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AN INVESTIGATION OF THE ODD-PARITY STATES OF ^{40}Ca WITH THE TABAKIN INTERACTION AND THE MSDI

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Abstract: The realistic Tabakin nucleon-nucleon interaction, and the effective modified surface delta interaction (MSDI) are applied to determine the energy spectrum and wave functions of the low-lying odd-parity states in ^{40}Ca. The calculations are performed in the TDA and the RPA. The wave functions are used to determine γ-transition rates and mean lifetimes.

1. Introduction

In recent years considerable interest has been aroused in the application of realistic interactions to nuclear spectroscopy and Hartree-Fock calculations.

The local type of realistic interaction possesses, in order to fit the high-energy scattering data, a singular behaviour at short distances. This hard-core repulsion embodies the principal difficulty in the application to nuclear spectroscopy. Though remarkable progress has been made by Kuo and Brown [1,2]) in the development of methods [3,4]) to deal with the infinities caused by the hard core, the evaluation of shell-model matrix elements still remains quite complicated.

Attempts to avoid this problem have led to the construction of several non-local or velocity-dependent interactions [5,6]). Among these the one constructed by Tabakin has been frequently used in shell-model calculations [7-9]) and in Hartree-Fock calculations. The latter have been applied [10,11]) to ^{16}O and ^{40}Ca. Satisfactory values for the binding energy could be obtained only if second-order contributions were included.

Most of the shell-model calculations with the Tabakin interaction have been performed for singly closed-shell nuclei [7,8]). In these cases only $T = 1$ particle-particle matrix elements are involved, for which second-order corrections turn out to be small; it has been shown that a reasonable agreement with the experimental energies could be obtained by including enough configuration mixing [9]). However, for the $^3S_1$ and $^1P_1$ parts of the interaction that occur in the calculation of $T = 0$ particle-particle matrix elements, "second-order Born" corrections have been found (refs. [9,11])) to be relatively important. These corrections are mainly due to the excitations of two nucleons to high-energy intermediate states, which for simplicity were taken to be plane waves. In a recent shell-model calculation [9]) on ^{18}F, these second-order Born corrections have been included. The energies thus calculated with the

Tabakin interaction agree as well with experiment as those determined from the Hamada-Johnston potential [1,2]).

Only a few particle-hole calculations [12-14]) with realistic interactions have been performed. The results appear to be less satisfactory than those obtained for particle-particle calculations. In this paper we shall include second-order Born corrections in order to calculate the odd-parity spectrum of ^{40}Ca.

The surface delta interaction (SDI) is of quite different character. The results obtained with the SDI are very remarkable in view of the simplicity of the model. Initially the SDI was used for heavy nuclei by Moszkowski *et al.* [15,16]). Application of this interaction to light and medium-weight nuclei [17,18]) showed some systematic deviation that could be avoided by adding two terms that produce an extra energy shift between $T = 0$ and $T = 1$ states. The new interaction [19]), named modified surface delta interaction (MSDI), gives results that are in much better agreement with the experimental data.

Thus far the MSDI has been applied to light nuclei only in the particle-particle matrix elements of a standard shell-model calculation [19]). (The SDI has been applied by Letourneux and Eisenberg [20]) to the spectrum of ^{208}Pb in the particle-hole model as well in the Tamm-Dancoff approximation (TDA) as in the random-phase approximation (RPA).) In this paper, the results of the MSDI on the ^{40}Ca spectrum will be compared in the TDA and the RPA.

The spectrum of the odd-parity states, mean lifetimes and γ-branching ratios will be calculated with the two interactions and compared with experiment. The configuration space will allow for particle excitations from the 2s–1d shell into the 2p–1f shell.

In sect. 2 an outline is given of the particle-hole formalism used in this paper. In sect. 3 the calculation of particle-particle matrix elements of the Tabakin interaction in first and second order is indicated. The MSDI is treated in sect. 4. In sect. 5 the numerical results of the energy calculations are given and compared with experiment, and in sect. 6 this is done for the wave functions and electromagnetic transition rates. A general discussion finally is given in sect. 7.

2. Particle-hole formalism

With the customary assumption of two-body forces described by a potential V, the Hamiltonian can be written

$$H = \sum_{\alpha\beta} \langle\alpha|T|\beta\rangle \eta_\alpha^\dagger \eta_\beta + \tfrac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|V|\gamma\delta\rangle \eta_\alpha^\dagger \eta_\beta^\dagger \eta_\delta \eta_\gamma, \tag{2.1}$$

where $\eta_\alpha^\dagger$ and η_α denote the creation and annihilation operators, respectively, of a nucleon in state $|\alpha\rangle$, T is the kinetic-energy operator, and $\langle\alpha\beta|V|\gamma\delta\rangle$ represents the normalized and antisymmetrized matrix element of the nucleon-nucleon interaction. Here the Greek letters denote the single-particle quantum numbers in the j–j coupling scheme: n, l, j, m and the third component t of the isospin vector. The corresponding Latin letters denote the same quantum numbers with the exclusion of m and t. The convention will be used that $t = -\frac{1}{2}$ represents a proton and $t = +\frac{1}{2}$ a neutron. For the phases of the matrix elements to be given explicitly later, it is relevant to remark that orbital angular momentum l and spin s are coupled to j (in that order).

Particle-particle states are defined by

$$|ab; JM, TM_T\rangle = (1+\delta_{ab})^{-\frac{1}{2}} \sum_{m_a m_b t_a t_b} \langle j_a m_a j_b m_b|JM\rangle\langle \tfrac{1}{2}t_a \tfrac{1}{2}t_b|TM_T\rangle \eta^\dagger_{j_a m_a t_a}\eta^\dagger_{j_b m_b t_b}|\rangle, \tag{2.2}$$

and particle-hole states by

$$|ab^{-1}; JM, TM_T\rangle = \sum_{m_a m_b t_a t_b} \langle j_a m_a j_b -m_b|JM\rangle\langle \tfrac{1}{2}t_a \tfrac{1}{2}-t_b|TM_T\rangle(-1)^{j_b-m_b+\frac{1}{2}-t_b} \times \eta^\dagger_{j_a m_a t_a}\eta_{j_b m_b t_b}|\rangle, \tag{2.3}$$

where $|\rangle$ denotes the bare vacuum.

The energy spectrum and the wave functions of ^{40}Ca will be calculated in the TDA as well as in the RPA. Therefore the following matrix elements [21]), real and independent of M and M_T, will have to be evaluated (ζ denotes J and T)

$$\mathscr{A}^{(\zeta)}_{mi,nj} = (\varepsilon_m - \varepsilon_i)\delta_{mn}\delta_{ij} + \langle mi^{-1}; JM, TM_T|V|nj^{-1}; JM, TM_T\rangle, \tag{2.4a}$$

$$\mathscr{B}^{(\zeta)}_{mi,nj} = (-1)^{j_j+j_n+1}\langle mi^{-1}; JM, TM_T|V|jn^{-1}; JM, TM_T\rangle. \tag{2.4b}$$

The convention is used to label states above the Fermi surface by indices m, n and below the Fermi surface by indices i, j. The energies ε_m and ε_i are the energies of the single-particle states $|m\rangle$ and $|i\rangle$, respectively.

The particle-hole matrix elements in eqs. (2.4) can be expressed in terms of particle-particle matrix elements if one substitutes the definition (2.3) and subsequently applies Wick's theorem. One obtains (suppressing the irrelevant quantum numbers M and M_T)

$$\langle ab^{-1}; JT|V|cd^{-1}; JT\rangle = -\{(1+\delta_{ad})(1+\delta_{bc})\}^{\frac{1}{2}} \sum_{J'T'}(2J'+1)(2T'+1) \times \begin{Bmatrix} j_a & j_b & J \\ j_c & j_d & J' \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & T \\ \frac{1}{2} & \frac{1}{2} & T' \end{Bmatrix} \langle ad; J'T'|V|cb; J'T'\rangle. \tag{2.5}$$

The choice of the phase of the matrix elements $\mathscr{B}^{(\zeta)}_{mi,nj}$ differs from Baranger's definition [22]) (if the latter is extended to comprise isospin), but concurs with the one that is in use for other particle-hole computations [12]). [It should be noted though that still another phase convention is used by Gillet and coworkers [23]) and also in ref. [20]).] It affects the phase of the components $y^{(\zeta)}_{mi}$ of the state vectors, and therefore the relation (2.7). If in the matrix in eq. (2.6) the two off-diagonal blocks, $\mathscr{B}$ and $-\mathscr{B}$, are interchanged [24]) an overall minus sign should be added to all components $y^{(\zeta)}_{mi}$.

The random-phase approximation is obtained from the eigenvalue problem [21])

$$\begin{pmatrix} \mathscr{A} & \mathscr{B} \\ -\mathscr{B} & -\mathscr{A} \end{pmatrix} \begin{pmatrix} X^{(\zeta)} \\ Y^{(\zeta)} \end{pmatrix} = E^{(\zeta)} \begin{pmatrix} X^{(\zeta)} \\ Y^{(\zeta)} \end{pmatrix}, \tag{2.6}$$

where the components $x_{mi}^{(\zeta)}$ and $y_{mi}^{(\zeta)}$ of the column vectors $X^{(\zeta)}$ and $Y^{(\zeta)}$, respectively, specify the wave function of the state $|JM, TM_T\rangle$:

$$|JM, TM_T\rangle = \sum_{j_m j_i} \{x_{mi}^{(\zeta)} A^\dagger(mi; JM, TM_T) - y_{mi}^{(\zeta)}(-1)^{J+T} \bar{A}(mi; JM, TM_T)\}|\tilde{0}\rangle. \tag{2.7}$$

Here the quasi-boson operators are defined by

$$A^\dagger(mi; JM, TM_T) \equiv \sum_{m_m m_i t_m t_i} \langle j_m m_m j_i - m_i|JM\rangle\langle \tfrac{1}{2}t_m \tfrac{1}{2} - t_i|TM_T\rangle \times (-1)^{j_i - m_i + \frac{1}{2} - t_i} \eta^\dagger_{j_m m_m t_m} \eta_{j_i m_i t_i}, \tag{2.8}$$

$$\bar{A}(mi; JM, TM_T) \equiv (-1)^{J+M+T+M_T} A(mi; J-M, T-M_T) = (-1)^{J+M+T+M_T} \times \{A^\dagger(mi; J-M, T-M_T)\}^\dagger. \tag{2.9}$$

The correlated ground state $|0\rangle$ is defined by $A(mi; JM, TM_T)|\tilde{0}\rangle = 0$ for all m, i, J, M, T and M_T. The Tamm-Dancoff approximation is obtained if the $\mathscr{B}$-matrix is ignored, the $y_{mi}^{(\zeta)}$ all set equal to zero, and the correlated ground state $|\tilde{0}\rangle$ replaced by the bare vacuum $|\rangle$.

The low-lying odd-parity states in ^{40}Ca will be described in terms of excitations of particles from the 2s–1d shell into the 2p–1f shell. In the TDA only one-particle–one-hole excited states will be considered; in the RPA the correlated ground state $|\tilde{0}\rangle$ will contain even numbers of particle-hole pairs, the excited states therefore contain odd numbers of particle-hole pairs.

3. The Tabakin interaction

The Tabakin interaction in terms of the relative coordinates is defined as a sum of separable potentials of the form [6])

$$V(\boldsymbol{r}|\boldsymbol{r}') = \frac{\hbar^2}{m_N} \sum_{\alpha \mathscr{M} l l'} [s_{\alpha l l'} \tilde{g}_{\alpha l}(r) \tilde{g}_{\alpha l'}(r') + t_{\alpha l l'} \tilde{h}_{\alpha l}(r) \tilde{h}_{\alpha l'}(r')] \mathscr{Y}_{\alpha l}^{\mathscr{M}}(\hat{r}) \mathscr{Y}_{\alpha l'}^{\mathscr{M}}(\hat{r}'). \tag{3.1}$$

Here $\boldsymbol{r}$ is the relative coordinate of the two particles, $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$; the subscript† α denotes the quantum numbers S, $\mathscr{J}$ and T, i.e., the total spin, the total angular momentum in the centre-of-mass system and the total isospin, respectively; the relative angular momentum is represented by l; $\mathscr{M}$ denotes the z-component of $\mathscr{J}$,

† No confusion should arise with α used in eq. (2.1).

and m_N the nucleon mass. The function $\mathscr{Y}^{\mathscr{M}}_{\alpha l}(\hat{r})$ is defined by

$$\mathscr{Y}^{\mathscr{M}}_{\alpha l}(\hat{r}) = \sum_{m_l M_S} \langle l m_l S M_S | \mathscr{J}\mathscr{M} \rangle Y_{l m_l}(\hat{r}) \chi_{S M_S} P^T, \tag{3.2}$$

where $Y_{lm_l}(\hat{r})$ denotes a spherical harmonic, χ_{SM_S} the spin wave function of the two-nucleon system and P^T the isospin projection operator that selects an eigenstate of total isospin T.

TABLE 1a

First- and second-order $T = 0$ matrix elements of the Tabakin interaction in the relative coordinate system (in MeV)

	l	l'	n	n'	$V^{(1)}$	$\bar{V}^{(2)}$	$V^{(1)}+\bar{V}^{(2)}$
$^3S_1$	0	0	0	0	−5.99	−1.30	−7.29
			1	1	−5.43	−1.18	−6.61
			2	2	−4.57	−0.99	−5.56
$^3D_1$	2	2	0	0	3.55	−0.58	2.97
			1	1	3.75	−0.65	3.10
$^3D_2$	2	2	0	0	−1.35	−0.22	−1.57
			1	1	−2.13	−0.38	−2.51
$^3D_3$	2	2	0	0	−0.46	−0.03	−0.49
			1	1	−0.74	−0.05	−0.79
$^1P_1$	1	1	0	0	1.14	−0.28	0.86
			1	1	2.64	−0.87	1.77
			2	2	3.75	−1.36	2.39
$^3D_1$–$^3S_1$	2	0	0	1	−2.13	−0.36	−1.77
			1	2	−2.26	−0.27	−1.99

TABLE 1b

First- and second-order $T = 1$ matrix elements of the Tabakin interaction in the relative coordinate system (in MeV)

	l	l'	n	n'	$V^{(1)}$	$\bar{V}^{(2)}$	$V^{(1)}+\bar{V}^{(2)}$
$^1S_0$	0	0	0	0	−5.13	−0.40	−5.53
			1	1	−4.06	−0.20	−4.26
			2	2	−2.78	−0.08	−2.86
$^1D_2$	2	2	0	0	−0.29	−0.03	−0.32
			1	1	−0.60	−0.06	−0.66
$^3P_0$	1	1	0	0	−1.80	−0.05	−1.85
			1	1	−1.25	−0.03	−1.28
			2	2	−0.25	−0.12	−0.37
$^3P_1$	1	1	0	0	1.80	−0.34	1.46
			1	1	3.14	−0.78	2.36
			2	2	3.86	−1.07	2.79
$^3P_2$	1	1	0	0	−0.60	−0.07	−0.67
			1	1	−1.11	−0.16	−1.27
			2	2	−1.49	−0.24	−1.73

The functions $\tilde{g}_{\alpha l}(r)$, $\tilde{h}_{\alpha l}(r)$ and coefficients $s_{\alpha l l'}$, $t_{\alpha l l'}$ are given in ref. [9]). The new set of parameters for the $^1P_1$ case has been employed.

All necessary matrix elements in the present calculation can be expressed, up to second order, in terms of the antisymmetrized two-particle matrix elements

$$\langle ab; JT|V - \sum_{\zeta}{}' V \frac{|\zeta\rangle\langle\zeta|}{E_\zeta - E_0} \cdot V|cd; JT\rangle.$$

Harmonic-oscillator wave functions have been used for the single-particle wave functions. With the use of Brody-Moshinsky brackets, the first-order matrix elements can be expressed in terms of the matrix elements in the relative-coordinate system $V^{(1)} \equiv \langle nl\alpha|V|n'l'\alpha\rangle$. The latter are integrals [7]) over the functions $\tilde{g}_{\alpha l}(r)$ and $\tilde{h}_{\alpha l}(r)$.

For the evaluation of the second-order matrix elements, the method of Clement and Baranger [9]) has been closely followed. The intermediate two-particle states $|\zeta\rangle$ are represented by plane-wave states and the angle-averaged Pauli operator is used. In good approximation, these so-called second-order Born corrections can also be obtained in the relative-coordinate system as an average $\overline{V^{(2)}}$ over centre-of-mass states [9]). In tables 1 the matrix elements $V^{(1)}$, $\overline{V^{(2)}}$ and $V^{(1)} + \overline{V^{(2)}}$ are given. The following parameters are used in the present calculation: harmonic-oscillator parameter $\nu \equiv m\omega/2\hbar = 0.143\ \mathrm{fm}^{-2}$, fermi momentum $k_F = 1.3\ \mathrm{fm}^{-1}$, energy of the unperturbed bound-state pairs $E_0 = -20$ MeV.

The latter two values are the same as previously used for Hartree-Fock calculations (refs. [10,11])) in ^{40}Ca. As a test of our computer codes, we have recalculated the first- and second-order matrix elements for ^{18}O as given in ref. [9]). We obtained agreement within 3 %.

The particle-hole matrix elements $\mathscr{A}^{(\zeta)}_{mi,nj}$ and $\mathscr{B}^{(\zeta)}_{mi,nj}$ defined in eqs. (2.4) can now be calculated with the aid of eq. (2.5) from the particle-particle matrix elements.

4. The modified surface delta interaction

In this section the residual nuclear interaction will be considered from a different point of view. Instead of taking a potential derived from high-energy free-nucleon scattering data, a phenomenological interaction will be used that is fitted to nuclear spectroscopic data, the modified surface delta interaction (MSDI), which is defined [19]) by

$$V(|\boldsymbol{r}-\boldsymbol{r}'|) = -4\pi A_T \delta(\Omega)\delta(r-R)\delta(r'-R) + B_T. \tag{4.1}$$

Here Ω denotes the angle between the interacting particles and R the nuclear radius. The parameters $A_T(T = 0, 1)$ represent the strength of the interaction for a nucleon pair coupled to isospin T; the parameters $B_T(T = 0, 1)$ have been added to improve the energy spacing between the centres of mass of the $T = 0$ and $T = 1$ states.

When the matrix elements † of the MSDI are calculated, one finds that the radial integrals of the first term of the right-hand side of eq. (4.1) reduce to the product of the radial parts of the four single-particle wave functions evaluated at the nuclear radius R. For the evaluation of the matrix elements of the MSDI, one then assumes that all radial integrals are positive and equal to unity (the non-vanishing matrix elements of the second term of the right-hand side of eq. (4.1) contain radial integrals that are equal to unity anyway, because of the normalization of the wave functions). This assumption is plausible as long as all four single-particle states belong to one major shell.

The matrix elements of the MSDI between one-particle–one-hole states will contain contributions from $A_{T=0}$, $B_{T=0}$ as well as from $A_{T=1}$, $B_{T=1}$. Therefore the parameters A_T and B_T in eq. (4.1) should be replaced by $\sum_{T'} A_{T'} P^{T'}$ and $\sum_{T'} B_{T'} P^{T'}$, respectively, where $P^{T'}$ denotes the projection operator for T'-coupled particle-particle states.

Explicit evaluation of the particle-hole matrix elements defined in eqs. (2.4) yields

$$\mathscr{A}^{(\zeta)}_{mi,nj} = (\varepsilon_m - \varepsilon_i)\delta_{mn}\delta_{ij} + \mathfrak{A}^{(\zeta)}_{mi,nj}, \tag{4.2a}$$

$$\mathscr{B}^{(\zeta)}_{mi,nj} = (-1)^{j_j+j_n+1}\{(1+\delta_{ij})(1+\delta_{mn})\}^{\frac{1}{2}}\mathfrak{A}^{(\zeta)}_{mi,jn}, \tag{4.2b}$$

with

$$\begin{aligned}\mathfrak{A}^{(\zeta)}_{mi,nj} = {} & \tfrac{1}{4}\{(2j_m+1)(2j_n+1)(2j_i+1)(2j_j+1)\}^{\frac{1}{2}}(-1)^{j_i+j_j+l_m+l_i+J+T} \\ & \times \left[\left\{A_0\{1+2(-1)^{l_j+l_n+J}\}+A_1\{1+2(-1)^T\}\right\}\begin{pmatrix} j_m & j_i & J \\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}\begin{pmatrix} j_j & j_n & J \\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}\right. \\ & \left. +(-1)^{l_i+l_n+j_i+j_n}\{A_0 - A_1\{1+2(-1)^T\}\}\begin{pmatrix} j_m & j_i & J \\ \frac{1}{2} & \frac{1}{2} & -1\end{pmatrix}\begin{pmatrix} j_j & j_n & J \\ \frac{1}{2} & \frac{1}{2} & -1\end{pmatrix}\right] \\ & \times \delta^{\text{even}}_{l_m+l_n+l_i+l_j} + \tfrac{1}{2}(-1)^T[B_0 - B_1\{1+2(-1)^T\}]\delta_{mn}\delta_{ij}.\end{aligned} \tag{4.2c}$$

The expression for $\mathfrak{A}^{(\zeta)}_{mi,jn}$ is obtained from eq. (4.2c) by interchanging n and j. The values to be taken for the single-particle energies ε_i and ε_m will be derived from the single-particle and single-hole states of the neighbouring nuclei (see subsect. 5.1). The constants A_0, A_1, B_0 and B_1 are determined from a least-squares fit to the experimental excitation energies [25]) of low-lying states in ^{40}Ca given in table 2.

TABLE 2

Levels in ^{40}Ca used for the least-squares fit of the MSDI

$J^\pi; T$	E_x (MeV)	$J^\pi; T$	E_x (MeV)
$3^-; 0$	3.73	$4^-; 1$	7.66
$5^-; 0$	4.48	$3^-; 1$	7.69
$4^-; 0$	5.61	$2^-; 1$	8.46
		$5^-; 1$	8.55

† It should be noted that in ref. [19]) the single-particle states are obtained by coupling $s+l \rightarrow j$ (in that order).

5. Calculations and resulting energies

5.1. CHOICE OF THE UNPERTURBED PARTICLE-HOLE ENERGIES

The unperturbed energies of the one-particle–one-hole configurations with respect to the ground state of ^{40}Ca are usually derived from experimental single-particle states observed in ^{39}Ca, ^{39}K and ^{41}Ca. With the assumption that the ground states of ^{39}Ca, ^{40}Ca and ^{41}Ca are described by pure shell-model states, the difference between the $1f_{\frac{7}{2}}$ and $1d_{\frac{3}{2}}$ single-neutron energies is determined from the ground-state binding energies [25] E_b as

$$E(1f_{\frac{7}{2}}\,1d_{\frac{3}{2}}^{-1}) = E_b(^{39}\mathrm{Ca}) + E_b(^{41}\mathrm{Ca}) - 2E_b(^{40}\mathrm{Ca}) = 7.3 \text{ MeV}.$$

Recently Gerace [26]) has re-evaluated this particle-hole energy difference taking into account admixtures of deformed states in the ground states and obtained a value of 5.4 MeV. For the MSDI – being an effective interaction – the influence of deformed states will be absorbed into the interaction parameters obtained from the fitting procedure. For a realistic interaction, an appropriate choice of the $1f_{\frac{7}{2}}\,1d_{\frac{3}{2}}^{-1}$ splitting is important; the position and wave function of the lowest $J^\pi = 3^-$ $T = 0$ level, for instance, in the RPA are very sensitive to this choice. Application of the value of 5.4 MeV for the $1f_{\frac{7}{2}}\,d_{\frac{3}{2}}^{-1}$ splitting in the present calculation with the Tabakin interaction would lead to an imaginary energy for the lowest octupole state. For this reason we have used for the $1f_{\frac{7}{2}}\,1d_{\frac{3}{2}}^{-1}$ splitting the value of 7.3 MeV.

The set of single-particle energies used in the present calculation is given in table 3.

TABLE 3
Single-particle energies near $A = 40$ used in the present calculation

Label	1	2	3	4	5	6	7
Orbit	$1d_{\frac{5}{2}}^{-1}$	$2s_{\frac{1}{2}}^{-1}$	$1d_{\frac{3}{2}}^{-1}$	$1f_{\frac{7}{2}}$	$2p_{\frac{3}{2}}$	$1f_{\frac{5}{2}}$	$2p_{\frac{1}{2}}$
$E_{\mathrm{s.p.}}$ (MeV)	−21.6	−18.1	−15.6	−8.3	−6.3	−2.1	−4.2

5.2. COULOMB EFFECTS

The Coulomb force should appear in the calculation in two ways [27]).

(i) The unperturbed proton-proton-hole energies differ from the corresponding neutron-neutron-hole energies. Although the experimental information about the positions of the single-proton-hole states is poor, recent data seem to lead to values [25]) less than 0.2 MeV for these deviations.

(ii) There exists a residual Coulomb interaction between proton-proton-hole configurations. We have estimated the Coulomb contribution to the proton-proton-hole matrix elements by calculating the particle-hole matrix elements of the Coulomb potential. The contributions to the diagonal matrix elements turn out to be rather constant at −0.3 MeV, and those to the off-diagonal matrix elements are smaller than 0.1 MeV in magnitude.

We may conclude that the effect of the Coulomb force on the calculated level energies is small. Therefore in the present calculation the effects (i) and (ii) have been neglected.

5.3. CALCULATED ENERGIES AND COMPARISON WITH EXPERIMENT

The calculations were performed with the two interactions in the TDA and in the RPA. For the Tabakin interaction only the results in the RPA are given, which is a better approximation than the TDA for a realistic interaction. For the MSDI it turned out that the energy levels could be fitted equally well in the TDA and the RPA, but with different sets of parameters.

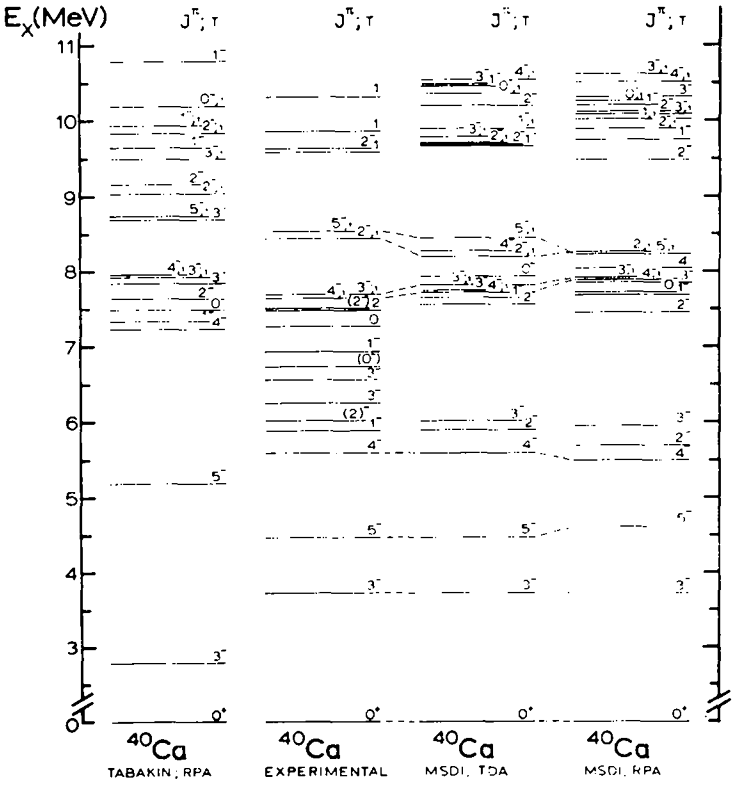

Fig. 1. Odd-parity spectra of ^{40}Ca. For the $T = 0$ levels, the isospin label has been suppressed. The results for the Tabakin interaction are drawn for the 1 $\hbar\omega$ space only.

In fig. 1, the level energies calculated with the Tabakin interaction and the MSDI are displayed and compared with the experimental spectrum. The lowest $J^\pi = 1^-$ $T = 0$ state in the TDA was calculated at 1.27 MeV and turned out to be 95 % spurious; therefore it is not drawn. In both RPA calculations the spurious states yielded imaginary energies.

It is evident that a reasonable agreement is obtained between the two calculations and experiment, although there are also some notable differences. The great resemblance between the two MSDI calculations is remarkable, although it should be kept in mind that the best-fit parameters are different. In the TDA the parameters are $A_0 = 0.70$ MeV, $A_1 = 0.42$ MeV, $B_0 = -1.30$ MeV and $B_1 = 0.70$ MeV, in the RPA $A_0 = 0.45$ MeV, $A_1 = 0.29$ MeV, $B_0 = -1.66$ MeV and $B_1 = 0.72$ MeV.

The position of the lowest $J^\pi = 0^- \ T = 0$ level is not known. The 7.30 MeV level possesses spin [25] $J = 0$, but the parity has not yet been determined. Another candidate for the $J^\pi = 0^-$ level may be the level at 6.75 MeV that is strongly excited in the ^{39}K(^{3}He, d)^{40}Ca reaction [28,29]. Both the Tabakin interaction and the MSDI predict the lowest $J^\pi = 0^-$ level in the region of 7 MeV. On the contrary Gillet [30], using an effective central residual interaction, obtained the lowest $J^\pi = 0^-$ level at 9 MeV. In the Tabakin case the RPA causes a rather large depression of 400 keV of the $J^\pi = 0^-$ state with respect to the TDA result.

The lowest known $J^\pi = 1^- \ T = 0$ level at 5.90 MeV seems to belong to a rotational band [31] as is also indicated by ^{39}K(^{3}He, d)^{40}Ca experiments [28,29]. Therefore the $J^\pi = 1^-$ level at 6.95 MeV probably may be identified with the lowest non-spurious $J^\pi = 1^-$ state with a one-particle–one-hole character. Both interactions give the lowest non-spurious $J^\pi = 1^-$ state approximately 0.5 MeV too high. The second and third non-spurious $J^\pi = 1^-$ states are predicted in the region near 10 MeV and might correspond with the $J = 1$ resonances at $E_x = 9.61$ MeV and 10.33 MeV seen [18,32] in (p, γ) work.

The characteristics of the experimental $J^\pi = 2^- \ T = 0$ levels are unclear. Spin and parity assignments $J^\pi = 2^-$ have been made [25] to the levels at 6.03 MeV and 7.47 MeV. Recent information obtained from the ^{39}K(^{3}He, d)^{40}Ca reaction [29] indicates that the 7.53 MeV level rather than the 6.03 MeV level has mainly a $1f_{\frac{7}{2}}\, 1d_{\frac{3}{2}}^{-1}$ configuration. The MSDI predicts a low-lying $J^\pi = 2^-$ level which might correspond with the level at 6.03 MeV. The Tabakin interaction does not yield a $J^\pi = 2^-$ state below 7.5 MeV.

The lowest $J^\pi = 3^- \ T = 0$ state lies at 3.73 MeV and is strongly excited in (α, α') experiments [25]. It is a complex mixture of one-particle–one-hole states. For the Tabakin interaction the position of the unperturbed $|1f_{\frac{7}{2}}\, 1d_{\frac{3}{2}}^{-1}; J^\pi = 3^- \ T = 0\rangle$ state at 7.3 MeV, is lowered in the TDA to 5.3 MeV. In the RPA, a further depression of 2.5 MeV to 2.8 MeV is introduced. If, however, one employs a smaller value of the $1f_{\frac{7}{2}}\, 1d_{\frac{3}{2}}^{-1}$ splitting (e.g. 6 MeV), the energy of the $J^\pi = 3^- \ T = 0$ level soon becomes imaginary in the RPA. The second and third $J^\pi = 3^-$ states observed at 6.28 MeV and 6.59 MeV probably are mixtures of a one-particle–one-hole state and a deformed state [31].

The Tabakin interaction predicts the lowest $J^\pi = 4^- \ T = 0$ state more than 1 MeV higher than the experimental $J^\pi = 4^- \ T = 0$ level at 5.61 MeV. This may be partially due to the neglect of $3\,\hbar\omega$ configurations, which can depress the $J^\pi = 4^- \ T = 0$ level by approximately 0.5 MeV (see subsect. 5.4).

The $J^\pi = 5^-$ $T = 0$ level at 4.48 MeV is reasonably reproduced in the present calculations (0.71 MeV too high for the Tabakin interaction and 0.27 MeV too high for the MSDI in the RPA).

The calculated energies of $T = 1$ levels compare fairly well with experiment. The spacings of the $1f_{\frac{7}{2}}1d_{\frac{3}{2}}^{-1}$ quadruplet calculated with the MSDI are found to be too small, but no acceptable variation of the interaction parameters could improve this. With the Tabakin interaction the position of the $J^\pi = 2^-$ level is approximately 0.5 MeV too high. Above the $1f_{\frac{7}{2}}1d_{\frac{3}{2}}^{-1}$ quadruplet no $T = 1$ states are known with certainty.

5.4. EFFECT OF 3 $\hbar\omega$ CONFIGURATIONS

Thus far we have taken into account only $1\hbar\omega$ configurations explicitly but allowed for ground-state correlations in the RPA. Calculations performed with realistic interactions on nuclei with two nucleons outside a closed-shell core have shown that renormalization effects due to core polarization are very important [1,2,33]). The renormalization of the particle-hole force has been investigated less thoroughly [34]). In the following we shall include, using perturbation theory, the effect of $3\hbar\omega$ configurations for the Tabakin interaction.

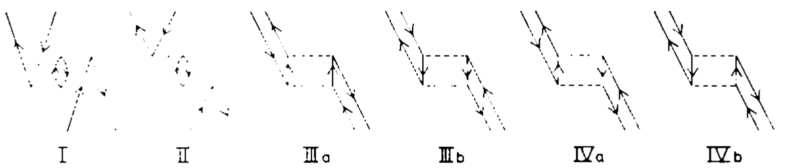

Fig. 2. Second-order diagrams contributing to the particle-hole force.

All corrections to the first-order particle-hole matrix elements are drawn in fig. 2. Exchange diagrams are not shown.

Explicit expressions for the diagrams of types III and IV have been given by de Takacsy [39]). The backward going diagrams I already have been included in the RPA up to all orders and therefore should not be taken into account again. The intermediate states are restricted to all $3\hbar\omega$ states; an average energy denominator of 22 MeV is used. All matrix elements required for the evaluation of the second-order corrections have been obtained as indicated in sect. 3.

It was found that the contributions from each type of diagram may be quite large. However, in general these contributions do not possess the same sign and an appreciable cancellation results. Some of the $3\hbar\omega$ second-order corrections to the matrix elements $\langle 1f_{\frac{7}{2}}1d_{\frac{3}{2}}^{-1}; JT|V|1f_{\frac{7}{2}}1d_{\frac{3}{2}}^{-1}; JT\rangle$ could be compared with those obtained for the Kallio-Kolltveit force [35]) given in ref. [36]). There is complete agreement as for the signs, whereas the magnitudes agree within 50 %, which seems reasonable in view of the different interactions employed.

The energies of the lower states obtained after inclusion of $3\hbar\omega$ excitations are given in table 4, where they are compared with the $1\hbar\omega$ RPA and TDA results. It is worthwhile noting that the effect of complete inclusion of the $3\hbar\omega$ configurations on the excitation energies is in general as important as the effect of the ground-state correlations that are taken into account in the RPA. In most particle-hole calculations, however, the $3\hbar\omega$ configurations are accounted for only partly, i.e. as far as they are contained in the RPA.

TABLE 4

Excitation energies (in MeV) of low-lying states of ^{40}Ca calculated with the Tabakin interaction

JT	00	10	20	30	40	50	01	11	21	31	41	51
TDA $1\hbar\omega$	7.91	—	7.86	5.40	7.31	5.96	10.26	9.99	9.20	7.97	7.99	8.79
RPA $1\hbar\omega$	7.54	7.36	7.65	2.80	7.25	5.20	10.22	9.96	9.08	7.94	7.95	8.75
RPA $1\hbar\omega$ and $3\hbar\omega$	8.35	7.27	7.61	0.91 i	6.81	4.70	10.67	9.90	9.11	7.78	7.76	9.09

In the first line the results of the TDA are given; as the lowest $J = 1$, $T = 0$ state is mainly spurious, its calculated energy is not listed. The second line contains the standard RPA results and the third line those with $3\hbar\omega$ configurations taken into account.

In most cases the inclusion of $3\hbar\omega$ configurations leads to an improved agreement with experiment.

The $J^\pi = 4^-$ $T = 0$ level is depressed by 0.5 MeV into the right direction, but it still lies more than 1 MeV too high.

The $J^\pi = 5^-$ $T = 1$ level is raised by 0.35 MeV, which is rather unexpected for a $T = 1$ level.

For the $J^\pi = 3^-$ $T = 0$ states the inclusion of $3\hbar\omega$ corrections has the undesired consequence that the energy of the lowest state becomes imaginary in the RPA for all acceptable choices of the $1f_{\frac{7}{2}}\,1d_{\frac{3}{2}}^{-1}$ energy splitting. The depression in this case is mainly due to an increase of the magnitude of the off-diagonal matrix elements.

The contributions from the diagrams of types II and III to the off-diagonal matrix elements appear to possess the same sign as the first-order matrix elements, while those from the diagrams of type IV possess the opposite sign. The net effect tends to increase the magnitude of the off-diagonal matrix elements.

6. Wave functions and electromagnetic transitions

6.1. WAVE FUNCTIONS

Before one can compare the wave functions obtained with the MSDI and with the Tabakin interaction, it is necessary to check the relative signs of the single-particle wave functions of the two sets.

TABLE 5

The RPA wave functions of the lowest odd-parity $T = 0$ states in ^{40}Ca for the Tabakin interaction and the MSDI

p-h states			53	61	72
$J^\pi = 0^-$	Tab	x	+1.00	−0.05	−0.11
		y	−0.10	+0.09	+0.06
	MSDI	x	+1.00	+0.06	+0.04
		y	−0.05	+0.01	+0.02

p-h states			41	51	52	53	61	63	72	73	$E_x^{exp} = 6.95$ MeV
$J^\pi = 1^-$	Tab	x	−0.29	+0.45	+0.16	+0.52	−0.02	−0.12	−0.12	+0.62	$E_x = 7.36$ MeV
		y	−0.04	+0.04	+0.03	+0.02	+0.02	−0.04	+0.04	+0.03	
	MSDI	x	−0.06	−0.09	−0.06	+0.97	+0.03	−0.05	+0.07	−0.20	$E_x = 7.67$ MeV
		y	−0.02	−0.03	−0.01	−0.02	+0.01	−0.01	+0.02	−0.03	

p-h states			41	43	51	52	53	61	62	63	71	73
$J^\pi = 2^-$	Tab	x	+0.14	+0.89	+0.00	+0.16	−0.38	+0.08	+0.04	+0.05	+0.03	+0.17
		y	+0.02	−0.07	+0.00	+0.02	+0.01	−0.03	+0.00	−0.01	+0.00	−0.03
	MSDI	x	−0.03	+0.99	−0.01	−0.01	−0.10	+0.02	+0.04	−0.02	+0.00	−0.01
		y	+0.02	−0.01	+0.02	+0.02	−0.01	+0.01	+0.00	+0.02	−0.02	+0.02

p-h states			41	42	43	51	53	61	62	63	71
$J^\pi = 3^-$	Tab	x	+0.50	+0.75	+0.66	+0.23	+0.39	−0.28	−0.38	+0.42	−0.25
		y	+0.37	+0.50	+0.42	+0.18	+0.28	−0.21	−0.27	+0.31	−0.19
	MSDI	x	+0.28	+0.56	+0.62	+0.16	+0.50	−0.08	−0.15	+0.21	−0.13
		y	+0.15	+0.22	+0.11	+0.09	+0.17	−0.04	−0.08	+0.11	−0.08

p-h states			41	42	43	51	61	63
$J^\pi = 4^-$	Tab	x	+0.11	+0.02	+0.99	+0.03	+0.02	+0.10
		y	+0.03	+0.02	−0.02	−0.01	+0.00	+0.01
	MSDI	x	+0.00	+0.01	+1.00	+0.00	+0.00	+0.00
		y	+0.01	+0.01	−0.01	+0.01	+0.00	+0.01

p-h states			41	43	61
$J^\pi = 5^-$	Tab	x	+0.19	+0.99	−0.24
		y	+0.09	+0.24	−0.09
	MSDI	x	+0.09	+1.00	−0.06
		y	+0.04	+0.12	−0.03

The signs of the MSDI wave functions have been changed as described in subsect. 6.1. The labels of the particle-hole states are given in table 3.

The harmonic-oscillator wave functions are defined as in ref. 7). For the MSDI calculation, however, no radial wave functions are specified; instead all radial matrix elements of the MSDI are defined to be positive and equal to unity (see sect. 4). The use of the surface delta interaction is justified [15]) by taking into account single-particle wave functions that are peaked at one and the same distance only, i.e. at the nuclear surface. In the present calculations, however, 2s, 1d, 1f and 2p states are taken into account; the radial parts of the 2s and 2p harmonic-oscillator wave functions possess a zero between $r = 0$ and $r = \infty$, so that their values are negative [if they are defined as in ref. [7])] at the second extremum and in the asymptotic region, that appear to be responsible for the sign of the radial integrals. Thus for a proper comparison of the wave functions that are determined with the MSDI and with the Tabakin interaction, it is necessary to invert the sign of the wave functions of the 2s and 2p states in one of the two cases. Therefore we have added a minus sign in table 5 to all components that refer to the 2s and 2p components calculated from the MSDI, so that their signs agree in the asymptotic region with the harmonic-oscillator wave functions employed in the present paper.

In table 5 the calculated RPA wave functions for ^{40}Ca are given; for each spin value the level with the lowest energy is considered. It is seen from the table that in general the Tabakin wave functions show more configuration mixing than those obtained from the MSDI. The lowest $J^\pi = 0^-, 2^-, 4^-$ and 5^- $T = 0$ states are almost pure j-j coupling states in the MSDI case. However, the lowest $J^\pi = 3^-$ $T = 0$ state is even in the MSDI case strongly mixed. The Tabakin RPA wave function of this state is very sensitive to the energy taken for the $1f_{\frac{7}{2}}\,1d_{\frac{3}{2}}^{-1}$ splitting. The components of the wave functions of the non-spurious $J^\pi = 1^-$ $T = 0$ states determined from the MSDI and the Tabakin interaction differ not only in magnitude but also in relative sign.

It is found that in general in the RPA the amplitudes $y_{mi}^{(\zeta)}$ are much larger for the Tabakin interaction than for the MSDI. For the latter interaction the amplitudes $y_{mi}^{(\zeta)}$ are always smaller than 0.07, except for the lowest $J^\pi = 3^-$ and 5^- $T = 0$ states. Although the parameters A_T and B_T of the MSDI are not the same for the RPA and TDA, the differences of the amplitudes $x_{mi}^{(\zeta)}$ in the RPA with the corresponding amplitudes in the TDA are smaller than 0.1. Therefore only the RPA wave functions are tabulated.

6.2. ELECTROMAGNETIC TRANSITIONS

In ^{40}Ca some experimental information is available concerning mean lifetimes, branching ratios and E2/M1 mixing of electromagnetic transitions. It is interesting to compare predictions that follow from the present calculations and experiment.

The reduced transition probabilities for a 2^λ-pole transition of parity π from an initial state $|JTM_T\rangle$ to the ground state $|\tilde{0}\rangle$ and to a final state $|J'T'M_T\rangle$ are given by [the particle-hole states are defined in eq. (2.3)]

$$B(\pi\lambda; JTM_T = 0 \to \tilde{0}) = \frac{1}{2(2J+1)} \,|\sum_{mi\lambda_t} (x_{mi}^{(\zeta)} + y_{mi}^{(\zeta)})\delta_{T\lambda_t}\langle j_m||\boldsymbol{\Omega}_{\lambda_t}(\pi\lambda)||j_i\rangle|^2\delta_{\lambda J}, \quad (6.1)$$

$$B(\pi\lambda;\, JTM_T \to J'T'M_T) = \tfrac{1}{2}(2J'+1)(2T+1)\Bigg|\sum_{i\,\lambda_t}(2\lambda_t+1)^{\frac{1}{2}}\langle TM_T\lambda_t 0|T'M_T\rangle$$
$$\times \sum_{minj}(-1)^{j_m+j_j}\Bigg[(x_{nj}^{(\zeta')}x_{ni}^{(\zeta)}-\phi y_{nj}^{(\zeta')}y_{ni}^{(\zeta)})\delta_{mn}(-1)^{J'+T'}\langle i||\boldsymbol{\Omega}_{\lambda_t}(\pi\lambda)||j\rangle$$
$$\times \begin{Bmatrix} J' & \lambda & J \\ j_i & j_n & j_j \end{Bmatrix}\begin{Bmatrix} T' & \lambda_t & T \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} -(x_{mj}^{(\zeta')}x_{nj}^{(\zeta)}-\phi y_{mj}^{(\zeta')}y_{nj}^{(\zeta)})\delta_{ij}(-1)^{J+T+\lambda+\lambda_t}$$
$$\times \langle m||\boldsymbol{\Omega}_{\lambda_t}(\pi\lambda)||n\rangle \begin{Bmatrix} J' & \lambda & J \\ j_n & j_j & j_m \end{Bmatrix}\begin{Bmatrix} T' & \lambda_t & T \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}\Bigg]\Bigg|^2, \tag{6.2}$$

where for brevity a phase factor $\phi \equiv (-1)^{J'+T'+\lambda+\lambda_t+J+T}$ has been introduced, and where the transition operator $\Omega(\pi\lambda\mu)$ has been expanded into its iso-irreducible parts

$$\Omega(\pi\lambda\mu) = \Omega_{\lambda_t=0}(\pi\lambda\mu)\tfrac{1}{2}+\Omega_{\lambda_t=1}(\pi\lambda\mu)t, \tag{6.3}$$

where t denotes the third component of the single-particle isospin operator. For the derivation of eq. (6.2), the Hermitean conjugates of the transition operators $\Omega(\pi\lambda\mu)$ were assumed to be given by

$$\Omega^\dagger(\pi\lambda\mu) = (-1)^\mu\Omega(\pi\lambda-\mu). \tag{6.4}$$

A different phase in eq. (6.4) will affect the phase factor ϕ in eq. (6.2).

The reduced matrix elements can be evaluated by standard methods [37]). For the Tabakin case, of course the harmonic-oscillator wave functions have been used to calculate the radial integrals; for the case of the MSDI we took, in the spirit of the surface delta interaction $\int r^\lambda R'(r)R(r)r^2\mathrm{d}r = R^\lambda$, where $R'(r)$ and $R(r)$ represent the radial parts of the single-particle wave functions and the nuclear radius is given by $R = 1.2\times A^{\frac{1}{3}}$ fm. For the present calculations the free-nucleon g-factors have been used. No effective-charge assumption has been made.

In table 6 mean lifetimes are given for electromagnetic transitions to odd-parity states and the ground state for levels with $E_x < 9$ MeV. The experimental excitation energies, mean lifetimes and branching ratios [25]) are shown together with the same quantities calculated with the Tabakin interaction in the RPA, with the MSDI in the RPA and with the MSDI in the TDA. In general the results of the latter calculation are very similar to those calculated with the MSDI in the RPA, and therefore they are given only when the difference in transition rates in these two approximations amounts to more than 25 %.

The transition rates are strongly dependent on the excitation energies. Therefore the calculated levels were, when possible, identified with the corresponding experimental states, and the experimental energies were used to calculate the mean lifetimes and branching ratios.

TABLE 6

Decay of odd-parity levels in ^{40}Ca with $E_x < 9$ MeV

J^π; T	Case [a])	E_x (MeV)	τ_m [b])	Decay in (percents) to lower levels [MeV] 0	3.73	4.48	5.61	6.03	6.28	other levels
3⁻; 0	Exp	3.73	85 ps	100						
	Tab	2.80	58 ps	100						
	MR	3.72	76 ps	100						
	MT	3.73	160 ps	100						
5⁻; 0	Exp	4.48	>1.5 ps [c])		100					
	Tab	5.20	19 ns		100					
	MR	4.64	38 ns		100					
	MT	4.49	20 ns		100					
4⁻; 0	Exp	5.61			65	35				
	Tab	7.25	9 ps		55	45				
	MR	5.46	4 ps		60	40				
2⁻; 0	Exp	6.03			100					
	MR	5.70	940 fs	65	35					
3⁻; 0	Exp	6.28	0.39 ps			100				
	Tab	7.85	1.4 ps		80	17	3			
	MR	5.94	0.9 ps		91	6	3			
2⁻; 0	Exp	(7.53)								
	Tab	7.65	400 fs	55	30				15	
	MR	7.47	700 fs		73		18	1	15	[6.95] 3
0⁻; 0	Tab	7.50	40 ps							[6.95]100
	MR	7.72	2.2 ps					45		[6.95] 55
4⁻; 1	Exp	7.66			35	25	40			
	Tab	7.95	1.6 fs		7	26	65		2	
	MR	7.90	1.0 fs		20	27	50		3	
3⁻; 1	Exp	7.69			100					
	Tab	7.94	1.6 fs		72		19		9	
	MR	7.92	0.8 fs		71		14	5	10	
3⁻; 0	MR	7.84	105 fs		5	95				
4⁻; 0	MR	8.05	250 fs		61	23	9	3	4	
2⁻; 1	Exp	8.47								
	Tab	9.08	2.1 fs	40	44				8	[7.53] 6
	MR	8.27	0.5 fs	11	16			64	9	
5⁻; 1	Exp	8.54								
	Tab	8.75	0.21 fs			90	10			
	MR	8.24	0.26 fs			94	6			

[a]) Exp – Experiment; the mean lifetimes and transition rates are reduced for transitions to odd-parity states and to the ground state only. Tab – Tabakin interaction in the RPA. MR – MSDI in the RPA. MT – MSDI in the TDA; these results are only given, if the transition strength differs more than 25 % from the corresponding strength found in the RPA.

[b]) If experimental energies are given, these values are used in the calculation of the mean lifetimes and transition rates.

[c]) Ref. [38]).

The strength of the octupole transition from the lowest $J^\pi = 3^-$ level to the ground state (experimentally 22±5 W.u.) cannot be reproduced in the TDA (11 W.u.); in the RPA the agreement both for Tabakin (31 W.u.) and for the MSDI (24 W.u.) is good.

The branching ratio of the transitions from the $J^\pi = 4^-$ $T = 0$ level at 5.61 MeV is in good agreement with experiment for both interactions employed. The large E2/M1 mixing ratio δ of the transition to the $E_x = 4.48$ MeV level is reasonably reproduced (experimentally [18]) $\delta = 0.7 \pm 0.2$, calculated with the Tabakin interaction $\delta = 0.55$ and with the MSDI $\delta = 0.45$, both in the RPA). The mixing ratio of the transition to the level at 3.73 MeV is in better agreement with experiment [18]) ($\delta = 0.27 \pm 0.05$) for the Tabakin interaction ($\delta = 0.31$) than for the MSDI ($\delta = 0.04$).

TABLE 7

Decay of odd-parity levels in ^{40}Ca with 9 MeV $< E_x <$ 11 MeV

J^π; T	Case [a])	E_x (MeV)	Γ_γ (eV)	Decay (in percents) to lower levels (MeV) 0	3.73	4.48	5.61	6.03	6.28	7.53	other levels
2^-, 1	Exp	9.65	>1		80					20	
	MR	9.91	0.65	8	53		1	5	8	25	
	Exp	9.87	>5	99						(1)	[6.95] 1
1^-, 1	MR	10.03	60	99						1	
	MT	9.89	10	100							
3^-, 1	MR	10.17	1.6		80	3	2		5		[7.84] 10
0^-, 1	MR	10.28	1.6					1			[6.95] 99
4^-, 1	MR	10.52	2.8		20	1			3		[7.84] 5; [8.05] 69
3^-, 1	MR	10.65	1.8		65	2	1		6		[7.84] 7; [8.05] 19

[a]) Exp — Experiment; the mean lifetimes and transition rates are reduced for transitions to odd-parity states and to the ground state only. MR — MSDI in the RPA. MT — MSDI in the TDA; these results are only given, if the transition strength differs more than 25 % from the corresponding strength found in the RPA.

The second $J^\pi = 3^-$ state at 6.28 MeV experimentally shows a transition only to the $J^\pi = 5^-$ state at 4.48 MeV (we consider the odd-parity levels solely). If we identify the second calculated $J^\pi_c = 3^-$ state with the level at 6.28 MeV, the transition to the $J^\pi = 3^-$ state at 3.73 MeV is 5 or 10 times stronger than that to the $J^\pi = 5^-$ state in the present calculations. The M1 strength of the $J^\pi = 3^-$ to $J^\pi = 3^-$ transition results from several contributions that partially cancel. A slight change in the wave functions (e.g. due to the presence of components of deformed states) already may give a much weaker (or stronger) strength than in the present case.

In ^{40}Ca some strong (p, γ) resonances have been observed [18, 32]) in the region 9 MeV $< E_x <$ 11 MeV. For these states, some experimental lower limits [32]) of the widths Γ_γ have been derived. In the same region several $J^\pi = 1^-$ and/or $T = 0$ states are predicted by the MSDI calculation, that show relatively strong γ-decay to other odd-parity states or to the ground state. The γ-widths and branching ratios of these states are given in table 7.

TABLE 8

Decay of the lower odd-parity states in ^{40}K

J^π; T	Case [a])	E_x (MeV)	τ_m	Decay (in percents) to lower levels (MeV): 0	0.03	0.80	(2.10)	$J^\pi = 1^-$; $T = 1$
3^-; 1	Exp	0.03	7.2 ns [b])	100				
	Tab	−0.01	11 ns	100				
	MR	0.02	10 ns	100				
2^-; 1	Exp	0.80			100			
	Tab	1.13	2100 fs		100			
	MR	0.37	460 fs		100			
	MT	0.49	830 fs		100			
5^-; 1	Exp	0.89		100				
	Tab	0.81	4 ps	100				
	MR	0.34	1.1 ps	100				
	MT	0.79	2.1 ps	100				
2^-; 1	Exp	(2.10)			(100)			
	Tab	1.96	800 fs		80	20		
	MR	2.01	1500 fs	2	98			
	MT	2.05	400 fs	1	76	23		
1^-; 1	Tab	2.01	480 fs			94	6	
	MR	2.13	230 fs			99	1	
	MT	2.23	2400 fs		4	66	30	
3^-; 1	Exp	(2.05)		(45)	(25)	(30)		
	Tab	1.58	500 fs	79	19	2		
	MR	2.27	160 fs	64	35		1	
	MT	2.13	1100 fs	11	57	32		
0^-; 1	Tab	2.26	1 ps			14		86
	MR	2.38	3.4 ps			2		98
	MT	2.69	4.5 ps			3		97

[a]) Exp — Experiment; the mean lifetimes and transition rates are reduced for transitions to odd-parity states only. Tab — Tabakin interaction in the RPA. MR — MSDI in the RPA. MT — MSDI in the TDA; these results are only given, if the transition strength differs more than 25 % from the corresponding strength found in the RPA.

[b]) This mean lifetime has been reduced for internal conversion.

The decay of $J^\pi = 1^-$ $T = 0$ states to the ground state cannot be accounted for in the present model without isospin-mixing. In view of the calculated energies and radiation strengths the $T = 1$ doublet at 9.87 MeV might correspond to the $J^\pi = 1^-$ $T = 1$ level. Experimentally one further has the relatively strong $J^\pi = 2^-$ resonance at

$E_x = 9.65$ MeV, that likely possesses isospin $T = 1$. This state is rather well reproduced in the present calculation.

Finally we have calculated the energies and transition strengths of the lower odd-parity states in ^{40}K. The results are shown in table 8. The only spins experimentally known in this nucleus belong to the four lower levels with $J^\pi = 4^-, 3^-, 2^-$ and 5^-. The mean lifetime of the first excited state is known; the two interactions give a mean life approximately 1.4 times too long.

7. Discussion

In this paper we have investigated odd-parity states in ^{40}Ca that are described as particle-hole excitations using two different types of interactions.

The energies obtained with the Tabakin interaction in the RPA for the $1\hbar\omega$ space do not reproduce (even with the second-order Born corrections included) the experimental data very well. This is in line with the findings from other calculations [1,8,9]) with realistic interactions. A very large model space is required in order to obtain satisfactory results.

When $3\,\hbar\omega$ excitations are included in second-order perturbation theory, the agreement with experiment is improved except for the lowest $J^\pi = 3^-$ $T = 0$ level that is depressed to an imaginary energy. This effect may be partly due to the fact that the Tabakin interaction is a little too strong. A comparison of the two-body matrix elements that were obtained for $A = 18$ nuclei from the Hamada-Johnston interaction [1,2]) and from the Tabakin interaction [9]) shows that the Tabakin $T = 0$ off-diagonal matrix elements are generally larger in magnitude. In the former calculation, four-particle–two-hole excitations had to be included in order to obtain agreement with experiment, whereas in the latter calculation such a procedure would depress, e.g. the $J^\pi = 1^+$, $T = 0$ energy level much too far. However, Mavromatis *et al.* [12]) have reported that a similar effect, i.e. too large a depression of the $J^\pi = 3^-$ $T = 0$ state in the RPA, is found after inclusion of $3\,\hbar\omega$ configurations in a calculation on ^{16}O with the Hamada-Johnston interaction.

As for the MSDI, it is remarkable that this schematic interaction reproduces the experimental energies as well as more complicated effective interactions. Similar satisfactory results for the MSDI have been found in other light nuclei [19]). However, the strengths A_0 and A_1 determined in the present particle-hole calculation appreciably differ from those obtained for the particle-particle interaction [19]). This is in agreement with the evidence [34]) that the effective particle-hole interaction is weaker than the effective particle-particle interaction. The parameters B_0 and B_1 that produce an over-all shift between the $T = 0$ and $T = 1$ levels have nearly the same values for particle-hole and particle-particle calculations.

The wave functions obtained in the present calculations with the Tabakin interaction and the MSDI differ qualitatively. The Tabakin wave functions show more configuration mixing than those calculated from the MSDI; in general the latter interaction produces almost pure *j*–*j* coupling states, except for the $J^\pi = 1^-$ and 3^-

$T = 0$ states. However, the scarce experimental data preclude a meaningful discrimination as to the quality of the wave functions obtained from these different interactions.

For the MSDI, there is little difference between the results of the TDA and RPA; however, calculation of transition rates shows that the RPA better reproduces the γ-width of the lowest $J^\pi = 3^- \ T = 0$ state.

In this paper, the customary multipole moment operators in the long-wavelength limit have been used. The non-locality of the Tabakin interaction presumably will cause some correction terms to be added to the customary transition operators †. A rough estimate of this correction has been made [39]) only for the magnetic dipole moment for a particular non-local potential [the Green potential [5])]; the correction turned out to be small but not negligible. Using the customary M1 operator we obtained for the magnetic dipole moment of the ground state of ^{40}K the values: $\mu_{\text{Tab}} = -1.86$ n.m., $\mu_{\text{MSDI}} = -1.91$ n.m. ($\mu_{\text{exp}} = -1.30$ n.m.).

Another point to be investigated more carefully is the double counting that may be involved in the Tabakin calculation. A complete set of intermediate two-particle planewave states was included for the evaluation of $\overline{V^{(2)}}$. Then the application of the RPA and the inclusion of $3\,\hbar\omega$ configurations may involve the intermediate states that already were taken into account in the evaluation of $\overline{V^{(2)}}$. However, it has been argued (ref. [40])) that contributions from low-lying intermediate states are underestimated because of the plane-wave approximation.

A difficult problem in calculations like the present one remains the appearance of low-lying deformed states that may considerably complicate a fair comparison of the results of shell-model calculations on a restricted basis with experiment. In ^{40}Ca, several deformed states seem to be strongly mixed [31]) with one-particle–one-hole states. The construction of deformed states and the calculation of matrix elements between these states and shell-model states have been recently attempted [31]). The results look very promising, although they are strongly dependent on the single-particle energies.

Several other particle-hole calculations on ^{40}Ca have been reported †† with various effective interactions. However, a direct comparison of these results is complicated by the fact that different sets of unperturbed one-particle–one-hole energies have been used. An extensive calculation with an effective interaction with adjustable parameters has been performed by Gillet and coworkers [30]). These calculations predict a strong isospin mixing, caused by the assumed differences (0.5 MeV) between the unperturbed proton-proton-hole and neutron-neutron-hole energies. These large differences are in disagreement with the experimental values of the $1f_{\frac{7}{2}}1d_{\frac{3}{2}}^{-1}$ and $2p_{\frac{3}{2}}1d_{\frac{3}{2}}^{-1}$ energy spacings as determined from the binding energies of states in neighbouring nuclei (assumed to be pure *j-j* coupling states). A recent study of the ^{39}K(^{3}He, d)^{40}Ca reaction [29]) does not give any indication for such strong isospin

† We are grateful to Professor E. Werner for drawing our attention to this problem.
†† Ref. [25]), p. 321.

mixing in the lower excited states, in particular such an indication is lacking for the lowest $J^{\pi} = 4^{-}$ $T = 0$ level in ^{40}Ca.

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