PHYSICAL BASIS OF BALLISTOCARDIOGRAPHY. V

The Distortion of the Ballistocardiogram Caused by the Movement of the Heart Inside the Body

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INTRODUCTION

THE problem of the difference in movement of a ballistocardiograph (bcg) and the subject lying on it has been treated quantitatively for various types of bcg's.[1,2] A quantitative treatment of the difference in movement of the heart and a part of its surrounding tissues with respect to the skeleton has not yet been given. In the present paper the latter treatment will be given while taking the former into account. The necessary data, concerning the binding of the heart to the skeleton, will be taken from the literature.[3]

THEORY

The problem that will be treated in this paper is a generalization of that in which only the difference in movement of subject and bcg has been considered. The latter has been schematized as follows[1,2,4]: The subject and the bcg are coupled by a directive force and a frictional force. The bcg itself is coupled not only to the subject but also to the surroundings. The latter coupling is also represented by a directive force and a frictional one.

In the above intended schema the subject is represented by a single mass. In reality, the body must be represented by a great many masses in the three-dimensional space, all coupled mutually. It has been shown that the binding between the larger outer parts, namely, the legs and the head, is, as a first approximation, strong enough to be assumed infinitely strong.[2]

A very important coupling is that of the heart to the skeleton.[2,3,4,8,9] The coupling appears not to be strong,[3] so that they can move independently to some extent. As the ballistocardiographic effect is caused by the action of the heart, it will be of importance to realize the influence of the weakness of this binding upon the ballistocardiogram (BCG). As a result of this weak binding, the heart, with a part of the tissues surrounding it, will move with respect to the rest of the body. The former will be referred to as the "heart," the latter as the "subject." The "heart" will not move as a single mass. It is a problem of a movement

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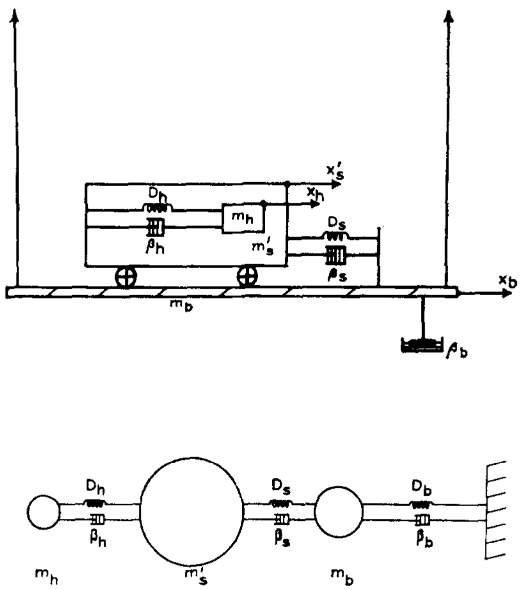
within an almost continuous medium. However, to make the problem at issue solvable, the "heart" is assumed to move as a single mass as is done for the "subject."

The more general schema in which the binding between the "heart" with mass m_h and the "subject" with mass m'_s, as well as that between the "subject" and the bcg with mass m_b, and that between the bcg and the surroundings are represented as shown in Fig. 1,*A*. Fig. 1,*B* gives a somewhat simpler illustration of the same system, all composing parts being drawn beside each other. This way of schematization has been proposed already,[4] just like the couplings, each of them being built up from a directive force and a frictional one.

A.



B.

Fig. 1.—*A*, Schematic representation of the ballistocardiographic system. The "heart" of the subject is coupled to the subject by a directive force and a frictional one. The couplings between subject and bcg and between bcg and surroundings are assumed as being built up in the same way. The masses of the "heart," the "subject," and the bcg are marked by m_h, m'_s, and m_b, respectively. The different couplings are represented schematically. *B*, A simpler illust ation of the system of *A*.

To find the influence of the various couplings upon the behavior of the different parts, it is necessary to write down the differential equations in which the forces acting on the different parts of the system are included. The way in which these equations are to be found has been described in previous papers[2,5] and will not be repeated here. In the case that there are two masses movable with respect to each other, two simultaneous differential equations have been found.[2] In this case there are three masses movable with respect to each other. Then three simultaneous differential equations describe the movement.

The driving force is assumed to work only inside the "heart." It equals the product of the body mass m_s and the acceleration of the center of gravity $\ddot{x}_c$.

The displacement x_c, the velocity $\dot{x}_c$ and the acceleration $\ddot{x}_c$ of the center of gravity of the subject with mass m_s ($m_s = m'_s + m_h$) are reckoned with respect to the skeleton and chosen positive to the left in Fig. 1 (footward). The respective quantities for the "heart" (index h), the "subject" (index s), and the bcg (index b) are calculated with respect to the surroundings and chosen positive to the right (headward). Every frictional force acting between two masses is assumed to be proportional to the difference in velocity of those two masses. The proportion factors are called β and provided by corresponding indexes as is shown in Fig. 1. In an analogous way every directive force is assumed to be proportional to the difference in displacement of the masses. The proportion factors are called D and are indexed in the same way. The three differential equations holding for this system then read as follows:

$$m_s\ddot{x}_c - \beta_h(\dot{x}_h - \dot{x}'_s) - D_h(x_h - x'_s) = m_h\ddot{x}_h \tag{1}$$

$$\beta_h(\dot{x}_h - \dot{x}'_s) + D_h(x_h - x'_s) - \beta_s(\dot{x}'_s - \dot{x}_b) - D_s(x'_s - x_b) = m'_s\ddot{x}'_s \tag{2}$$

$$\beta_s(\dot{x}'_s - \dot{x}_b) + D_s(x'_s - x_b) - \beta_b\dot{x}_b - D_bx_b = m_b\ddot{x}_b \tag{3}$$

In the case that the binding between the "heart" and the "subject" is infinitely strong ($D_h = \infty$) both the differential equations holding for the problem that there is only difference in movement between subject and bcg are found again.[2]

The center of gravity of the subject is assumed to move periodically and, hence, can be developed in a Fourier series. A term arbitrarily chosen from this series can be written in the usual exponential form

$$x_c = |x_c|\, e^{j\omega t}, \tag{4a}$$

in which x_c is the displacement and $|x_c|$ is the amplitude of this displacement of the center of gravity, $\omega = 2\pi\nu$, ν the frequency and $j^2 = -1$.

The solution of the differential equations (1), (2), and (3), valid if the movement of the various masses is stationary, can be represented by

$$x_h = |x_h|\, e^{j(\omega t + \varphi_h)} \tag{4b}$$

$$x'_s = |x'_s|\, e^{j(\omega t + \varphi_s)} \tag{4c}$$

$$x_b = |x_b|\, e^{j(\omega t + \varphi_b)} \tag{4d}$$

$|x_h|$, $|x'_s|$ and $|x_b|$ represent the amplitude of the displacement of the "heart," the "subject," and the bcg, respectively. But their values must still be calculated. The phase shift (time-lag) between the mass movement within the subject and between the movement of the "heart," the "subject," and the bcg, respectively, is indicated by φ_h, φ'_s, and φ_b.

By substituting the formulas (4a), (4b), (4c), and (4d) in the three differential equations, the amplitude distortion and the time-lag can be derived by a simple but somewhat elaborate calculation. Moreover, the difference in movement of two arbitrarily chosen masses follows from this calculation. It gives *a* for the amplitude of the "heart"

$$|x_h| = m_s\, \omega^2\, L_h\, |x_c| \tag{5}$$

b for the amplitude of the "subject"

$$|x_s| = m_s\, \omega^2\, L_s\, |x_c| \tag{6}$$

c for the amplitude of the bcg

$$|x_b| = m_s\, \omega^2\, L_b\, |x_c| \tag{7}$$

The abbreviations L_h, L_s, and L_b stand for:

$$L_h = \left[\frac{h_1^2 + h_2^2}{n_1^2 + n_2^2}\right]^{\frac{1}{2}},\ L_s = \left[\frac{s_1^2 + s_2^2}{n_1^2 + n_2^2}\right]^{\frac{1}{2}},\ L_b = \left[\frac{b_1^2 + b_2^2}{n_1^2 + n_2^2}\right]^{\frac{1}{2}} \qquad (5a), (6a), (7a)$$

in which

$$h_1 = -\omega^4 m_s' m_b + \omega^2 (m_s' D_s + m_s' D_b + m_b D_h + m_b D_s + \beta_h\beta_s + \beta_h\beta_b + \beta_s\beta_b) - (D_h D_s + D_h D_b + D_s D_b)$$

$$h_2 = \omega^3 (m_s'\beta_s + m_s'\beta_b + m_b\beta_h + m_b\beta_s) - \omega(\beta_h D_s + \beta_h D_b + \beta_s D_h + \beta_s D_b + \beta_b D_h + \beta_b D_s)$$

$$s_1 = \omega^2 (m_b D_h + \beta_h\beta_s + \beta_h\beta_b) - (D_h D_s + D_h D_b)$$

$$s_2 = \omega^3 m_b\beta_h - \omega(\beta_h D_s + \beta_h D_b + \beta_s D_h + \beta_b D_h)$$

$$b_1 = \omega^2 \beta_h\beta_s - D_h D_s$$

$$b_2 = -\omega(\beta_h D_s + \beta_s D_h)$$

$$n_1 = -\omega^6 (m_h m_s' m_b) +$$

$$+\omega^4 (m_s m_b D_h + m_h m_b D_s + m_h m_s' D_s + m_h m_s' D_b + m_h\beta_s\beta_b + m_s\beta_h\beta_b + m_s\beta_h\beta_s + m_b\beta_h\beta_s) -$$

$$-\omega^2 (m_h D_s D_b + m_s D_h D_b + m_b D_h D_s + m_s D_h D_s + \beta_h\beta_s D_b + \beta_h\beta_b D_s + \beta_s\beta_b D_h) + (D_h D_s D_b)$$

$$n_2 = \omega^5 (m_h m_s'\beta_s + m_h m_s'\beta_b + m_h m_b\beta_s + m_s m_b\beta_h) -$$

$$-\omega^3 (m_h\beta_b D_s + m_h\beta_s D_b + m_s\beta_h D_b + m_s\beta_s D_h + m_s\beta_b D_h + m_s\beta_h D_s + m_b\beta_h D_s + m_b\beta_s D_h + \beta_h\beta_s\beta_b)$$

$$+\omega(\beta_h D_s D_b + \beta_s D_h D_b + \beta_b D_h D_s)$$

d for the ratio between the amplitude of the difference in displacement of "heart" and "subject" and the amplitude of the displacement of the "heart"

$$\left|\frac{x_h - x_s'}{x_h}\right| = \left[\frac{(h_1 - s_1)^2 + (h_2 - s_2)^2}{h_1^2 + h_2^2}\right]^{\frac{1}{2}} \tag{8}$$

We calculated this ratio because of the reason that it is interesting to know whether the difference in movement of "heart" and "subject" is strongly dependent on the type of bcg that is made use of. It would be obvious to calculate for that purpose the amplitude of the difference in displacement of "heart" and "subject" $|x_h - x_s'|$ as a function of frequency. However, it will be clear that this amplitude does not give a reliable answer to the question. For the displacements of both types of bcg's are widely different for the same subject. Consequently we took the ratio between the above-mentioned amplitude and the amplitude of the displacement of the "heart."

e for the phase shift (time-lag) between the displacement of the center of gravity and the displacement of the "heart" φ_h

$$tg\varphi_h = \frac{h_2 n_1 - h_1 n_2}{h_1 n_1 + h_2 n_2} \tag{9}$$

f for the phase shift (time-lag) between the displacement of the center of gravity and the displacement of the "subject" φ'_s

$$tg\varphi'_s = \frac{s_2 n_1 - s_1 n_2}{s_1 n_1 + s_2 n_2} \tag{10}$$

g for the phase shift (time-lag) between the displacement of the center of gravity and the displacement of the bcg φ_b

$$tg\varphi_b = \frac{b_2 n_1 - b_1 n_2}{b_1 n_1 + b_2 n_2} \tag{11}$$

If the BCG is obtained by recording the displacement of a *low-frequency* bcg (natural frequency of the loaded instrument low with respect to the heart frequency) the recorded curve is intended to approximate the displacement of the center of gravity x_c. The above given formulas (5) to (11) can be applied on this type of bcg.

If the BCG is obtained by recording the displacement of a *high-frequency* bcg (natural frequency of the loaded bcg high with respect to the heart frequency) the recorded curve is intended to approximate the acceleration of the center of gravity $\ddot{x}_c$. So, in these formulas $\ddot{x}_c$ is to be considered as required, as x_c was in the cases described above. For the latter type of bcg the formulas below hold, which follow from those given above by a simple transformation. It gives

a for the amplitude of the "heart"

$$|x_h| = m_s L_h |\ddot{x}_c| \tag{12}$$

b for the amplitude of the "subject"

$$|x_s| = m_s L_s |\ddot{x}_c| \tag{13}$$

c for the amplitude of the bcg

$$|x_b| = m_s L_b |\ddot{x}_c| \tag{14}$$

d for the ratio between the amplitude of the difference in displacement of "heart" and "subject" and the amplitude of the displacement of the "heart" formula (8)

e the phase shift (time-lag) between the acceleration of the center of gravity and the displacement the "heart," the "subject," and the bcg φ_h^*, φ'^*_s and φ_b^*, respectively, are to be found from φ_h, φ'_s and φ_b (formulas (9), (10), and (11)) by a relation derived earlier.[5]

RESULTS

To make the results of the calculations given in the formulas more easily accessible, the amplitude distortion and the phase shift can be represented graphically. For that purpose, it is necessary to know the numerical values of the constants that are met with in these formulas.

For the values of $m_s = m'_s + m_h$, m_b, β_s, β_b, D_s, and D_b, the same numbers will be used as in a preceding paper.[2] They are found in Table I. The improvements yielded by Starr[10] could not yet be taken into account.

TABLE I

	LOW-FREQUENCY bcg (BURGER)			HIGH-FREQUENCY bcg (STARR)		
	I	II	III	IV	V	VI
m_h (Kg.)	0.5	1.5	4.5	0.5	1.5	4.5
m'_s (Kg.)	73.5	72.5	69.5	73.5	72.5	69.5
m_s (Kg.)	74	74	74	74	74	74
m_b (Kg.)	6.0	6.0	6.0	15	15	15
D_h (10^3Kg. s^{-2})	1.3	3.8	12	1.3	3.8	12
D_s (10^4Kg. s^{-2})	6.5	6.5	6.5	6.5	6.5	6.5
D_b (10^2Kg. s^{-2})	2.9	2.9	2.9	80.10^2	80.10^2	80.10^2
β_h (Kg. s^{-1})	8.5	26	77	8.5	26	77
β_s (10^2Kg. s^{-1})	9.8	9.8	9.8	9.8	9.8	9.8
β_b (10 Kg. s^{-1})	12	12	12	9.6	9.6	9.6

The numerical values of the constants appearing in the formulas given above which hold good for the low-frequency bcg with a (loaded or not loaded) natural frequency of 0.3 c/s and for the high-frequency bcg with a loaded natural frequency of 15 c/s. The values of the constants determining the binding between "subject" and bcg (with index s) are used in case a footplate is applied. Except those concerning the binding between "heart" and "subject," all constants are taken from an earlier paper.[2]

It is more difficult to trace numerical data concerning m_h, β_h, and D_h. For m_h, the mass of the "heart," we have chosen three values, namely,

$$m_h = 0.5, 1.5, \text{ and } 4.5 \text{ Kg.}$$

These values have been chosen for two reasons. In the first place, to find out what will be the influence of the numerical value of the "heart" mass on the reliability of the recorded curve. In the second place, because we are convinced that this region includes the correct values of m_h occurring in practice. Data concerning D_h are given in the literature.[3,4] Von Wittern[4] gives for the natural frequency of the "heart" about 5 c/s, but that number might have been deduced from a single measurement. Grandell[3] gives as a result of a series of measurements, done in another way (though at necropsy), a mean value of 8 c/s. For a reason that will be mentioned below, we prefer the use of the latter number. With the aid of the approximating formula

$$4\pi^2\nu_h^2 = D_h/m_h,$$

holding for not too large dampings, the following data for D_h are found, using the above given data of m_h respectively:

$$D_h = 1.3\ 10^3,\ 3.8\ 10^3, \text{ and } 12\ 10^3 \text{Kg. s}^{-2}.$$

We found data concerning β_h only in the above paper of Grandell. He gives for the experimentally determined damping about 65 per cent per cycle as a mean value. We interpreted that this meant (Fig. 2)

$$\frac{x_0 - x_2}{x_0} = 0.65.$$

From this the damping coefficient β_h can be derived in the following way. As for a free vibration $x_1/x_0 = x_2/x_1$ holds, it follows for the amplitude ratio in two successive inversion points:

$$\frac{x_1}{x_0} = (1 - 0.65)^{\frac{1}{2}} = 0.59.$$

From this ratio, making use of a formula deduced earlier,[6] it follows for the ratio δ_h of the damping β_h and the critical damping $(\beta_h)crit$:

$$\delta_h = 0.17.$$

The critical damping equals: $(\beta_h)crit = 2\sqrt{m_h D_h}$, so β_h can be calculated for the three above-mentioned cases in which the value of δ_h is kept constant. It results in:

$$\beta_h = 8.5,\ 26,\ 77\ Kg.\ s^{-1}.$$

These numbers complete the necessary numerical data. They are tabulated in Table I.

From the given formulas (5) to (11) all characteristics are to be found when use is made of the data given in Table I. But as the calculations are rather extensive, only a few representative characteristics have been worked out numerically. The following characteristics are represented:

1. The amplitude characteristics of the *low-frequency* bcg if the displacement of the center of gravity x_c is found by recording the displacement x'_s of the "subject." They are calculated for the cases that the mass of the "heart" equals 0.5, 1.5, and 4.5 Kg. (Fig. 3). For frequencies higher than 4 c/s the heights of these characteristics differ only a few percentages. Therefore, only one line could be drawn in this region. Also a phase characteristic holding for the case that $m_h = 1.5$ *Kg.* has been drawn as a representative for the three cases corresponding to the three values of m_h (Fig. 4).

2. The amplitude characteristics of the *low-frequency* bcg if the displacement of the center of gravity x_c is found by recording the displacement x_b of the bcg. The mass of the "heart" equals 0.5, 1.5, and 4.5 Kg. (Fig. 5). For frequencies higher than 4 c/s the heights of the characteristics again differ only a few percentages. In the case of $m_h = 1.5$ *Kg.* the phase characteristic is represented too (Fig. 6).

3. The amplitude characteristic of the *high-frequency* bcg if the acceleration of the center of gravity $\ddot{x}_c$ is found by recording the displacement of the "subject" x'_s. The mass of the "heart" equals 1.5 Kg. (Fig. 7).

4. The amplitude characteristic of the *high-frequency* BCG if the acceleration of the center of gravity $\ddot{x}_c$ is found by recording the displacement of the bcg x_b. The mass of the "heart" equals 1.5 Kg. (Fig. 8).

5. The ratio between the amplitude of the difference in displacement of "heart" and "subject" $[x_h - x'_s]$ and the amplitude of the displacement of the "heart" $|x_h|$, if a *low-frequency* bcg is used (Fig. 9, *lf*).

6. The ratio between the amplitude of the difference in displacement of "heart" and "subject" $|x_h - x'_s|$ and the amplitude of the displacement of the "heart" $|x_h|$ if a *high-frequency* bcg is used (Fig. 9, *hf*).

DISCUSSION

1. The given formulas have been checked by the examination of extreme cases. For instance, for the case drawn schematically in Fig. 10, in which the binding between "heart" and "subject" is supposed to be extremely strong

(D_h and/or $\beta_h = \infty$), must hold the formula given for a more simple case.[2] Moreover, $|x_h|$ from formula (5) must then be equal to $|x'_s|$ from formula (6), for two rigidly coupled masses must move in the same way. In the same case, the phase shift φ'_s in formula (10) must be equal to the phase calculated earlier.[2]

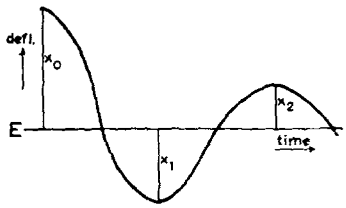


Fig. 2.—The deflection from the equilibrium position E of an unforced damped system that is brought out of that position plotted with respect to time.

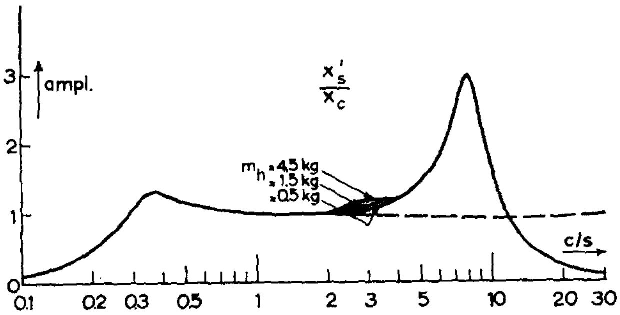


Fig. 3.—The amplitude characteristics of a *low-frequency* bcg if the displacement of the center of gravity x_c is found by recording the displacement of the "subject" x'_s. The difference between the characteristics is only more than a few percentages in the neighborhood of 3 c/s. The broken line holds if the binding between "heart" and "subject" were infinitely strong.

In this way a number of other special cases are to be found. All checks gave the results that were predicted for the corresponding simpler cases.

2. In Figs. 3 and 5 a high resonance peak appears in the place where the frequency ν equals the natural frequency of the "heart" with respect to the fixed "subject" ($\nu = 8$ c/s). In a preceding paper,[2] in which only the relative movement of subject and bcg was taken into account, this high resonance peak did not occur. To find out when such a peak occurs and when it does not, the behavior of the system in two simplified cases will be represented graphically.

In both simplified cases the binding between "subject" and bcg is assumed to be infinitely strong (D_s and/or $\beta_s = \infty$). Therefore, the masses inside the outlined area of Fig. 11, *A* move as a whole. Fig. 11, *B* gives a simpler drawing of the same system. The amplitude characteristic of this system is represented in Fig. 13, curve 1. It is assumed that the displacement of the center of gravity is found by recording the displacement of the mass $m'_s + m_b$. The second simplified system equals the first except that the masses m_h and $m'_s + m_b$ have been

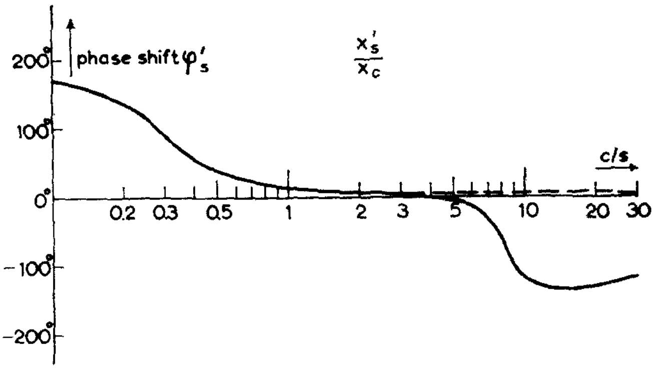


Fig. 4.—The phase characteristic of a *low-frequency* bcg if the displacement of the center of gravity x_c is found by recording the displacement of the "subject" x'_s, belonging to a value of the mass of the "heart" of 1.5 Kg. The broken line gives the characteristic if there were no difference in movement between "heart" and "subject."

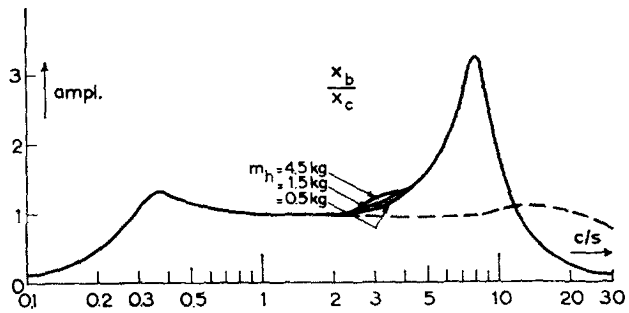


Fig. 5.—The amplitude characteristics of a *low-frequency* bcg if the displacement of the center of gravity x_c is found by recording the displacement of the bcg x_b. The characteristic representing the case that "heart" and "subject" were infinitely strongly bound to each other is given by the broken line.

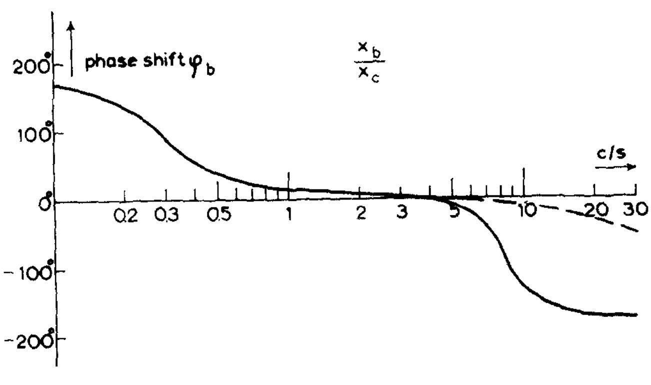


Fig. 6.—The phase characteristic of a *low-frequency* bcg if the displacement of the center of gravity x_c is found by recording the displacement of the bcg x_b belonging to a value of the mass of the "heart" of 1.5 Kg. The characteristic representing the case that the compliance of the "heart" would be zero is shown by the broken line.

interchanged (Fig. 12). The amplitude characteristic of this system is represented in Fig. 13, curve 2. The results are scarcely dependent on the choice of m_h. In this case it is assumed that the displacement of the center of gravity is found by recording the displacement of the mass m_h. In both cases the numerical values of Table I have been used.

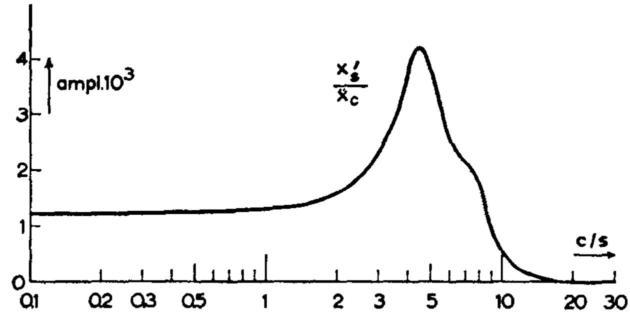


Fig. 7.—The amplitude characteristic of a *high-frequency* bcg if the acceleration of the center of gravity $\ddot{x}_c$ is found by recording the displacement of the "subject" x'_s. The mass of the "heart" has been chosen 1.5 Kg. The ordinates are given in square seconds.

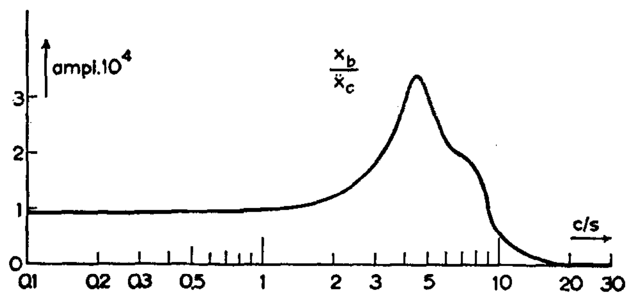


Fig. 8.—The amplitude characteristic of a *high-frequency* bcg if the acceleration of the center of gravity $\ddot{x}_c$ is found by recording the displacement of the bcg x_b. The ordinates are given in square seconds.

From the characteristics of Fig. 13 it appears that the height of the abovementioned resonance peak is highly dependent on the sequence of the masses if the coupling constants remain constant. From this it is understandable that in the characteristics represented in Figs. 3 and 5 the height of the resonance peak may be great. Moreover, it is understandable why the resonance peak in an earlier paper[2] was low.

3. In the calculations to obtain the characteristics represented in Figs. 3 and 5 the mass m_h of the "heart" equals 0.5, 1.5, and 4.5 Kg. Grandell[3] gives for the mass of the proper heart about 0.5 Kg. But not only the heart will move with respect to the skeleton; a part of the tissues surrounding it will be made to move too. The tissue close to the heart will perform the same movement as the heart, while the tissue will carry out a smaller movement as it is more removed from it. We introduced therefore an effective mass m_h. The radius of this effective mass is assumed to be about 2 cm. larger than that of the heart. From this it follows for the value of m_h about 1.5 Kg.

To examine the importance of the choice of the numerical value of m_h the characteristics belonging to somewhat different values of m_h are here represented in a few cases. Knowing the exact value of the "heart" mass appears not to be very important. Therefore in all cases except those of Figs. 3 and 5 m_h is taken 1.5 Kg. only.

4. From the above it follows that it is understandable why the height of the peaks in Figs. 3 and 5 is so great for the given binding. The resonance peak at about 8 c/s means that occurrences appearing with this frequency will be represented more conspicuously than those closer to the heart frequency. However, in the experimentally determined BCG this frequency is hardly to be found. Because of this reason we suppose that the damping found by Grandell[3] from his experiments might be smaller than it is in the intact human body. Perhaps this is caused by the removal of the 2 costal cartilages.

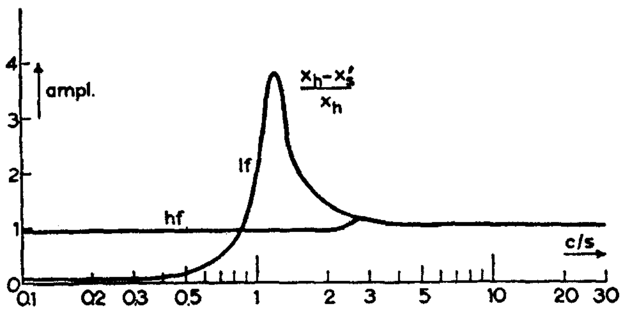


Fig. 9—The ratio of the amplitude of the difference in displacement of "heart" and "subject" $|x_h - x_s'|$ and the amplitude of the displacement of the "heart" $|x_h|$, when a *low-frequency* bcg is used (curve *lf*) and when a *high-frequency* bcg is used (curve *hf*).

From our calculations it followed that the height of the peak is very strongly dependent on the value of δ_h, the ratio between the damping coefficient β_h as it presents itself in reality and its critical value. From Grandell's paper[3] we deduced δ_h to equal 0.17. We erroneously made calculations also with $\delta_h = 0.11$, and the height of the resonance peak for $\nu = 8$ c/s appeared to be a few times greater than in the cases $\delta_h = 0.17$, represented in Figs. 3 and 5.

As we supposed the damping determined by Grandell to be too small, we calculated in one case the amplitude characteristic for a larger damping of the "heart," namely $\delta_h = 0.29$ (Fig. 14). We believe that it would be a good plan to repeat the measurements concerning the natural frequency and the damping of the "heart" with respect to the "subject." These should then be done without the removal of parts of the thoracic cage. Perhaps people with a splinter in the heart wall would be very good subjects.

5. As has been mentioned above, von Wittern[4] found for the natural frequency of the heart with respect to the thoracic cage 5 c/s. He determined this value in the following manner: From roentgenograms it appeared that the heart in the upright position had shifted 1 cm. with respect to the ribs in the direction of the feet, as compared with the reclining position. However, in comparing these two cases the displacement is originated by the action of gravity on all organs in the trunk. Von Wittern may have found a larger displacement of the

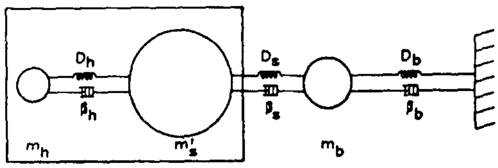
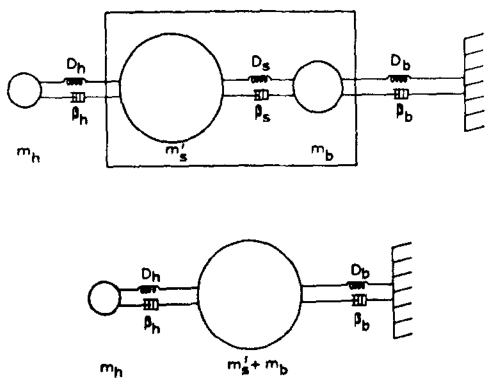


Fig. 10.—The system given in Fig. 1, *A* and *B*. The binding between the masses inside the outlined area is assumed to be infinitely strong in this case.

A.



B.

Fig. 11.—*A*, The system given in Fig. 1, *A* and *B*. The binding between the masses inside the outlined area is supposed to be infinitely strong in this case. *B*, A simpler illustration of the system given in *A*.

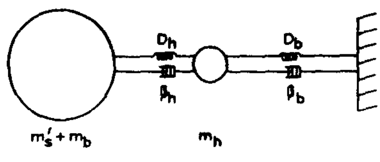


Fig. 12.—The same system as given in Fig. 11, *B*, except that the masses are interchanged.

heart than would be caused by a force working on the heart alone. Starting from this hypothesis and using von Wittern's formula[4] it is clear that he found a low natural frequency (5 c/s). For this reason we did not make use of the number given by von Wittern.

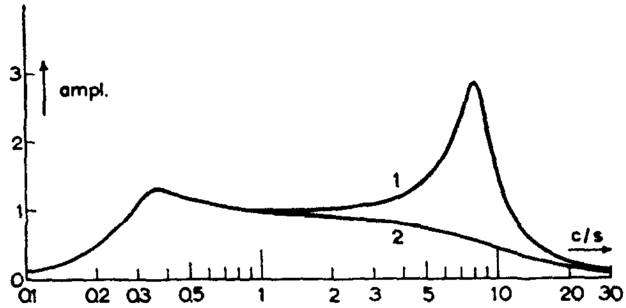


Fig. 13.—The amplitude characteristics of the systems represented in Figs. 11 and 12, if the displacement of the center of gravity x_c is found by recording the displacement of the mass to the right in Figs. 11, *B* and 12 (curves 1 and 2, respectively).

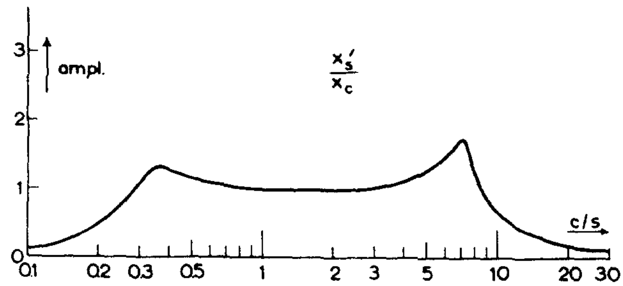


Fig. 14.—The amplitude characteristic of a *low-frequency* bcg if the displacement of the center of gravity x_c is found by recording the displacement of the "subject" x_s'. The mass of the "heart" is chosen 1.5 Kg. The difference between this characteristic and those represented in Fig. 3 is only that the damping of the "heart" is chosen somewhat larger in Fig. 14.

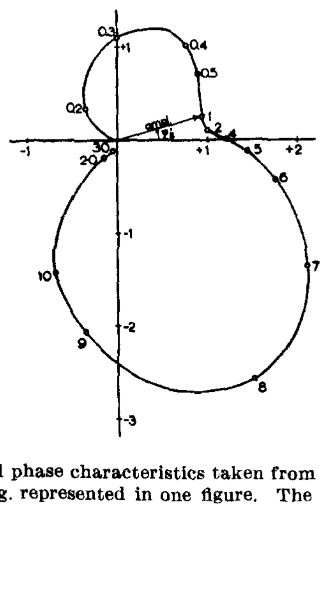


Fig. 15.—The amplitude and phase characteristics taken from Figs. 3 and 4 for the case that the mass of the "heart" equals 1.5 Kg. represented in one figure. The numbers along the curve represent the frequencies in c/s.

6. The representation of the displacement of the center of gravity within the subject, if it is recorded by a low-frequency bcg, is certainly better than is given by the characteristics of Figs. 3, 5, or 14. Not all displacements occur within a part of the body that is elastically bound to the skeleton. For instance, the arteries in the extremities may be assumed to be bound rigidly to the skeleton with respect to the binding between "heart" and skeleton. So the representation of the BCG will be better than is actually represented in the above-mentioned characteristics. This problem and the quantitative prediction of the BCG will be treated in a later publication.[7]

CONCLUSION

Records of the ballistocardiogram are distorted by the weak binding between the heart with its surrounding tissues and the skeleton, and by the binding between the skeleton and the ballistocardiograph.

From the numerical treatment given in the present paper it follows that only the components of higher frequencies are distorted if a ballistocardiograph of the low-frequency type, e.g., according to Talbot[1] or according to ours,[2,5,6] is made use of (Figs. 3 and 5).

Moreover, it follows from this treatment that the ballistocardiogram is also distorted for frequencies closer to the heart frequency if a ballistocardiograph of the high-frequency type (according to Starr) is used. This is caused by the gliding of the subject over the ballistocardiograph (Figs. 7 and 8).

SUMMARY

The influence of the binding between heart and skeleton and between skeleton and ballistocardiograph upon the representation of the human ballistocardiogram has been treated quantitatively for the low-frequency and the high-frequency ballistocardiograph.

It appeared that frequencies close to the frequency of the heart are more distorted when a high-frequency ballistocardiograph is used than when the low-frequency ballistocardiograph is the recording instrument. This is caused by the compliance between subject and ballistocardiograph as well as by that between the latter and the surroundings.

APPENDIX

There are a number of methods for indicating the quality of representation of a phenomenon. To represent the distortion of the ballistocardiogram by the transducer systems we have chosen the method in which the amplitude and phase distortion are given separately.

As an example of another method of representation of the distortion, the results given in Figs. 3 and 4 will be represented in another way for the case $m_h = 1.5$ Kg. A common method in techniques called the Nyquist diagram will be applied for this purpose.

From the differential equations (1) to (3) it followed that

$$\frac{x_s}{x_e} = m_s\omega^2 \frac{s_1 + js_2}{n_1 + jn_2} = \frac{m_s\omega^2}{n_1^2 + n_2^2}(s_1n_2 + s_2n_1) + j\frac{m_s\omega^2}{n_1^2 + n_2^2}(s_2n_1 - s_1n_2). \tag{15}$$

Formula (5) is to be found from this equation by calculation of the absolute values. Moreover, formula (10) follows from relation (15). Instead of evaluating the amplitude ratio $|x_s|/|x_e|$ and the phase shift φ'_s apart, both can be found from one figure. On the rectangular axes of Fig. 15

the real part of the right side of formula (15) is plotted on the horizontal axis while the imaginary part is plotted on the vertical axis. For each frequency a point in the coordinate system is found. The frequencies in the range from 0 to 30 c/s give the curve represented in Fig. 15. For an arbitrarily chosen frequency the amplitude representation is found by measuring the distance from the origin of the coordinate system to the point of the curve belonging to that frequency. The phase shift for that frequency is found by measuring the angle between the just mentioned line of communication and the positive real axis. The angle is calculated positive in the direction of the arrow in Fig. 15.

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