Boumans, A. A.
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STREAMING CURRENTS IN TURBULENT FLOWS AND METAL CAPILLARIES

IV. EXPERIMENT (2). RESULTS AND CONCLUSIONS

by A. A. BOUMANS

Stichting voor fundamenteel onderzoek der materie, Utrecht

Synopsis

Some charactistic results of the experiments on the streaming current are described with the help of graphs.

It is shown that in our case the streaming current in metal tubes is supplied by the wall of the tube; glass tubes give comparatively less streaming current.

The mean charge density $\bar{\rho}$ in the liquid is strongly dependent on temperature T, especially if $T > 20°C$; if the liquid is heated to 60°C its ability to produce charge is lost rapidly.

The experimental $\bar{\rho}(Re)$ and $\bar{\rho}(R, L)$ curves, describing the dependence of $\bar{\rho}$ on Reynolds number Re, tube length L and tube radius R, show a clear resemblance to the theoretical curves deduced in I.

It is found that wall roughness and entrance disturbance influence $\bar{\rho}$ strongly.

Finally with the obtained results a calculation is made of the components required for a 1 MV 0.1 mA h.t. generator in which the high tension is built up by the streaming current.

1. *General remarks.* Influence of wall current. With liq. 1 no measurements on wall current were carried out. But there is no doubt that with liq. 2, 3 and 4 the streaming current in the brass capillaries is supplied by the wall current. This follows from:

a. The fact that equ. (III-4) B are valied and (III-4) A are not, i.e. wall current is measured directly (situation c of fig. III-2).

b. If the capillary is not connected with earth or electrometer, the charge in the liquid is considerably less than if it is kept on earth potential.

c. The presence of saturation of the current described at the end of II and demonstrated by the found $\bar{\rho}(R, L)$ and $\bar{\rho}(Re)$ curves in IV.

From the fact that equ. (III-4) B are valid it follows that the streaming current is fully supplied by the wall current. Furthermore from the fact that condition (II-17) $R_h/2L \ll \kappa\eta L/\tau P$ is fulfilled (which will be shown later) it follows that this has no influence on charge distribution in the capillary as long as no saturation occurs. So in that case the ordinary streaming current formulae are valid. The currents involved are so small that a material transport, if any, between wall and solution cannot be detected within a reasonable time.

Comparing glass and metal tubes. The charge produced in metal tubes is found to be much larger than in glass tubes at the same Re. Combined with the fact that in situation (*a*) of fig. III-2 Q is the same for glass tube and capillary and therefore Re is much less in the glass tube (e.g. if in a capillary of 1 mm diameter $Re = 10\,000$, then in the glass tube it is only 1560, so the flow in the glass tube is laminar and the charge relatively low), this allows to neglect ω_g in respect with ω_m in case (*a*) of fig. III-2 and equ. (III-4). The disadvantage of the situations (*b*) and (*c*) above (*a*) is that in the cases (*b*) and (*c*) the insulation resistance of the glass tube lies between the line (29) and earth, which is only tolerable if the outside of the glass tube is cleaned very well, which results, however, in charging it, influencing the current and forcing the investigator to wait until the charge has leaked away. Circuit (*b*) of fig. III-2 can, however, be used with special advantage for measurements on the glass tube itself with low Re, because the pressure across it is much less than that across the capillary; this circuit (*b*) is applied in the measurements of fig. IV-1, for large Re (*d*) is used.

2. *$\bar{\rho}$ as a function of T.* The mean charge density $\bar{\rho}$ in the flow is strongly dependent on T. In a typical series of measurements on the glass tube with liq. 1 the sign of $\bar{\rho}$ altered at about 20°C; this phenomenon was reversible, since the measurements took place on two successive days from low to high and from high to low temperature.

In metal tubes relatively large values of $\bar{\rho}$ are obtained as soon as temperature exceeds 20°C, but if the temperature surpasses 50°C the charge producing ability is lost rapidly; series of measurements on this effect were performed several times. It appeared that after the first series of measurements, i.e. if the liquid had been heated to 60°C for some time, the second series gave lower $\bar{\rho}$ values, and the third series gave nearly no $\bar{\rho}$ at all. Fig. IV-2 shows some successive series on a brass capillary and liq. 2 at constant Re. Liq. 1 and 4 showed the same effect but the corresponding $\bar{\rho}$ values were 20 and 50 times higher respectively, liq. 1 and 4 produced also an appreciable charge at room temperature contrary to liq. 2 and 3.

If a series of measurements was made at a constant temperature of 60°C, the charge had at every subsequent measurement a lower value and decreased rapidly to zero.

If, however, the liquid was heated to 60°C in a closed vessel and kept at that temperature for a long time in that closed vessel, it did not lose its charge production ability.

The remarkable fact should be noted that boiling at room temperature had hardly any effect; thus it seems that the charging component is formed at elevated temperature and is lost only in contact with air. It is not quite clear yet how these phenomena may be explained.

Some years after these effects had been found, Ernsberger published similar results (ref. 5).

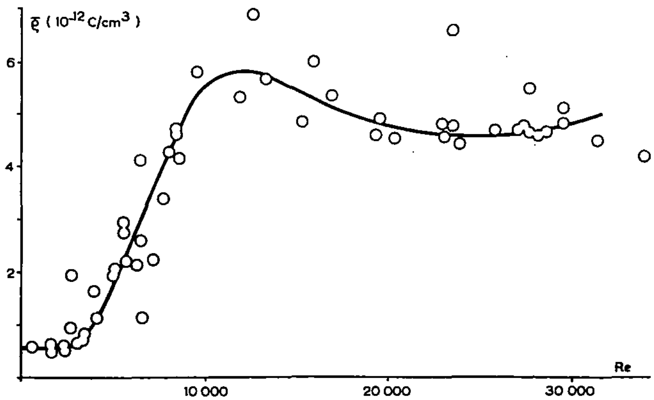

Fig. IV-1. Mean charge density $\bar{\rho}$ as a function of Reynolds number *Re* for liq. 2 flowing at room temperature through a glass tube of 700 mm length and 6.4 mm diameter. Compare with theoretical fig. II-1.

3. *$\bar{\rho}$ as a function of Re.* Typical $\bar{\rho}(Re)$ curves for liq. 2 and 3 are given in fig. IV-1 and fig. IV-4; curves typical for liq. 1 and 4 are shown in fig. IV-5. Some 50 of these curves have been measured for different capillaries and tubes.

Generally it can be said that at the transition from laminar into turbulent flow, $\bar{\rho}$ rises strongly, getting a maximum at *Re* about 5000 to 10 000, and in most cases a less pronounced minimum for a larger *Re* value and rises again somewhat at still larger *Re*. The maximum for narrow and for rough tubes is more pronounced and appears at lower *Re* than for smooth tubes. This corresponds with the fact that for short and for smooth tubes the transition from purely laminar into turbulent flow appears at larger *Re* than for narrow and for rough ones.

In the case of liq. 1 and 4 $\bar{\rho}$ is generally much higher than with liq. 2 and 3 and the maximum, if any, seems to lie at higher *Re* value.

From the experimental $\bar{\rho}(Re)$ curves of the figures IV-1, IV-2 and IV-4 some conclusions on the values of ζ and κ can be drawn. In the table below are given:

1°. the value of the involved liquid properties;

2°. the values of the mean charge density in the laminar flow $\bar{\rho}_l$ and its value in the turbulent flow $\bar{\rho}_t$ as found from the figures;

3°. ζ as found from $\bar{\rho}_l$ by formula (II-4): $\bar{\rho}_l = -2\varepsilon\zeta/\pi R_h^2$; this formula is only applicable if $\kappa R_h \gg 1$, but this condition is fulfilled since in the concerned figures $\bar{\rho}_t$ is much larger than $\bar{\rho}_l$ (see also fig. II-1);

4°. κ as found from the value of $\bar{\rho}_t/\bar{\rho}_l$ by formula (II-12): $\bar{\rho}_t/\bar{\rho}_l = \kappa R_h/4$;
5°. κ as estimated from the conductivity σ by formula (I-6):

$$\kappa^2/\sigma = 24\pi^2\eta\bar{\xi}_i/\varepsilon kT,$$

where $\bar{\xi}_i$ is taken 10^{-8} cm.

	Fig. IV-1	Fig. IV-4	Fig. IV-5	
T	290	320	290	°K
ϵ	2	2	2	
η	0.005	0.004	0.005	poise
σ	1.2×10^{-12}	10^{-10}	1.0×10^{-11}	Ω^{-1} cm^{-1}
R_h	0.3	0.05	0.075	cm
$\bar{\rho}_l$	0.5×10^{-12}	4×10^{-12}	-1×10^{-12}	C/cm^3
$\bar{\rho}_t$	5×10^{-12}	90×10^{-12}	-25×10^{-12} — 220×10^{-12}	C/cm^3
ζ from $\bar{\rho}_l$	—30	—7	4	mV
κ from $\bar{\rho}_t/\bar{\rho}_l$	130	1800	1300 12000	cm^{-1}
κ from σ	400	3100	1100	cm^{-1}

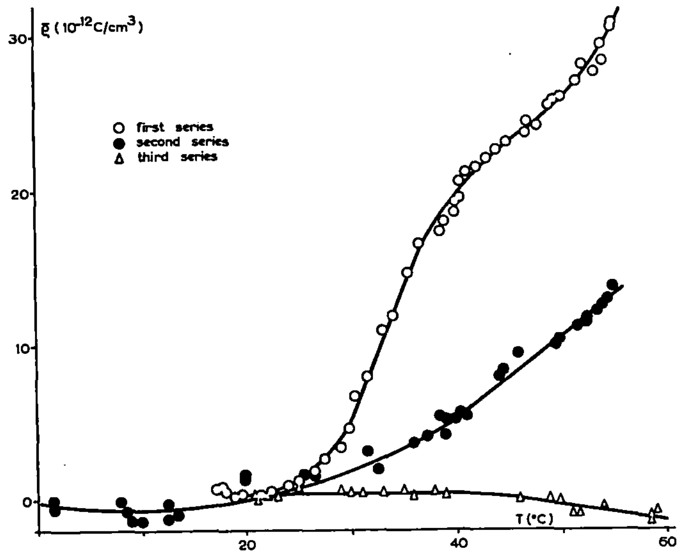

Fig. IV-2. Mean charge density $\bar{\rho}$ as a function of termperature T for liq. 2 flowing through brass capillaries of 40 mm length and 1 or 1.5 mm diameter at Reynolds number Re = 12000. Three successive series all taken from left to tight.

4. *$\bar{\rho}$ as a function of R and L.* Originally measurements of $\bar{\rho}(Re)$ curves would be used to find out this relation. The function $\bar{\rho}(R, L)$, however, appeared to be not so simple that this method would soon yield results. As many capillaries as possible had therefore to be measured in a relatively short time, since small alterations in the composition of the liquid which occur after a long time appeared to have not the same effect on all capillaries. On the other hand elimination of starting effects as described in III might interfere with quick working.

Finally it was decided to perform on every capillary, after attaining equilibrium, 4 measurements at $Re = 7000$, using one capillary for checking and calculating, if necessary, correction factors, provided that these would not grow too large. This resulted in fig. IV-3 where curves of constant $\bar{\rho}$ have been drawn in the R, L-plane for liq. 3 at a temperature of 45°C at $Re{=}7000$. The curves are obtained by logarithmic interpolation and extrapolation between the pairs of closest points. A similar graph was constructed before for liq. 2, the curves for constant $\bar{\rho}$ had the same shape, though all $\bar{\rho}$ values were lower by about a factor 5. The 3 dotted circles in fig. IV-3 represent capillaries that appeared to be rough as a consequence of the method of their construction and therefore gave too much charge, as some long and narrow capillaries were constructed by drilling from both sides which could result in a rough spot in the middle. Plotting $\lambda(Re)$ curves as in fig. I-7 gave an indication about this roughness. It appeared that the 3 capillaries represented in fig. IV-3 by dotted circles could be classified as rough and the other capillaries of that figure could not.

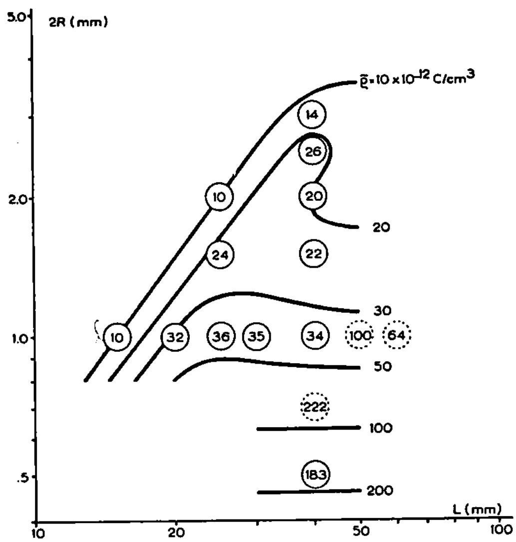

Fig. IV-3. Curves of constant mean charge density $\bar{\rho}$ in the length-diameter (L, R)-plane for liq. 3 at 45°C flowing in brass capillaries at Reynolds number $Re = 7000$. Every circle represents a capillary; the 3 dotted circles represent rough capillaries which give comparatively too high $\bar{\rho}$ values. Compare with the theoretical fig. II-3.

The shape of the curves for constant $\bar{\rho}$ agrees, apart from the hump at $R = 2.5$, $L = 40$, more or less with the expected ones of fig. II-3. This means that at high current densities saturation of wall current occurs and that at low current densities there is the same (R, L) dependence as with pure streaming current. This, however, is according to II, only possible if equ.

(II-17) is satisfied, i.e. if $R/2L \ll (\kappa\eta/\tau)L/P$. If this is true of the largest $(R/2L)P/L$ it is true of all capillaries and pressures.

With

$R = 0.15$ cm	$\sigma = 3 \times 10^{-10} \times 9 \times 10^{11}$ e.s.u.
$L = 1.5$ cm	$\varepsilon = 2$
$P = 30 \times 13.6 \times 981$ dyne/cm^2	$\tau = \varepsilon/4\pi\sigma$ sec
$\eta = 0.005$ poise	$\kappa^2 = (24\pi^2\eta\sigma/\varepsilon kT)3 \times 10^{-8}z$ cm^{-2} (according to equ. (I-6))

it is found that

$$R/2L = 0.05 \text{ and } (\kappa\eta/\tau)L/P = 0.3.$$

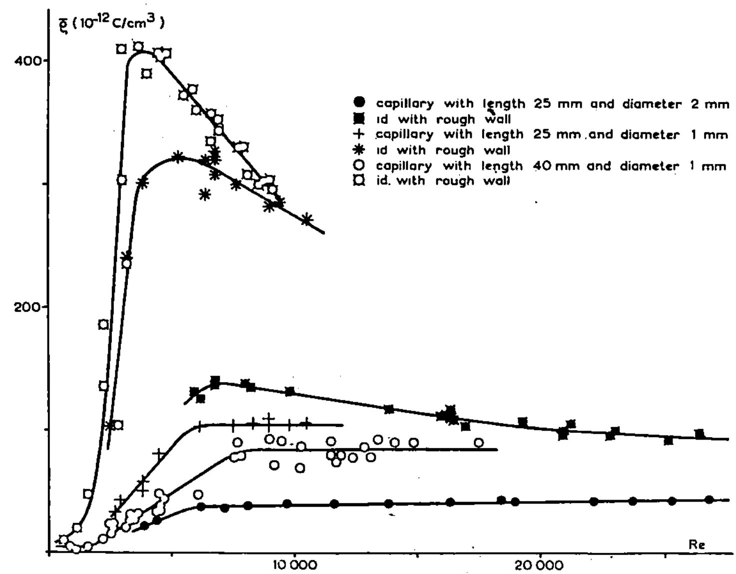

Fig. IV-4. Mean charge density $\bar{\rho}$ as a function of Reynolds number Re for liq. 3 at 45°C flowing through smooth and rough capillaries 40 — 1 (length $L = 40$ mm, diameter $2R = 1$ mm, 25 — 1 ($L = 25$ mm, $2R = 1$ mm) and 25 — 2 ($L = 25$ mm, $2R = 2$ mm). Compare $\bar{\rho}(Re)$ curves of smooth tubes with theoretical fig. II-1.

So (II-17) will be fulfilled for most capillaries and pressures, though influence of saturation may be perceptible in extreme cases.

Differences between fig. IV-3 and fig. II-3 may result from the fact that the method of comparing the capillaries at the same Re is not quite adequate, since the state of motion of the liquid in the capillary is not only a function of Re, but also of R/L, especially for comparatively short ones (ref. 4, p. 404) and so the shape of the $\bar{\rho}(Re)$ curves is dependent on L and R, as was found by experiment.

5. *Wall roughness and entrance disturbance.* Rough tubes. In fig. IV-4 it can be seen on 3 capillaries that the charge produced by rough tubes at

the same Re, is 2 to 4 times higher than for smooth ones of the same dimensions; this corresponds with the fact that the $\bar{\rho}$ in the 3 dotted circles (representing 3 other rough capillaries as mentioned in IV-4) of fig. IV-3 seems to be 2 to 3 times too high.

The rough capillaries were obtained by tapping a screw in a smooth tube. Transition from laminar into turbulent flow takes place more abruptly here than in smooth tubes. It is a well known fact that in rough tubes near parts projecting from the wall into the liquid, the liquid velocity increases much more rapidly with wall distance and the laminar sublayer is smaller than in smooth tubes. Furthermore at sharp points field strength and charge density are larger. Therefore the streaming current at the same liquid flow and the $\bar{\rho}$ in rough tubes are larger than in smooth ones.

Influence of entrance disturbance. The same effects described in the foregoing for rough tubes occur at the edge of tube entrance. For tubes with smooth entrance the $\bar{\rho}$ is smaller than for tubes with a sharp opening edge (fig. IV-5) and also the critical value of Re is larger.

Flat tubes. It was expected that flat and round tubes which have equal hydraulic radii (cf. I.5) would give the same $\bar{\rho}(Re)$ curves as explained in II.2. The experiments, however, showed for the flat tubes a larger $\bar{\rho}$ than for the round ones. This may be explained by taking into account that the diameter ratio of flat capillary to glass tube is half that of round capillary to glass tube, if flat and round capillary have equal hydraulic radius. So the entrance disturbance of flat tubes may be greater than that of corresponding round ones, i.e. the laminar sublayer at the opening edge might be smaller.

For this reason comparison experiments were carried out on a round and a flat capillary, both especially made with low entrance disturbance (fig. IV-5). The value of $\bar{\rho}$ was now with both capillaries of the same order, though the shape of their $\bar{\rho}(Re)$ curves was somewhat different.

A remarkable phenomenon with another flat capillary (length 40 mm, cross-section $1 \times 4\ mm^2$) was that it showed two $\lambda(Re)$ and two $\bar{\rho}(Re)$ curves at the same series of experiments, the higher $\bar{\rho}$ corresponding with the lower λ. This means that two different states of motion of the liquid in this tube were possible.

6. *Conclusions*. Comparing theory and experiment. The theory on dependence of $\bar{\rho}$ on R and L is qualitatively confirmed by experiment. The shape of the curves for constant $\bar{\rho}$ in the R, L-plane indicates that a saturation effect of wall current appears in comparatively small capillaries which corresponds with the fact that, according to the validity of (III-4) B the streaming current is in our case supplied by the wall current. The same thing happens in larger tubes at comparatively large Re-values.

In calculating from $\bar{\rho} = \rho_0 Re\lambda/64$ where $\rho_0 = -\ 2\varepsilon\zeta/\pi R_h^2$ (cf. II, 2) (if Re

is not too large), its value is in nearly all cases found to be of the order of 10 mV which is of the same order as normal ζ-potentials in water. Only in the case of the large brass tube (length 700 mm, diameter 6.4 mm) would a value of 850 mV be found if this formula is applied; it is quite obscure what this excessively high value means. In all cases $\kappa R_h \gg 1$. This follows from the measurement of σ, from the fact that $\bar{\rho}_t > \bar{\rho}_l$ (see fig. II-1) and from the calculation of ζ, for which much too large values are found if $\kappa R_h \approx 1$ or $\kappa R_h \ll 1$.

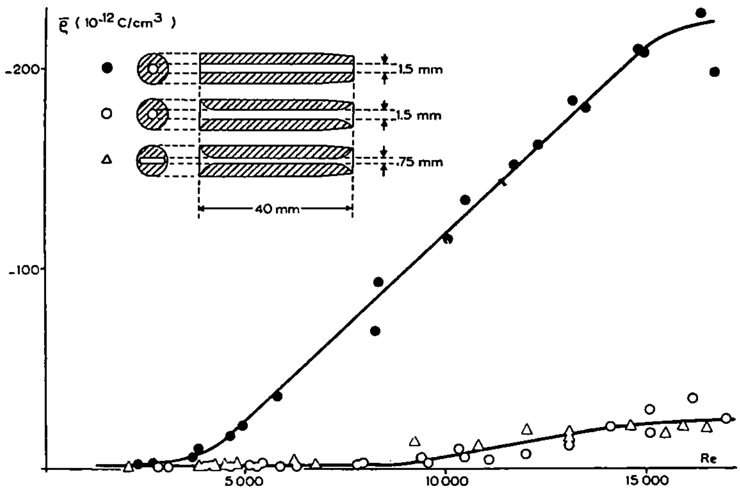

Fig. IV-5. Mean charge density $\bar{\rho}$ as a function of Reynolds number *Re* for liq. 4 at room temperature flowing through a round brass capillary with abrupt opening, a round brass capillary with low entrance disturbance and a flat brass capillary with low entrance disturbance, all of 40 mm length and 0.75 mm hydraulic radius. Compare $\bar{\rho}(Re)$ curves with theoretical fig. II-1.

Since $\bar{\rho}$ is strongly dependent on temperature, ζ must be strongly dependent on temperature (ε is found to decrese only 0.06%/°C); it cannot be a temperature effect of κ (i.e. of σ) which would have appreciable influence only if $\kappa R_h \ll 1$ since then $\bar{\rho}_t = -\varepsilon\kappa^2\zeta/4\pi$. The conclusion is that the adsorption of ions on the wall is strongly dependent on temperature.

The shape of the $\bar{\rho}(Re)$ curves for $Re > Re_{crit}$ found by experiment is not yet fully understood. The decrease for larger *Re* values could perhaps be connected with saturation of wall current and dependence on *Re* of the flow contraction just beyond tube entrance; the relatively large decrease in comparatively short tubes with small diameter confirms this, the contraction being largest in narrow tubes and having most influence in short ones. Sometimes, however, the decrease is absent.

With respect to the value of $\bar{\rho}$ it can be said that roughness of the tube and sharpness of tube opening cause a large $\bar{\rho}$ and a steep rise of the $\bar{\rho}(Re)$ curve just beyond Re_{crit}.

Conclusions relating with a h.t. generator. Finally with the obtained results a calculation was made for the components needed for a 1 MV 0.1 mA h.t. generator. These are: a 15 h.p. pump which delivers 150 l/sec at 10 m elevation and an exciter consisting of e.g. 200 parallel plates (with preferably a rough surface) of 5 cm length, 20 cm wide and with 0.5 mm distance. This calculation is based on $\bar{\rho} = 10^{-9}\ C/\text{cm}^3$ in a liquid with $\sigma = 10^{-12}\ \Omega^{-1}\ \text{cm}^{-1}$, which seems to be possible; it is further supposed that $\eta = 0.005$ poise and $s = 0.7\ \text{g/cm}^3$. The efficiency of the generator would be 0.9%.

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REFERENCES

1) Wien, W. and Harms, F., Handb. Experimentalphysik XII 2, Akad. Verlagsgesellschaft, Leipzig (1933).
2) Geiger, H. and Scheel, K., Hand. Physik XIII, Springer, Berlin (1928).
3) Rutgers, A. J., Physische scheikunde II, Noordhoff, Groningen (1947).
4) Lindgren, E. R., Physics Abstracts nr. 3166, **57** (1954) 404.
5) Ernsberger, F. M., J. appl. Phys. **27** (1956) 418.