various age groups. The numerical values of β_s and ν_s ($D_s = 4\pi^2\ \nu_s^2\ m_s$) are listed in Tables I, II, and III. (The natural frequency if there was no damping is called ν_s.) The measurements listed in Table I were done with a screen attached to a shin, while the subjects lay on a nonstretched canvas. In the columns indicated with I in Table II, the canvas was not stretched but in columns II and III it was. Columns I and II present measurements done with a screen attached to a shin, those in column III with a screen attached to the head.

TABLE I. SUBJECTS FROM 6 TO 14 YEARS OLD

SUBJECT	AGE (YEARS)	SEX	MASS (KG.)	ν_s (C/S)		$\beta_s \cdot 10^{-4}$ (GM. . SEC.$^{-1}$)	
				(F= 0)	(F= 25)	(F= 0)	(F= 25)
B	11	M	42	3.5	4.7	44	86
P	11	F	44	3.7	5.5	35	105
He	11	F	32	4.2	5.8	34	83
Ho	6	F	26	3.7	5.2	23	64
Ke	7	F	25	3.5	4.5	29	55
He	14	M	44	3.7	4.3	39	101
He	8	F	26	3.4	4.7	16	46
P	13	F	43	3.5	5.4	26	117
Ke	11	F	40	3.3	4.2	28	56
Kl	10	F	30	4.5	5.6	37	82
Ku	11	F	33	4.2	5.3	29	75
Mean	10		35	3.7	5.0	31	79

From the Tables I and II it appears that β_s and ν_s increase with increasing F. The increase on a stretched canvas is smaller than on a nonstretched one. Moreover, the differences in vibration properties of leg and head appear to be small, so they are strongly coupled.

Together with the measured values of β_s and D_s all constants necessary for the numerical calculations of amplitude and phase characteristics are known. We made calculations from these data for various circumstances for the following types of bcg's:

A. The *low-frequency*, critically ($\delta = 1$) and less than critically ($\delta = 0.4$) damped bcg with a (loaded or not loaded) natural frequency of 0.3 c/s (Figs. 3. to 16). (The ratio between the applied damping and the critical is called δ.)

TABLE II. SUBJECTS FROM 17 TO 54 YEARS OLD

SUBJECT	AGE (YEARS)	SEX	MASS (KG.)	ν_s (C/S)						$\beta_s \cdot 10^{-4}$ (GM. . SEC.$^{-1}$)					
				I (F = 0)	II (F = 0)	III (F = 0)	I (F = 25)	II (F = 25)	III (F = 25)	I (F = 0)	II (F = 0)	III (F = 0)	I (F = 25)	II (F = 25)	III (F = 25)
Me	24	M	65	3.2	—	—	4.5	—	—	40	—	—	180	—	—
Ho	18	F	55	3.3	4.1	4.5	4.6	5.6	5.1	25	46	98	130	64	65
Wa	20	F	80	3.3	3.9	4.6	4.8	—	4.4	71	62	70	170	67	160
Bu	22	M	74	3.2	—	5.5	4.4	4.2	4.3	90	70	105	170	86	85
He	24	M	76	2.9	—	—	4.0	—	—	53	—	—	150	—	—
V	22	M	76	3.0	—	—	4.3	—	—	43	—	—	200	—	—
Bo	18	M	85	2.9	4.2	4.9	4.3	5.1	—	55	78	95	185	100	—
T	18	M	66	3.2	4.4	4.0	4.1	5.1	4.4	65	78	83	115	110	50
J	20	F	51	3.0	—	—	4.0	—	—	48	—	—	85	—	—
Sc	17	M	65	3.4	4.4	4.0	4.6	5.8	5.7	48	75	45	62	83	95
Rd	22	M	93	—	3.9	3.8	—	4.2	5.4	—	67	54	—	120	72
A	21	M	77	—	3.8	4.3	—	3.8	5.0	—	58	45	—	87	90
We	21	M	60	—	3.7	3.6	—	5.1	4.1	—	65	50	—	130	110
Hm	22	M	70	—	4.1	4.0	—	6.1	5.7	—	73	64	—	78	51
Hk	26	M	81	3.2	4.0	4.0	4.2	4.1	3.9	64	69	69	145	120	100
Br	38	M	90	3.2	4.5	3.6	4.2	4.2	4.1	55	130	90	190	130	110
Be	54	M	68	3.5	4.7	4.0	4.9	4.9	4.5	27	70	48	205	110	90
St	41	M	74	3.4	—	4.0	4.4	4.4	4.1	39	—	54	100	94	65
Se	29	M	84	3.6	3.5	3.9	4.0	4.5	4.1	44	46	50	120	155	120
Ry	48	M	81	3.2	—	—	4.5	4.3	—	57	—	—	130	115	95
Bt	27	M	67	3.9	4.8	4.7	5.3	3.8	6.9	39	100	94	115	100	120
Ma	37	M	84	3.7	—	—	5.5	—	—	54	—	—	170	—	—
Mean			74	3.3	4.2	4.2	4.5	4.7	4.7	50	72	70	146	103	92

TABLE III. SUBJECTS FROM 17 TO 54 YEARS OLD. MEASUREMENTS ON A RIGID UNDERLAYER

SUBJECT	AGE (YEARS)	SEX	MASS (KG.)	ν_s (C/S)				$\beta_s \cdot 10^{-4}$ (GM. . SEC.$^{-1}$)			
				LEG (F = 0)	HEAD (F = 0)	LEG (F = 25)	HEAD (F = 25)	LEG (F = 0)	HEAD (F = 0)	LEG (F = 25)	HEAD (F = 25)
Ho	18	F	55	3.0	3.8	5.4	4.5	70	74	160	200
Wa	20	F	80	3.8	3.9	5.1	6.2	93	87	120	160
Bu	22	M	74	5.0	5.4	5.6	7.0	130	120	160	150
Bo	18	M	85	3.0	3.1	5.0	6.0	78	110	130	150
T	18	M	66	3.7	3.9	5.6	5.3	70	56	160	140
Sc	17	M	65	3.3	3.3	5.2	5.2	65	67	190	130
Rd	22	M	93	3.2	3.4	4.4	5.1	64	78	160	190
A	21	M	77	3.5	3.2	5.5	6.0	80	74	200	195
We	21	M	60	3.6	4.1	6.0	5.5	64	77	145	140
Hm	22	M	70	4.0	4.5	5.9	5.3	97	69	125	110
Hk	26	M	81	2.7	3.5	4.4	4.3	65	85	105	100
Br	38	M	90	3.4	3.2	5.7	4.1	95	120	150	165
Be	54	M	68	4.1	4.5	6.0	6.0	68	75	97	110
St	41	M	74	2.7	3.1	4.2	3.4	62	69	140	105
Se	29	M	84	3.4	3.2	5.8	5.1	82	90	210	155
Ry	48	M	81	3.0	3.1	4.2	4.3	73	—	—	—
Bt	27	M	67	3.5	3.3	5.0	5.5	77	72	215	185
Mean			75	3.4	3.7	5.2	5.2	78	82	154	149

TABLE IV

TYPE OF bcg	A. LOW-FREQUENCY							B. MIDDLE-FREQUENCY		C1. HIGH-FREQUENCY (STARR)		C2. HIGH FREQUENCY (DOCK)	
SUBJECTS	I (CHILDREN)	II (CHILDREN)	III (CHILDREN)	IV (ADULTS)	V (ADULTS)	VI (ADULTS)	VII (ADULTS)	VIII (ADULTS)	IX (ADULTS)	X (ADULTS)	XI (ADULTS)	XII (ADULTS)	XIII (ADULTS)
	←Canvas not stretched				→←			canvas stretched			→ rigid underlayer		
F (Kg.) (10^6 dynes)	0	0	25	0	25	0	25	0	25	0	25	0	25
m_b (Kg.)	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	6.0	15	15	—	—
m_s (Kg.)	35	35	35	74	74	74	74	74	74	74	74	74	74
D_b (10^5 Gm.. sec.$^{-2}$)	1.5	1.5	1.5	2.9	2.9	2.9	2.9	72	72	80.10^2	80.10^2	∞	∞
D_s (10^6 Gm.. sec.$^{-2}$)	19	19	35	33	60	52	65	52	65	52	65	37	80
β_b (10^3 Gm.. sec.$^{-1}$)	154	62	62	122	122	122	122	76.10^2	76.10^2	96	96	—	—
β_s (10^4 Gm.. sec.$^{-1}$)	31	31	79	50	146	71	98	71	98	71	98	80	150

B. The *middle-frequency*, more than critically damped ($\delta = 5$) bcg. The natural frequency, if there were no damping, has been chosen 1.5 c/s (Figs. 17 and 18).

C1. The *high-frequency*, far less than critically damped ($\delta = 6.10^{-3}$) bcg with a loaded natural frequency of 15 c/s, according to Starr (Figs. 19 and 20).

C2. The *high-frequency* bcg, according to Dock (Fig. 21).

All data used are tabulated in Table IV.* The value of β_b for the high-frequency bcg, according to Starr, was derived from measurements on the free vibration of a high-frequency bcg, loaded with dead weights, done by Bouhuys.[16]

DISCUSSION

As has been mentioned earlier in this paper we have assumed that an amplitude characteristic has to be flat with a deviation less than 10 per cent in the frequency range one is interested in (about 1 to 30 c/s). The corresponding phase characteristic is considered to be acceptable if the phase shift is less than 20 degrees.

A discussion of the characteristics presented yielded the following conclusions:

A. *The Low-Frequency bcg.*—(Figs. 3 to 16; in all figures the ordinates are represented in the C.G.S. system.)

1. The amplitude characteristic with critical damping of the loaded bcg does not meet the given requirement (Fig. 3). The damping will therefore further be chosen so small that there is 25 per cent overshooting ($\delta = 0.4$).

2. In all cases with $\delta = 0.4$, amplitude and phase characteristics concerning children and adults meet the requirements, provided the BCG is found by recording the displacement of the subject itself (Figs. 4 to 9). However, the characteristics improve by making use of a footplate that exerts a force (F = 25 Kg.) on the feet. The stretching of the canvas appeared to be of no importance in these cases.

3. When the BCG is found by recording the displacement of the bcg (Figs. 10 to 13) the characteristics do not fully meet the given requirements. Then the use of a footplate with $F = 25$ Kg. appears to be very important. The canvas being stretched or not appeared not to be important. If the footplate is used it is only for high frequencies (Fig. 13) that the characteristics do not meet the requirements.

4. Fig. 14 gives the characteristics of the bcg applying to the case when the bcg is not damped at all ($\beta_b = \delta = 0$). It turns out that the representation of the highest frequencies does not become better, whereas oscillations in the natural frequency (0.3 c/s) are very inconvenient. So, there is no reason to damp the low-frequency bcg to a lesser extent than corresponding to $\delta = 0.4$.

5. If the mass of the bcg m_b is made zero, there is still a small distortion of the higher frequencies (Fig. 15). The characteristics then meet the given requirements. So, if we wish to find the BCG by recording the displacement of

*In a preceding paper[4] the mass of the Starr bcg was erroneously stated to be 70 Kg.

Fig. 3.

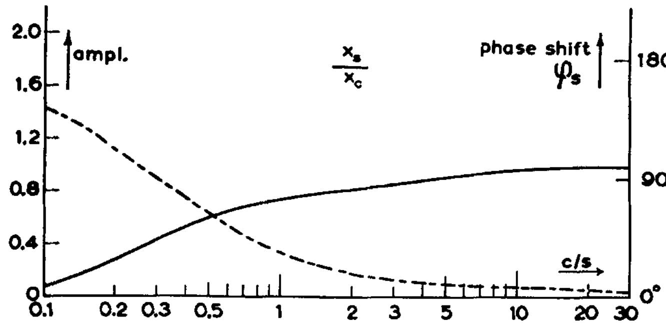
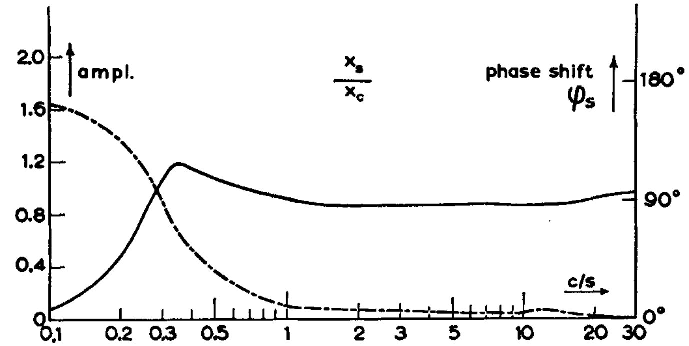


Fig. 5.



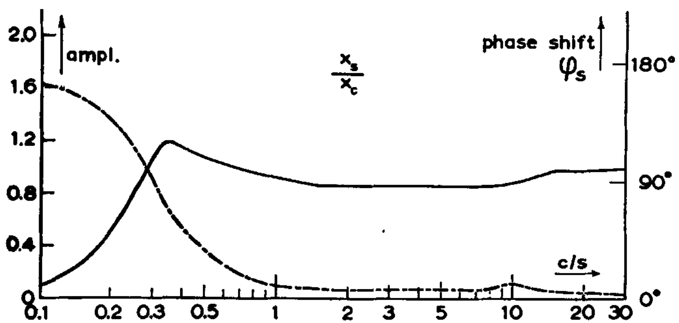


Fig. 4.

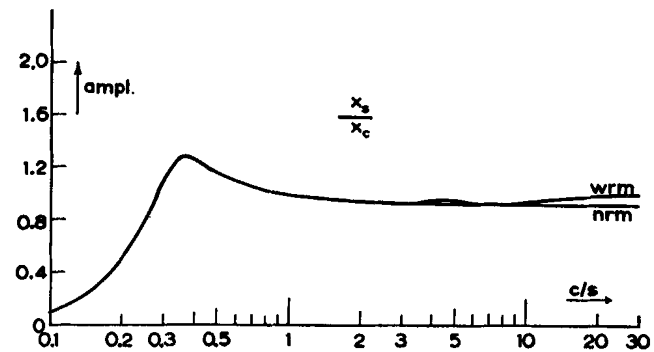


Fig. 6.

Fig. 3.—Amplitude and phase characteristics of the low-frequency, critically damped (δ = 1) bcg, if the BCG is found by recording the displacement of the body of a child. No footplate. Canvas not stretched.

Fig. 4.—Amplitude and phase characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the body of a child. No footplate. Canvas not stretched.

Fig. 5.—Amplitude and phase characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the body of a child. With footplate. Canvas not stretched.

Fig. 6.—Amplitude characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the body of an adult. No footplate. Canvas not stretched. *wrm* = with relative movement, *nrm* = if there were no relative movement.

Fig. 7.

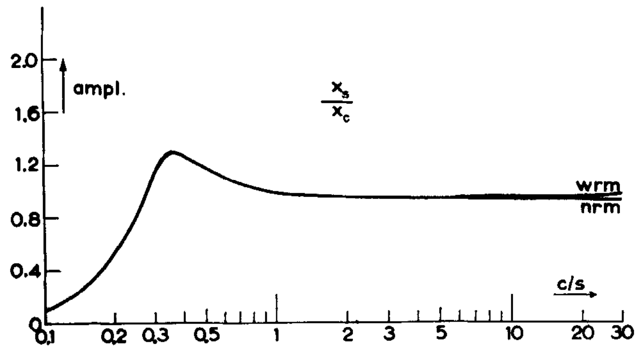
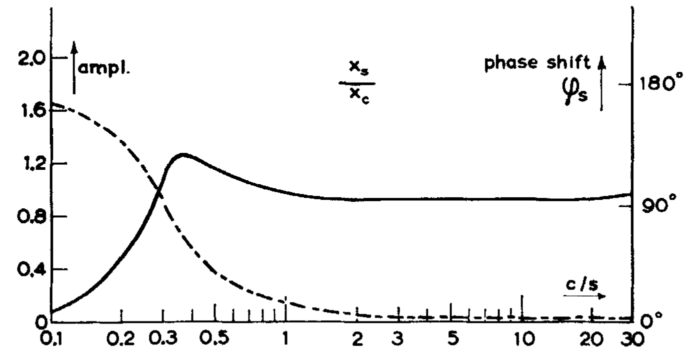


Fig. 9.



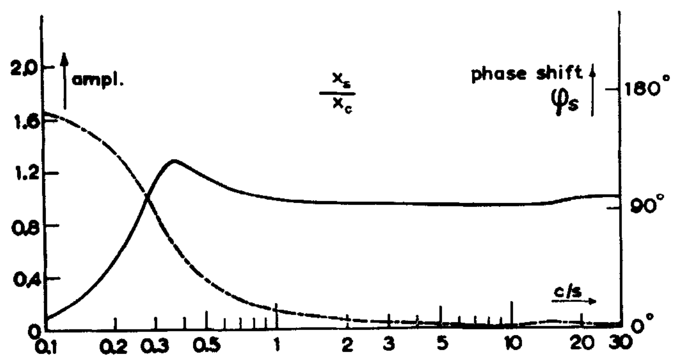


Fig. 8.

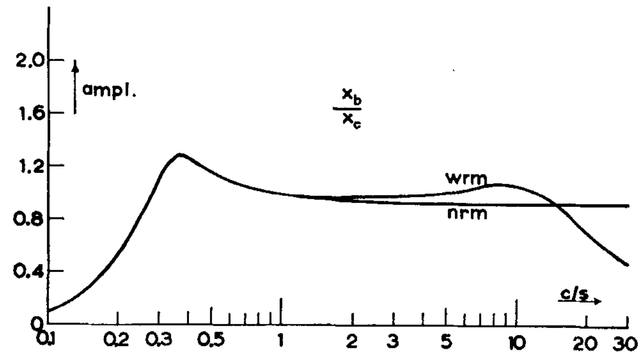


Fig. 10.

Fig. 7.—Amplitude characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the body of an adult. With footplate. Canvas not stretched. *wrm* = with relative movement, *nrm* = if there were no relative movement.

Fig. 8.—Amplitude and phase characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the body of an adult. No footplate. Canvas stretched.

Fig. 9.—Amplitude and phase characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the body of an adult. With footplate. Canvas stretched.

Fig. 10.—Amplitude characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the bcg. No footplate. Canvas not stretched. *wrm* = with relative movement, *nrm* = if there were no relative movement.

Fig. 11.

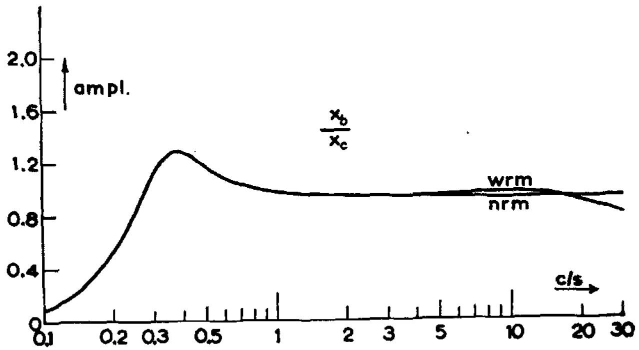
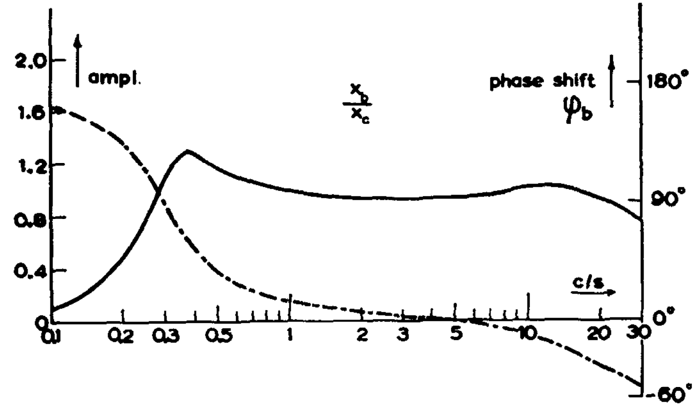


Fig. 13.



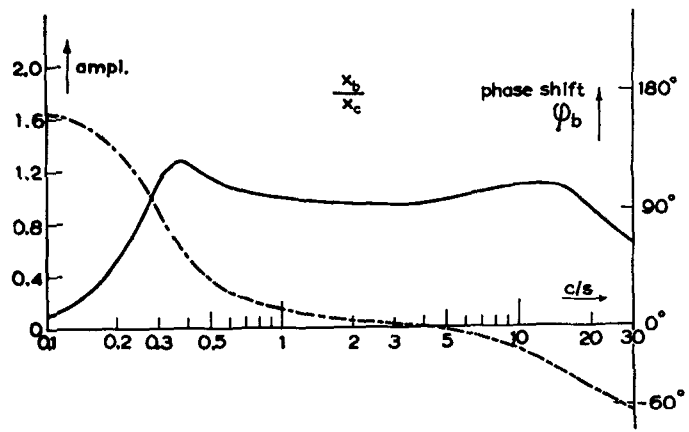


Fig. 12.

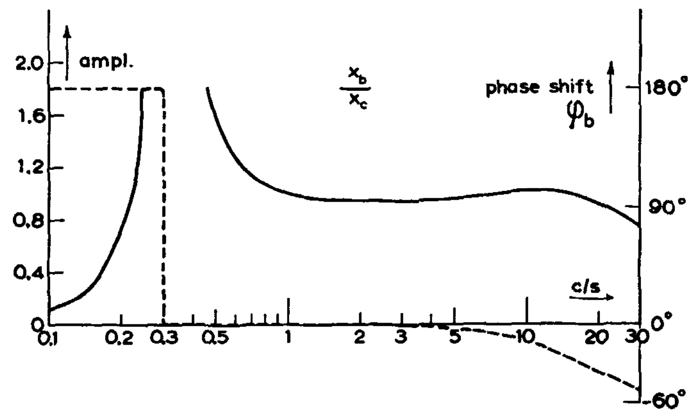


Fig. 14.

Fig. 11.—Amplitude characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the bcg. With footplate. Canvas not stretched. *wrm* = with relative movement, *nrm* = if there were no relative movement.

Fig. 12.—Amplitude and phase characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the bcg. No footplate. Canvas stretched.

Fig. 13.—Amplitude and phase characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the bcg. With footplate. Canvas stretched.

Fig. 14.—Amplitude and phase characteristics of the low-frequency, nondamped (δ = 0) bcg, if the BCG is found by recording the displacement of the bcg. With footplate. Canvas stretched.

the bcg, it is necessary to make the mass, the damping, and the directive force of the bcg small enough. Our bcg has a mass of 6 Kg. which is mainly responsible for the fact that there is not a fully correct representation of the higher frequencies. That the distortion of the higher frequencies is not zero, if the mass m_b is zero, is clear from the following consideration: the coupling between bcg and surroundings is principally determined by the directive force per centimeter D_b and can, in the case of a bcg suspending on wires, be found from

$$D_b = 4\,\pi^2\,\nu_b^2\,(m_s + m_b).$$

Fig. 15.

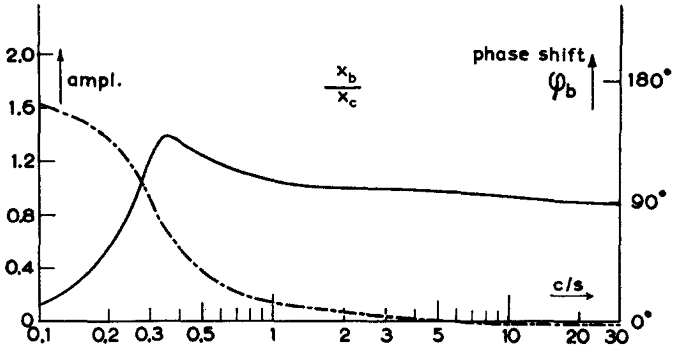


Fig. 16.

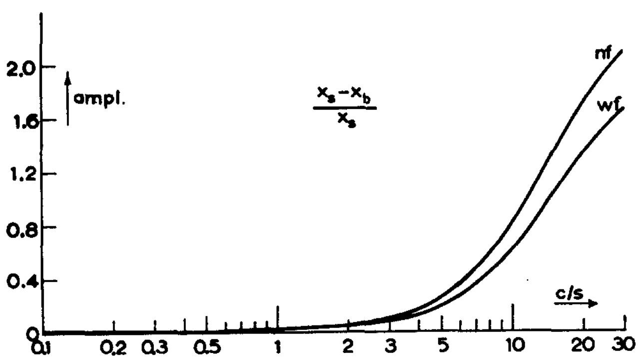


Fig. 15.—Amplitude and phase characteristics of the low-frequency, less than critically damped (δ = 0.4) bcg, if the BCG is found by recording the displacement of the bcg, while the mass m_b of the bcg equals zero. With footplate. Canvas stretched.

Fig. 16.—Amplitude characteristics of the relative movement occurring with the use of the low-frequency, less than critically damped (δ = 0.4) bcg. Canvas stretched. *nf* = no footplate, *wf* = with footplate.

For our bcg it holds that ν_b = 0.3 c/s, where m_s is the mass of the subject. So, D_b cannot be made zero by making m_b zero.

6. In his valuable paper, von Wittern,[14] discussing the relative movement, has chosen as the natural frequency of his loaded low-frequency bcg ν_b = 0.6 c/s. This choice has two drawbacks: (a) The phase shift in the neighborhood of 1 c/s is considerably greater than with ν_b = 0.3 c/s; (b) The directive force per centimeter D_b is four times greater if $m_s + m_b$ has the same value and, therefore, the relative movement for the higher frequencies is greater. How

much greater can be calculated with the aid of formulas (4) and (5). Moreover, his bcg has a mass of 10 Kg., also making the relative movement greater than in our case. The use of shoulder clamps works in the opposite direction as point b and the large mass m_b of the bcg. Von Wittern found a natural frequency of the subject on the fixed bcg ν_s of 9 c/s. Working without shoulder clamps, however, is much less disturbing for the subject. In exchange for the omission of the shoulder clamps the natural frequency of the bcg ν_b must then be chosen at least about two times lower. Moreover, the phase shift in the neighborhood of 1 c/s is then acceptable.

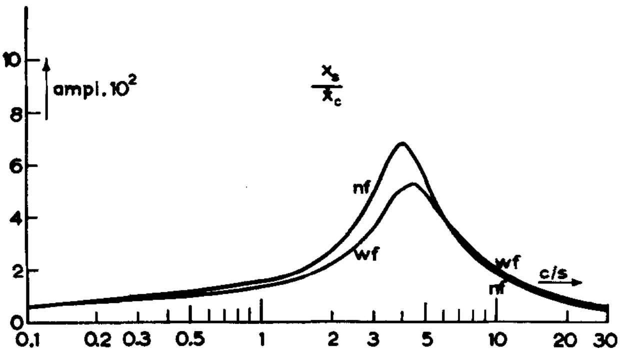


Fig. 17.

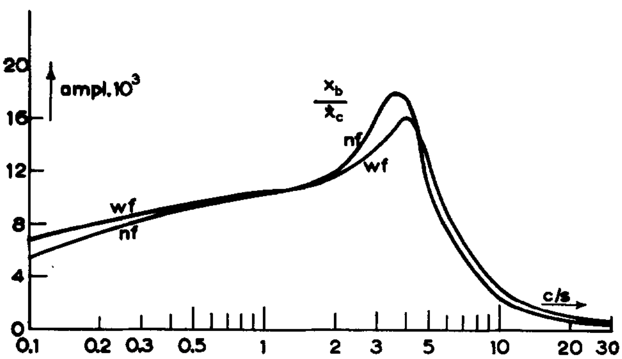


Fig. 18.

Fig. 17.—Amplitude characteristics of the middle-frequency, more than critically damped ($\delta = 5$) bcg, if the BCG is found by recording the displacement of the body of an adult. Canvas stretched. *nf* = no footplate, *wf* = with footplate.

Fig. 18.—Amplitude characteristics of the middle-frequency, more than critically damped ($\delta = 5$) bcg, if the BCG is found by recording the displacement of the bcg. Canvas stretched. *nf* = no footplate, *wf* = with footplate.

7. In Fig. 16 the amplitude of the difference in displacement of subject and bcg with respect to the amplitude of the subject is represented with the use of a footplate (curve *wf*) and without a footplate (curve *nf*). These curves cannot be found from the amplitude characteristics alone, for the phase difference between the movements must be taken into account. Formula (6) makes it possible to find the curves by substituting the known constants.

8. As can be seen, for instance, in Figs. 9 and 13 the flat parts of the amplitude characteristics are in the neighborhood of 3 c/s. The response, however,

is not 100 but about 93 per cent. This is caused by the fact that the mass m_b of the bcg is not zero, but 6 Kg. The amplitude in this range where the relative movement is negligible is therefore $m_s/m_s + m_b = 74/80$ times 100 per cent and equals 93 per cent. For proper calibration it is necessary to pay attention to this fact.

Fig. 19.

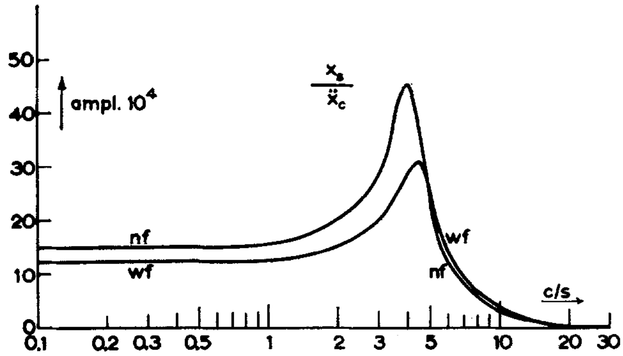


Fig. 20.

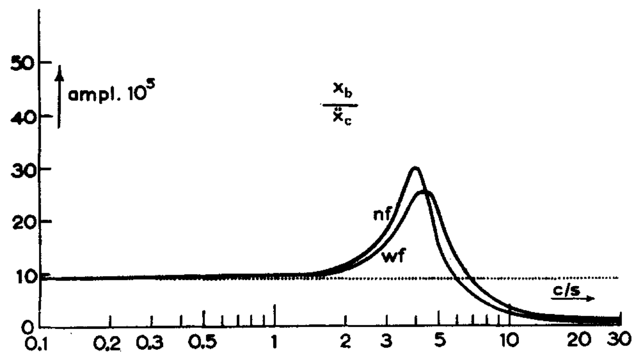


Fig. 21.

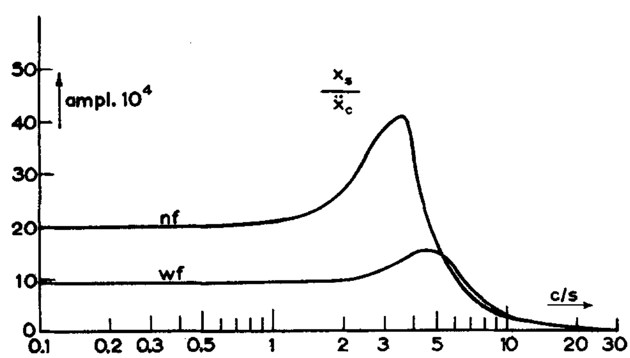


Fig. 19.—Amplitude characteristics of the high-frequency, far less than critically damped ($\delta = 6.10^{-3}$) bcg (Starr type), if the BCG is found by recording the displacement of the body of an adult. Canvas stretched. *nf* = no footplate, *wf* = with footplate.

Fig. 20.—Amplitude characteristics of the high-frequency, far less than critically damped ($\delta = 6.10^{-3}$) bcg (Starr type), if the BCG is found by recording the displacement of the bcg. Canvas stretched. *nf* = no footplate, *wf* = with footplate.

Fig. 21.—Amplitude characteristics of the high-frequency bcg (Dock type). *nf* = no footplate, *wf* = with footplate.

B. *The Middle-Frequency bcg.*—(Figs. 17 and 18.)

1. From the figures it appears that the amplitude characteristic whether a footplate is used or not, does not have a flat range in the frequency range one is interested in. In this calculation the damping is again chosen far more than critical ($\delta = 5$). The necessity of this appeared from an earlier investigation.[1] If the damping is chosen smaller, critical for instance, the amplitude distortion and the phase shift in the neighborhood of 1 c/s is not acceptable.

2. In point 1, it is shown how the displacement of the bcg and of the subject represents the velocity of the center of gravity $\dot{x}_c$.[1] Nickerson and Mathers[10] calculated the amplitude characteristic for the representation of the acceleration of the center of gravity $\ddot{x}_c$. They found a great distortion.[9,10]

C1. *The High-Frequency bcg According to Starr.*—(Figs. 19 and 20.)

1. From the figures it appears that the frequency characteristics are flat only for the lowest frequencies that are of interest and, thus, do not meet the assumed requirement. Both the characteristics holding good for recording the displacement of the subject and the displacement of the bcg have a maximum at about 4 c/s. The higher frequencies are scarcely represented. This can be seen from the BCG's recorded by a high-frequency bcg. The curves are very smooth; the so-called after vibrations have about the natural frequency of the subject with respect to the fixed bcg, i.e., about 4 to 5 c/s.

2. The sensitivity of this type of bcg is much smaller than that of the low-frequency bcg. This is caused by the strong binding of the bcg to the surroundings. If the acceleration $\ddot{x}_c$ is found by recording the displacement of the bcg, as is usual, the displacement of the bcg is less than 10 per cent of the displacement of the subject, caused by the large relative movement. For the lower frequencies (e.g., 1 c/s) this percentage equals the ratio D_s/D_b. In the latter case the sensitivity can be found by proper calibration, e.g., by exerting a constant external force of known amount on the subject and to measure the displacement of the bcg. The ratio D_s/D_b is then cancelled (dotted line in Fig. 20).

C2. *The High-Frequency bcg, According to Dock*[11].—(Direct-body method, Fig. 21.)

1. From the amplitude characteristics it appears that they greatly resemble the amplitude characteristics found for the Starr-type bcg (Figs. 19 and 20). So they, too, do not meet the requirement assumed for an amplitude characteristic.

2. When the Dock-type bcg is used, it is necessary to record the displacement of the subject, as the bcg is immovable. If the Starr-type bcg is used in the normal way, so that the displacement of the table is recorded, the Dock-type has a sensitivity of about 10 times that of the Starr-type.

3. The amplitude characteristic calculated by Smith and Rosenbaum[12] and by Smith, Rosenbaum, and Ostrich[13] for the Dock-type bcg, assuming that the leg-mounted transducer has no relative movement with respect to the legs, and our characteristics for this case (Fig. 21) agree fairly well. By making use of transducers with masses varying from 3 to 7 pounds, bound in several ways to the legs, these authors did not succeed in making the amplitude characteristic reliable.[12]

CONCLUSIONS

The amplitude and phase characteristics of the *low-frequency* bcg show that the deformation of a recorded BCG is negligibly small, especially when the displacement of the subject, instead of that of the bcg, is recorded. If a foot-plate is used the legs are fixed enough to record the movement of a shin. In this way the displacement of the center of gravity of the subject is recorded. The velocity and the acceleration of the center of gravity of the subject can be found by differentiating the displacement of the same *low-frequency* bcg once and twice, respectively. In the latter cases the amplitude and phase characteristics give a small deformation.[1] If the acceleration of the center of gravity is recorded in this way the respiration need not be held.

In contrast with the above the amplitude characteristics of the *middle-frequency* type and of the *high-frequency* types are not flat within the same limits.

Ballistocardiographic tracings with the same small deformation as occurring when making use of a *low-frequency* bcg, can possibly be found by the method proposed by Schwarzschild.[15] In that case, however, it is much simpler and easier to use a *low-frequency* bcg.

SUMMARY

Amplitude characteristics have been calculated numerically from formulas derived in this paper for the *low-frequency* bcg (Gordon, Henderson, Burger), the *middle-frequency* bcg (Nickerson), and the *high-frequency* bcg's (Starr and Dock), taking the difference in movement of subject and bcg into account. In a few cases corresponding phase characteristics have been calculated. Two constants of the system were determined experimentally.

From this it appears that it is only the *low-frequency* bcg that gives ballistocardiograms that are distorted slightly. Moreover, the velocity and the acceleration can be recorded with an analogously slight distortion.*

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*After the preparation of this paper we received the papers of Talbot and Harrison, who follow the same line of thought independently (S. A. Talbot and W. K. Harrison: Dynamic Comparison of Current Ballistocardiographic Methods. Part I, Circulation 12:577, 1955; Part II, Circulation 12:845 1955; Part III, Circulation 12:1022, 1955).

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