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SHELL-MODEL CALCULATIONS ON $A = 38$ NUCLEI

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Abstract: The shell model is used to investigate properties of levels in the $A = 38$ nuclei.

An inert ^{28}Si core with a residual two-particle interaction between the outer nucleons is assumed. The negative parity states are described by a closed $2s_{\frac{1}{2}}$ shell, five nucleons in the $1d_{\frac{3}{2}}$ shell and one nucleon in the $1f_{\frac{7}{2}}$ or $2p_{\frac{3}{2}}$ shell. For the positive parity states, all configurations in the $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ shell are taken into account. The effective two-particle interaction is chosen to be a modified surface delta interaction. The four parameters of this effective interaction are determined in a least-squares fit to the excitation energies of 15 nuclear levels. The rms deviation of the calculated energies from the experimental energies is 0.28 MeV. From the values of these parameters, the energies and wave functions of about 50 levels are derived.

The wave functions are used to determine γ-ray branching ratios, mean lives, *ft* values in allowed β-decay and spectroscopic factors for the ^{37}Cl(d, p)^{38}Cl reaction.

The computed branching ratios, mean lives and *ft* values agree generally within a factor of 2.5 with the experimental values. Good agreement with experiment is obtained for the spectroscopic factors.

1. Introduction

For the $A = 38$ nuclei only a few shell-model calculations have been performed on levels of odd parity. Amongst these is the well-known example [1,2]) of the $T = 2$ quadruplet in ^{38}Cl.

In a more recent calculation [3]), Erné has interpreted the negative parity states in ^{33}S$-^{41}$Ca as configurations with an inert ^{32}S core, one particle in the $1f_{\frac{7}{2}}$ shell and the remaining nucleons in the $1d_{\frac{3}{2}}$ shell. The residual two-particle interaction of the extra-core nucleons was described with 14 parameters, which were obtained from a least-squares fit to 60 nuclear levels.

The recent experimental information [4]) about transition probabilities for both $T = 2$ and $T = 1$ levels in ^{38}Ar stimulated a more detailed description.

In the present paper, theoretical excitation energies, wave functions, γ-ray branching ratios, mean lives, *ft* values and stripping spectroscopic factors are presented for negative-parity states in $A = 38$ nuclei.

The model used has an inert ^{28}Si core. Negative parity states are described by the configurations $(2s_{\frac{1}{2}})^4(1d_{\frac{3}{2}})^5\rho$ where ρ denotes a particle in the $1f_{\frac{7}{2}}$ or $2p_{\frac{3}{2}}$ shell. The positive parity states are described by $(2s_{\frac{1}{2}})^n(1d_{\frac{3}{2}})^m$ configurations, with $n+m = 10$.

To avoid a large number of parameters (there are 24 different two-body matrix elements) the modified surface delta interaction (MSDI) is used. This effective interaction contains only four parameters and is described in detail in ref. [5]).

2. Excitation energies and wave functions

A detailed theoretical treatment of the decomposition of the interaction matrix elements in terms of two-particle matrix elements has been given in refs. [6-8]).

In our model the wave functions for the negative-parity states are written as

$$\psi_{JT} = \sum_i (A_i^{\mathrm{f}} \phi_i^{\mathrm{f}} + A_i^{\mathrm{p}} \phi_i^{\mathrm{p}}), \tag{1}$$

where $\phi_i^{\mathrm{f}} = |1\mathrm{d}_{\frac{3}{2}}^5(J_1 T_1)1\mathrm{f}_{\frac{7}{2}}\rangle_{JT}$ and $\phi_i^{\mathrm{p}} = |1\mathrm{d}_{\frac{3}{2}}^5(J_2 T_2)2\mathrm{p}_{\frac{3}{2}}\rangle_{JT}$. If the A^{p} are taken to be zero, we are left with the model that was used by Erné in ref. [3]).

The necessary two-body matrix elements $\langle \mathrm{d}^2|V_{12}|\mathrm{d}^2\rangle$, $\langle \mathrm{df}|V_{12}|\mathrm{df}\rangle$, $\langle \mathrm{df}|V_{12}|\mathrm{dp}\rangle$ and $\langle \mathrm{dp}|V_{12}|\mathrm{dp}\rangle$ are calculated with the MSDI [see ref. [5])]. In this calculation, the coupling order $\boldsymbol{j} = \boldsymbol{l}+\boldsymbol{s}$ is used; this adds a factor $(-1)^{j_a+j_b+j_c+j_d}$ to eqn. (2) of ref. [5]).

The coefficients of fractional parentage are taken from ref. [8]). In order to have more levels for the fitting procedure than the known negative-parity states only, some positive-parity levels in $A = 38$, of which the energies can be well reproduced with $2\mathrm{s}_{\frac{1}{2}}1\mathrm{d}_{\frac{3}{2}}$ configurations, are also used.

In order to fit the four MSDI parameters A_0, A_1, B_0 and B_1 to the $A = 38$ levels, one has to know the values of the single-particle binding energies $E_{\mathrm{b}}(2\mathrm{s}_{\frac{1}{2}})$, $E_{\mathrm{b}}(1\mathrm{d}_{\frac{3}{2}})$, $E_{\mathrm{b}}(1\mathrm{f}_{\frac{7}{2}})$ and $E_{\mathrm{b}}(2\mathrm{p}_{\frac{3}{2}})$ with respect to the ^{28}Si core.

The binding energies of a $2\mathrm{s}_{\frac{1}{2}}$ and $1\mathrm{d}_{\frac{3}{2}}$ particle to the ^{28}Si core are taken to be $E_{\mathrm{b}}(2\mathrm{s}_{\frac{1}{2}}) = -9.29$ MeV and $E_{\mathrm{b}}(1\mathrm{d}_{\frac{3}{2}}) = -7.16$ MeV. These values were obtained from a separate MSDI fit to 26 even-parity states in $A = 35$–40 nuclei.

The $J^\pi = \frac{7}{2}^-$ and $\frac{3}{2}^-$ levels in ^{41}Ca and ^{41}Sc give information about the $2\mathrm{p}_{\frac{3}{2}}$–$1\mathrm{f}_{\frac{7}{2}}$ splitting. With the (d, p) stripping factors [11]) as weights, the centre of gravity of the two $\frac{3}{2}^-$ levels at 1.94 and 2.46 MeV in ^{41}Ca is at 2.06 MeV. In connection with the ^{41}Ca ground state, this yields 2.06 MeV for the p–f splitting. The same computation for the $\frac{3}{2}^-$ levels at 1.73 and 2.41 MeV in ^{41}Sc yields for the splitting 1.79 MeV. Therefore the difference $E_{\mathrm{b}}(2\mathrm{p}_{\frac{3}{2}}) - E_{\mathrm{b}}(1\mathrm{f}_{\frac{7}{2}})$ is kept fixed at the average 1.93 MeV. The values of the four MSDI parameters and one single-particle binding energy then can be determined from a least-squares fit to the 15 levels given between square brackets in table 1. No effort has been made to remove the influence of spurious states.

2.1. EXCITATION ENERGIES

The experimental and calculated level schemes for ^{38}Ar and ^{38}Cl are displayed in fig. 1.

The experimental and calculated energies for levels of ^{38}Ar, ^{38}Cl and ^{38}K are presented in table 1.

In this table are listed (i) the experimental binding energies E^{exp} relative to the binding energy of the ^{28}Si core, with the Coulomb energy [8]) of all particles outside the core subtracted, (ii) the computed binding energies E^{th}, (iii) the experimental

TABLE 1

Experimental and calculated energies (in MeV)

Nucleus isospin	J^{π} [a]	$E^{\exp}$ [b]	E^{th} [c]	$E_x^{\exp}$ [a]	E_x^{th} [d]	Erné [3] E_x^{th} [d]
^{38}Ar	0^+	[−115.74]	−115.49	0	0.25, 6.78	0
$T = 1$	1^+		−110.50		5.24	
	2^+	[−113.57]	−113.91	2.17	1.83	1.96
	2^+	[−111.80]		3.94	3.77	
	0^-		−108.71		7.03	6.97
	1^-		−109.96		5.78, 6.99	6.27, 7.21
	2^-		−110.71	(4.57)	5.03, 6.44,	4.78, 5.67, 6.79
	2^-				7.13, 7.32	
	3^-	[−111.93]	−111.42	3.81	4.32	3.68
	3^-	[−110.86]		4.88	5.08	4.89
	3^-			(5.51)	6.28, 6.83,	6.65, 7.42
					7.18, 7.48	
	4^-	[−111.26]	−111.42	4.48	4.32	4.12
	4^-	[−109.53]		6.21	5.88, 6.17	6.08
	4^-	[−109.14]		6.60	6.78	6.53, 7.35
	5^-	[−111.15]	−111.60	4.59	4.14	4.59
	5^-	[−110.08]		5.66	5.58	5.51
	5^-			(6.67)	6.14, 6.69	6.78
	6^-		−109.83		5.91, 6.19	7.00, 7.40
	7^-		−109.86		5.88	7.17
^{38}Cl	0^-		−102.84		2.20	
$T = 2$	1^-		−102.35		2.69	
	2^-	[−105.04]	−104.66	0	0.38, 3.36	0
	3^-	[−104.28]	−104.36	(0.76)	0.68, 2.85	0.67
	4^-	[−103.73]	−103.67	(1.31)	1.37	1.21
	5^-	[−104.37]	−104.61	0.67	0.43	0.40
^{38}K	3^+	[−115.93]	−115.68	0	0.25	0
$T = 0$	1^+		−116.34	(0.45)	−0.41	0.40
	1^+	−114.23		1.70	2.58	
	2^+		−112.72		3.21	
	0^-					
	1^-					3.69
	2^-		−113.16		2.77, 3.83	2.80, 3.70
	3^-		−113.83	$\geqq 2.61$	2.10	2.36, 3.92
	4^-		−113.99		1.94, 2.45, 3.47	1.90
	5^-		−113.54		2.39, 2.96	2.75
	6^-		−113.77		2.16, 3.80	3.90
	7^-		−112.31		3.62	

[a]) Ref. [11]).

[b]) The binding energies with respect to ^{28}Si are taken from ref. [8]).

[c]) Only the binding energy of the lowest level with given J, T value is listed.

[d]) For ^{38}Ar theoretical excitation energies larger than 7.5 MeV are omitted, while for ^{38}K only levels below 4.0 MeV are listed. For ^{38}Cl all theoretical excitation energies are given.

TABLE

Calculated energies and configuration amplitudes

38Cl($J = 0, T = 2$)		33p								
+2.20	-102.84	+1000								
38Cl($J = 1, T = 2$)		33p								
+2.69	-102.35	+1000								
38Cl($J = 3, T = 2$)		33f	33p							
+2.85	-102.19	-396	-919							
+0.68	-104.36	+919	-396							
38Cl($J = 4, T = 2$)		33f								
+1.37	-103.67	+1000								
38Ar($J = 0, T = 1$)		71f	31p	33p						
+7.03	-108.71	+824	-542	+162						
38Ar($J = 2, T = 1$)		31f	51f	71f	33f	11p	31p	51p	71p	33p
+7.32	-108.42	-189	-619	+324	-337	-478	-170	+312	+78	-31
+7.13	-108.61	-267	+348	-9	-569	-289	+412	-322	-28	-355
+6.44	-109.30	-692	-218	-552	+355	-163	+49	-106	-32	+41
+5.03	-110.71	+463	-507	-314	-8	-6	+648	-79	+60	+1
38Ar($J = 4, T = 1$)		11f	31f	51f	71f	33f	51p	71p		
+6.78	-108.96	+309	-89	+381	+81	-642	+16	-577		
+6.17	-109.57	+680	-40	-680	+196	-133	+92	+100		
+5.88	-109.86	+467	+86	+484	+415	+585	+157	-32		
+4.32	-111.42	-143	+928	-91	+220	-155	+182	-72		
38Ar($J = 6, T = 1$)		51f	71f							
+6.19	-109.55	+621	-784							
+5.91	-109.83	+784	+621							
38K($J = 2, T = 0$)		31f	51f	71f	11p	31p	51p	71p		
+3.83	-112.10	-473	-578	-343	-359	+430	-85	+54		
+2.77	-113.16	+802	-238	-174	+104	+508	-1	+34		
38K($J = 4, T = 0$)		11f	31f	51f	71f	51p	71p			
+3.74	-112.19	+376	-205	+414	-493	+352	-527			
+2.45	-113.48	-748	+148	+566	-215	-212	-87			
+1.94	-113.99	+215	+950	+114	+65	+170	-73			
38K($J = 6, T = 0$)		51f	71f							
+3.80	-112.13	+752	+659							
+2.16	-113.77	+659	-752							
38Ar($J = 0, T = 1$)		s4d6	s2d8							
+6.78	-108.96	-257	-967							
+0.25	-115.49	+967	-257							
38Ar($J = 1, T = 1$)		s3d7								
+5.24	-110.50	+100								
38K($J = 1, T = 0$)		s4d6	s3d7	s2d8						
+2.58	-113.35	+450	-889	+84						
-0.41	-116.34	+887	+456	+72						
38K($J = 2, T = 0$)		s3d7								
+3.21	-112.72	+100								

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(for the meaning of the symbols used see sect. 2).

38Cl($J = 2, T = 2$)		33f	33p
+3.36	−101.68	−149	−989
+0.38	−104.66	+989	−149

38Cl($J = 5, T = 2$)		33f
+0.43	−104.61	+1000

38Ar($J = 1, T = 1$)		51f	71f	11p	31p	51p	33p
+6.99	−108.75	−732	+312	−362	−399	+265	+ 79
+5.78	−109.96	+613	+538	− 62	−547	+171	− 60

38Ar($J = 3, T = 1$)		11f	31f	51f	71f	33f	31p	51p	71p	33p
+7.48	−108.26	+ 16	−317	−506	+500	− 2	+175	−117	+261	−530
+7.18	−108.56	+159	−166	+212	+383	+234	+381	+468	+356	+459
+6.83	−108.91	+665	−101	−276	+ 34	−584	−168	− 25	− 25	315
+6.28	−109.46	−305	+130	+431	+634	−469	−271	− 15	− 88	− 28
+5.08	−110.66	−586	−259	−225	−250	−490	+384	+ 2	+ 84	+286
+4.32	−111.42	− 8	−865	+299	−101	+115	−349	−121	− 45	+ 25

38Ar($J = 5, T = 1$)		31f	51f	71f	33f	71p
+6.69	−109.05	−135	+640	+594	−418	+210
+6.14	−109.60	− 20	+667	−724	− 78	−155
+5.58	−110.16	−592	+226	+122	+758	+ 99
+4.14	−111.60	+753	+306	+253	+483	−205

38Ar($J = 7, T = 1$)		71f
+5.88	−109.86	+1000

38K($J = 3, T = 0$)		11f	31f	51f	71f	31p	51p	71p
+2.10	−113.83	+153	+928	−4	−17	+315	+121	+ 24

38K($J = 5, T = 0$)		31f	51f	71f	71p
+2.96	−112.97	+918	+121	+365	−100
+2.39	−113.54	+163	+606	−702	−335

38K($J = 7, T = 0$)		71f
+3.62	−112.31	+1000

38Ar($J = 2, T = 1$)		s4d6	s3d7
+3.77	−111.97	+595	−804
+1.83	−113.91	+804	−595

38K($J = 3, T = 0$)		s4d6
+0.25	−115.68	+1000

excitation energies E_x^{exp} and (iv) the excitation energies E_x^{th}, which are given relatively to the experimental ground-state energy. The rms deviation of the 15 theoretical binding energies with respect to the fitted binding energies amounts to 0.28 MeV.

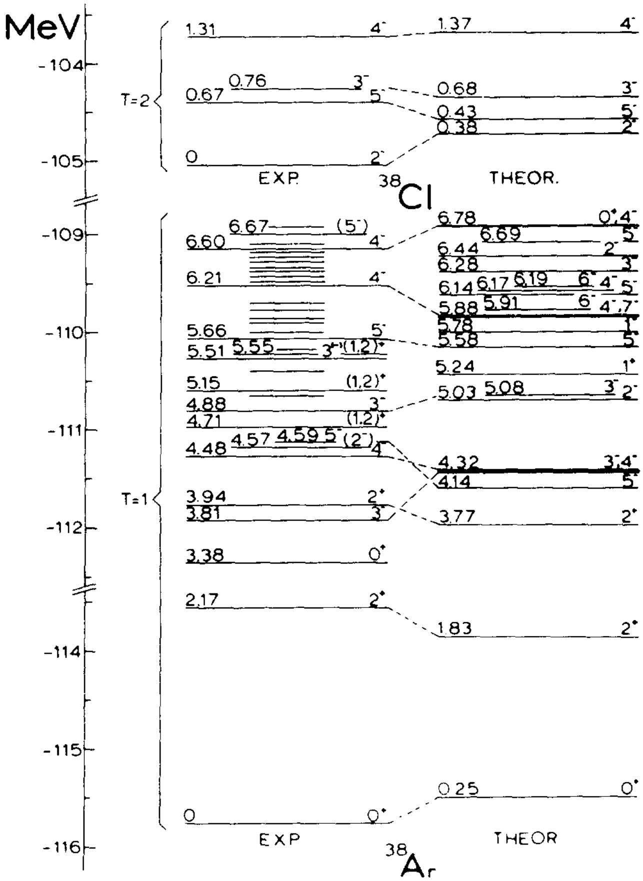

Fig. 1. Comparison of the experimental and theoretical level schemes for ^{38}Ar and ^{38}Cl. The experimental information is taken from refs. [4,11,26]). The calculated excitation energies are taken from table 1. For ^{38}Ar all experimental and theoretical levels below 6.8 MeV are given. The binding energies are given with respect to the ^{28}Si core.

2.2. WAVE FUNCTIONS

The coefficients A^{f} and A^{p}, defined in eq. (1), which denote the amplitudes of the pure configurations for the negative-parity states in ^{38}Ar, ^{38}Cl and ^{38}K are given in table 2.

Each matrix in the table is labelled by a heading indicating the mass number, the chemical symbol and the pair of (J, T) values. For negative parity states the pure configurations are given as, e.g.

$$2J_a 2T_a \rho \equiv (\text{d}_{\frac{3}{2}})^5_{J_a T_a} \rho,$$

where ρ stands for $1f_{\frac{7}{2}}$ or $2p_{\frac{3}{2}}$. Similarly for positive-parity states the pure configurations $(2s_{\frac{1}{2}})^n(1d_{\frac{3}{2}})^m$ are denoted by *sndm*. Amplitudes of the various pure wave functions (indicated at the top of each column) constituting the mixed configuration of a particular state are given in tenths of a percent. The two columns of numbers that label the rows of the amplitude matrices represent the excitation energies (first column) and the computed binding energies (second column). The wave functions of states in ^{38}Ar with theoretical excitation energies larger than 7.5 MeV are omitted, while for ^{38}K only levels below 4.0 MeV are listed. The largest matrices have order 9.

2.3. THE MSDI PARAMETERS

The values for the parameters obtained in the least-squares fit as described in sect. 2 are

$$A_0 = +0.69 \text{ MeV}, A_1 = +1.15 \text{ MeV}, B_0 = -1.44 \text{ MeV}, B_1 = +0.66 \text{ MeV},$$
$$E_b(1f_{\frac{7}{2}}) = -2.84 \text{ MeV; this results in } E_b(2p_{\frac{3}{2}}) = -0.91 \text{ MeV}.$$

With the four MSDI parameters the 12 two-body matrix elements, which were used by Erné [3]) as free parameters, can be computed. The results are shown in table 3.

TABLE 3

Comparison of some two-body matrix elements (in MeV)

			Computed from the 4 MSDI parameters	Erné [3]) Free parameters
$\langle 1d_{\frac{3}{2}}^2\|V\|1d_{\frac{3}{2}}^2\rangle$	$J = 0$	$T = 1$	−1.64	−1.71
	2	1	+0.20	+0.26
	1	0	−2.27	−2.11
	3	0	−2.27	−2.51
$\langle 1d_{\frac{3}{2}}1f_{\frac{7}{2}}\|V\|1d_{\frac{3}{2}}1f_{\frac{7}{2}}\rangle$	$J = 2$	$T = 0$	−3.81	−3.65
	3	0	−2.23	−1.85
	4	0	−1.96	−1.77
	5	0	−2.44	−2.90
	2	1	+0.66	+0.38
	3	1	+0.22	−0.08
	4	1	+0.66	+0.92
	5	1	−0.73	+0.43

There is good agreement between the two sets except for the matrix element $\langle 1d_{\frac{3}{2}}1f_{\frac{7}{2}}|V|1d_{\frac{3}{2}}1f_{\frac{7}{2}}\rangle_{51}$. However, the MSDI value of this matrix element is in good agreement with the value of −0.59 MeV calculated with the Hamada-Johnston potential [12]).

The surface delta interaction yields $\langle df|V|df\rangle_{21} = \langle df|V|df\rangle_{41} = B_1$, since a diagonal two-body matrix element $\langle ab|V|ab\rangle_{JT=1}$ is equal to B_1 if l_a+l_b+J is odd.

3. Transition probabilities

3.1. M1 TRANSITIONS

Experimentally, one observes [4]) strong M1 transitions in ^{38}Ar with $\Delta T = 1$ between states having the same J values. If initial and final wave function have the form $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{J_i 2}$ and $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{J_f 1}$, respectively, and the M1 operator is restricted to the $1f_{\frac{7}{2}}$ particle then the transition strength can be given by

$$\Gamma_\gamma(\mathrm{M1}) = C(2J_f+1)\begin{Bmatrix} J_i & J_f & 1 \\ \frac{7}{2} & \frac{7}{2} & \frac{3}{2} \end{Bmatrix}^2,$$

with $C = 15.04$ W.u.

Apart from C this formula has been given already by Erné [13]), although his formula contains an erroneous factor $(2J_i+1)$.

Erné has pointed out that the 6-j symbol is responsible for the enhancement of $J_i \rightarrow J_f = J_i$ transitions over $J_i \rightarrow J_f = J_i \pm 1$ transitions. It is worthwhile to investigate whether this simple explanation still holds when more complicated wave functions, especially for the final states, are used. Experimentally it is observed also that the strong $J \rightarrow J$ $\Delta T = 1$ decay may take place to several final states. With the model presented here a comparison can be made between such observed and calculated branching ratios.

In the present model the initial states $J^\pi_i = 4^-$ or 5^- with $T = 2$ can only be described by the pure configuration $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{J_i 2}$. From the calculations it follows that the strength of an M1 transition with $J^\pi_i = J^\pi_f = 4^-$ or 5^- is almost completely determined by the $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{J_f 1}$ component in the final wave function, even if this admixture has an intensity as low as 25 %.

This can be simply explained since a M1 transition from $f_{\frac{7}{2}}$ to $p_{\frac{3}{2}}$ orbitals is not allowed and transitions within the $(d_{\frac{3}{2}})^5$ configurations are weak, due to the small value of the reduced single-particle matrix element $\langle d_{\frac{3}{2}}||\mathrm{M1}||d_{\frac{3}{2}}\rangle$ compared to $\langle f_{\frac{7}{2}}||\mathrm{M1}||f_{\frac{7}{2}}\rangle$. The reduced matrix elements of the isovector parts of the M1 operator are approximately an order of magnitude larger, if one or both nucleons are in a $j = l+\frac{1}{2}$ orbit; see also ref. [24]).

3.2. E2 TRANSITIONS

For the calculation of E2 single-particle matrix elements in the surface delta model the expectation value of r^2 is taken to be R^2, where $R = r_0 A^{\frac{1}{3}}$ and $r_0 = 1.2$ fm. Effective charges are not taken into account.

4. Decay of the $T = 2$ states

The theoretical results for the ($\Delta T = 1$) M1 transitions from the $T = 2$ analogue states with $J^\pi = 5^-, 4^-$ and 3^- to the low-lying $T = 1$ negative-parity states in ^{38}Ar are given in fig. 2. The computed branching percentages are rounded off to one percent. The experimental spins and branching ratios are taken from ref. [4]). The Γ_γ values were derived from refs. [4,14]).

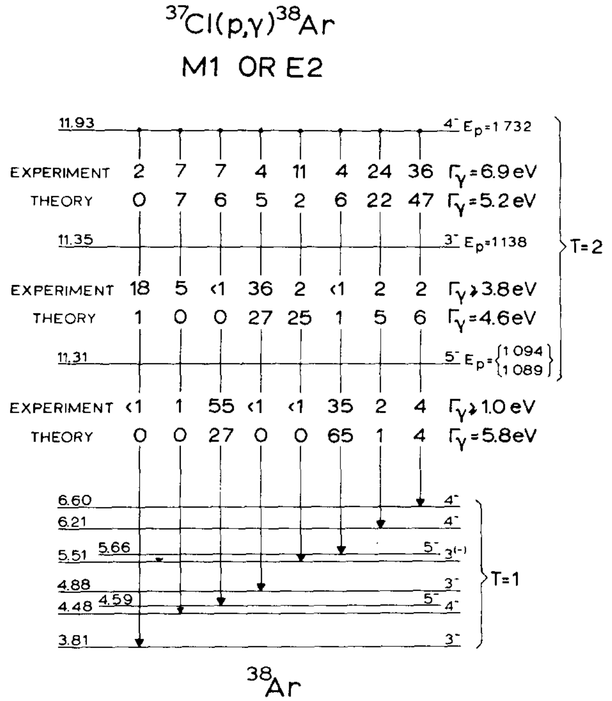

Fig. 2. Shell-model calculations of branching ratios and radiation widths for the γ-decay of analogue states in ^{38}Ar, as compared to experiment. The experimental Γ_γ value for the 5^- analogue represents the sum of the Γ_γ values for the two components into which this state is split. The branching indicated for the 5^- analogue is the weighted average of those for the $E_p = 1\ 089$ and $1\ 094$ keV resonances.

4.1. THE $J^\pi = 5^-$, $T = 2$ STATES AT $E_x = 11.31$ MeV

The experimental Γ_γ value for the 5^- analogue state represents the sum of the Γ_γ values for the two components into which this state is split. The branching indicated for the 5^- analogue state in fig. 2 is the weighted average of those for the $E_p = 1089$ and 1094 keV resonances in the ^{37}Cl(p, γ)^{38}Ar reaction.

In our model, the initial wave function consists of the component $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{52}$ only. The wave function of the 5.66 MeV level has 57 % intensity of the component $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{51}$ and the 4.59 MeV level has 23 % intensity. The M1 transitions to the 5.66 and 4.59 MeV levels are predominantly determined by this component.

The large fractions of the component $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{41}$ in the 6.60 and 6.21 MeV levels (41 % and 34 % intensity, respectively) do not give rise to strong M1 transitions, due to the effectiveness of the $J \rightarrow J$ rule. For the third $J^\pi = 5^-$ level (theoretically at approximately $E_x = 6.1$ MeV) a weak M1 transition is expected, due to the 0.6 % intensity of the $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{51}$ component. Moreover here the E_γ^3 rule is also effective.

4.2. THE $J^\pi = 4^-$ $T = 2$ STATE AT $E_x = 11.93$ MeV

The initial wave function only consists of the component $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{42}$. The component $(d^5_{\frac{3}{2}\frac{3}{2}}f_{\frac{7}{2}\frac{1}{2}})_{41}$ is present in the 4.48, 6.21 and 6.60 MeV levels with intensities of 2 %, 34 % and 41 %, respectively, which explains the weak M1 transition to the 4.48 MeV level compared to the strong M1 transitions to the 6.21 and 6.60 MeV levels. The M1 transitions to the $J^\pi = 5^-$ levels at 4.59 and 5.66 MeV and to the $J^\pi = 3^-$ level at 4.88 MeV are weakened by the $J \to J$ rule. The theoretical M1 strengths for the $\Delta T = 1$ transitions to the 6.21 and 6.60 MeV levels are 0.3 and 0.8 W.u., while the experimental strengths are 0.4 and 0.8 W.u., respectively.

4.3. THE $J^\pi = 3^-$ $T = 2$ STATE AT $E_x = 11.35$ MeV

The initial wave function contains the component $(d^5_{\frac{3}{2}\frac{3}{2}}p_{\frac{3}{2}\frac{1}{2}})_{32}$ with 16% intensity. If this admixture is ignored, the decay as given in fig. 2 is changed only slightly. The strong theoretical M1 transitions to the 4.88 and 5.51 MeV levels are due to the $(d^5_{\frac{3}{2}\frac{3}{2}}f_{\frac{7}{2}\frac{1}{2}})_{31}$ component in the wave functions of these levels.

The theoretical M1 strengths for the $\Delta T = 1$ transitions to the 3.81 and 4.88 MeV levels are 0.01 and 0.22 W.u., while the experimental strengths are $\geqq 0.08$ and $\geqq 0.25$ W.u., respectively.

4.4. THE LOWEST $J^\pi = 2^-$ $T = 2$ STATE

Not much pertinent experimental information is known about this state which probably is formed [4]) in the ^{37}Cl(p, γ)^{38}Ar reaction at a proton energy of $E_p = 427$ keV ($E_x = 10.66$ MeV).

The calculated wave function for this state possesses a $(d^5_{\frac{3}{2}\frac{3}{2}}p_{\frac{3}{2}\frac{1}{2}})_{22}$ admixture of only 2 % in intensity.

Since the penetrabilities for $l = 1$ and $l = 3$ capture at $E_p = 427$ keV differ by a factor 355, the almost pure $(d^5_{\frac{3}{2}\frac{3}{2}}f_{\frac{7}{2}\frac{1}{2}})_{22}$ character of this $J^\pi = 2^-$, $T = 2$ state explains the experimental weakness of the (p, γ) resonance.

The lowest three $J^\pi = 2^-$, $T = 1$ levels are theoretically expected at $E_x = 5.0$, 6.4 and 7.1 MeV, with $(d^5_{\frac{3}{2}\frac{3}{2}}f_{\frac{7}{2}\frac{1}{2}})_{21}$ components of 0.01, 13 and 32 % intensity, respectively. Therefore, the decay of the $J^\pi = 2^-$, $T = 2$ level is expected to proceed predominantly to $J^\pi = 2^-$, $T = 1$ levels near 6.4 and 7.1 MeV excitation energy.

5. Decay of $T = 1$ states

The mean lives and branching ratios are taken from ref. [4]). The data about the mean lives of the 2.17 MeV and 4.59 MeV levels are taken from refs. [15]) and [4,16]), respectively.

The results for the decay of the lowest two $J^\pi = 5^-$, $T = 1$ levels are given in fig. 3. The strong M1 decay of the 5.66 MeV level to the 4.59 MeV level (experimentally 0.40 W.u., theoretically 0.60 W.u.) is due to the constructive adding of the isovector contributions from the $(d^5_{\frac{3}{2}\frac{1}{2}}f_{\frac{7}{2}\frac{1}{2}})_{51}$ and $(d^5_{\frac{3}{2}\frac{3}{2}}f_{\frac{7}{2}\frac{1}{2}})_{51}$ components, which are present in the wave functions with large amplitudes (see table 2). The difference in intensity between the transitions 5.66 → 4.48 MeV ($5^- \to 4^-$, experimentally 0.02 W.u., theoretically 0.02 W.u.) and 4.59 → 4.48 MeV ($5^- \to 4^-$, experimentally >0.22 W.u., theoretically 0.19 W.u.) is due to the different signs of the $(d^5_{\frac{3}{2}\frac{1}{2}}f_{\frac{7}{2}\frac{1}{2}})_{51}$ components

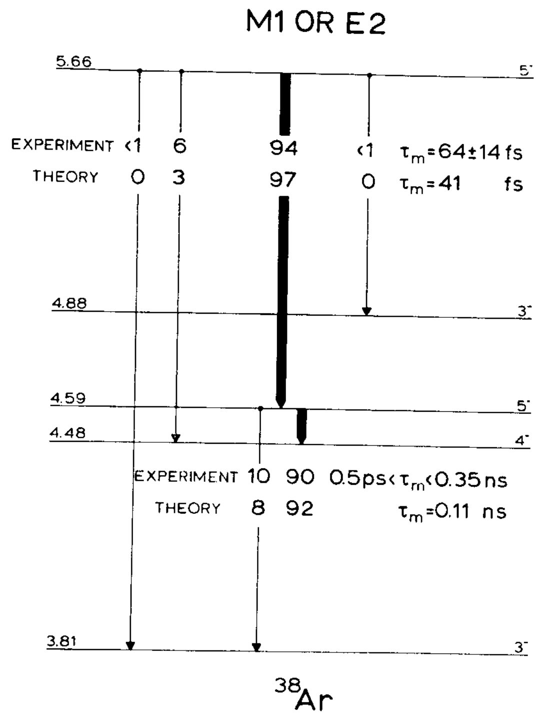

Fig. 3. Shell-model calculations of branching ratios and mean lives for the γ-decay of 5^- bound states in ^{38}Ar.

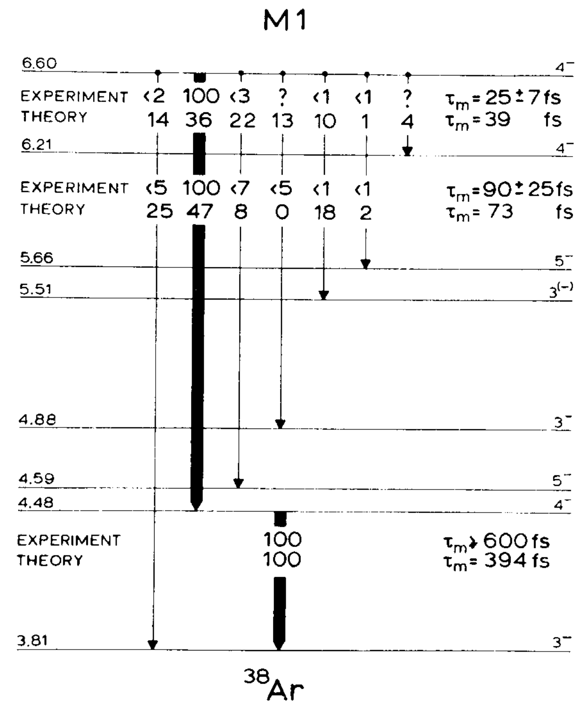

Fig. 4. Shell-model calculations of branching ratios and mean lives for the γ-decay of 4^- bound states in ^{38}Ar.

with respect to the $(d_{\frac{3}{2}}^{5} f_{\frac{7}{2}})_{51}$ components in the wave functions of the 5.66 and 4.59 MeV levels.

The results for the decay of the lowest three $J^\pi = 4^-$, $T = 1$ levels are given in fig. 4. Experimentally the decay of the $J^\pi = 4^-$ level at 4.48 MeV only proceeds to the $J^\pi = 3^-$ level at 3.81 MeV excitation energy. Other decay modes to lower levels would involve M2 or M4 transitions.

The results for the decay of the second and third $J^\pi = 3^-$, $T = 1$ levels are given in fig. 5. The agreement between experiment and calculation is rather poor for these levels.

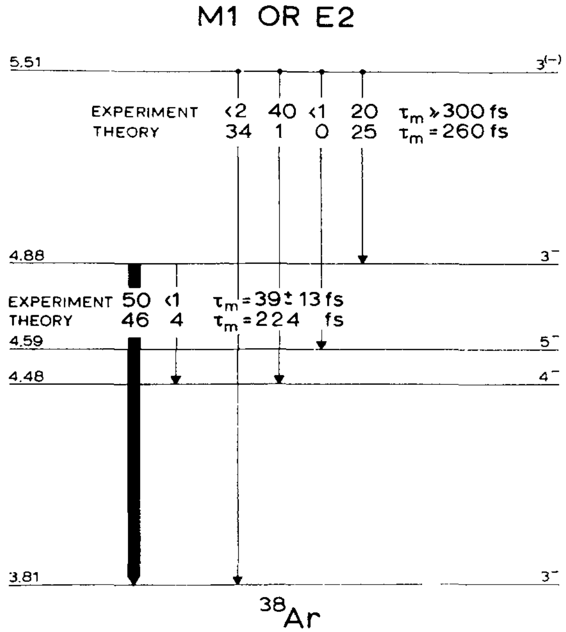

Fig. 5. Shell-model calculations of branching ratios and mean lives for the γ-decay of 3^- bound states in ^{38}Ar.

5.1. MIXING RATIOS

Theoretical E2/M1 mixing ratios are given in table 4.

The calculated mixing ratios are all small except for the $11.35 \rightarrow 3.81$ MeV and $11.93 \rightarrow 4.59$ MeV $\Delta T = 1$ transitions, in agreement with experiment. The mixing ratio for the $6.21 \rightarrow 4.48$ MeV $\Delta T = 0$ transition shows a large discrepancy with the experimental value.

5.2. THE E2 TRANSITIONS FROM THE FIRST AND SECOND $J^\pi = 2^+$ LEVELS

The levels at 2.17 and 3.94 MeV excitation energy with $J^\pi = 2^+$ experimentally decay only to the $J^\pi = 0^+$ ^{38}Ar ground state. The mean lives 700 ± 110 fs for the 2.17 MeV level [15]) and 105 ± 16 fs for the 3.94 MeV level [4]) correspond to E2 transitions with strengths of 3.2 and 1.1 W.u., respectively. The calculated values are 2.0 and 0.12 W.u., respectively. The experimental value for the M1 transition from the

3.94 MeV level to the 2.17 MeV level is smaller than 0.002 W.u. With the wave functions from table 2, however, one obtains 0.35 W.u. The experimental ratio $B(\mathrm{E2}, 3.94 \rightarrow 0)/B(\mathrm{E2}, 2.17 \rightarrow 0)$ has the large value 0.34 which also indicates [23]) that other configurations strongly contribute.

Admixtures of $1d_{\frac{5}{2}}$ hole states are expected to be less than 10 % in intensity [10]) in the low-lying states. There is evidence, however, for the presence of $(1f_{\frac{7}{2}})^2$ configurations: (i) the $J = 0^+$ level at 3.38 MeV excitation energy can be interpreted [3]) as a pure $(1d_{\frac{3}{2}})^4(1f_{\frac{7}{2}})^2$ configuration, (ii) a calculation [22]) of positive-parity levels in ^{38}Ar with the MSDI in the complete $1d_{\frac{5}{2}}$, $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ shell without truncation shows

TABLE 4

Comparison of E2/M1 amplitude mixing ratios in ^{38}Ar

Transition (E_x in MeV)	$J_i^\pi \rightarrow J_f^\pi$		Mixing ratios experimental [a])	theoretical
11.93 → 6.60	$4^- \rightarrow 4^-$	$\Delta T = 1$	-0.05 ± 0.08	-0.04
11.93 → 6.21	$4^- \rightarrow 4^-$	,,	$+0.02 \pm 0.08$	-0.03
11.93 → 5.51	$4^- \rightarrow 3^-$	,,	$+0.03 \pm 0.09$	$+0.15$
11.93 → 4.59	$4^- \rightarrow 5^-$	,,	$+0.20 \pm 0.10$	$+0.27$
11.93 → 4.48	$4^- \rightarrow 4^-$	,,	$+0.10 \pm 0.10$	$+0.03$
11.35 → 4.88	$3^- \rightarrow 3^-$	,,	$+0.16 \pm 0.10$	-0.06
11.35 → 3.81	$3^- \rightarrow 3^-$	,,	$+0.20 \pm 0.10$	$+0.58$
11.31 → 5.66	$5^- \rightarrow 5^-$	,,	$+0.13 \pm 0.06$	-0.08
11.31 → 4.59	$5^- \rightarrow 5^-$	,,	$+0.03 \pm 0.06$	$+0.03$
6.60 → 4.48	$4^- \rightarrow 4^-$	$\Delta T = 0$	$+0.05 \pm 0.08$	$+0.04$
6.21 → 4.48	$4^- \rightarrow 4^-$	,,	$+0.32 \pm 0.10$	$+0.04$
4.88 → 3.81	$3^- \rightarrow 3^-$	,,	-0.03 ± 0.07	$+0.03$
5.66 → 4.59	$5^- \rightarrow 5^-$	,,	$+0.10 \pm 0.09$	-0.03
4.59 → 4.48	$5^- \rightarrow 4^-$	,,	$+0.02 \pm 0.03$	$+0.00$
4.48 → 3.81	$4^- \rightarrow 3^-$	,,	-0.01 ± 0.02	$+0.01$

[a]) Ref. [4]).

only three levels with $J^\pi = 1^+$ or 2^+ below 9.5 MeV. Experimentally, there are at least [11]) five levels with $J^\pi = 1^+$ or 2^+ below 5.6 MeV; (iii) from the ^{38}Ar(d, p)^{39}Ar reaction experimental indications [17]) have been found for $(1f_{\frac{7}{2}})^2$ admixtures in the ^{38}Ar ground state with an intensity of approximately 10 %.

6. Allowed beta decay

The wave functions obtained can also be used to calculate some $\log ft$ values for the β^+ decay of ^{38}Ca and ^{38}K and for the β^- decay of ^{38}Cl. The theoretical $\log ft$ values are given in table 5, where they are compared with the experimental values [refs. [11,20])].

6.1. POSITON DECAY OF THE ^{38}K GROUND STATE

With the configuration space limited to the $2s_{\frac{1}{2}}$ and $1d_{\frac{3}{2}}$ shells the ^{38}K ground state has only the configuration d^6_{30}. The computed $\log ft$ values are in poor agreement with experiment. However, these log ft values are very sensitive to an admixture of $(1d_{\frac{5}{2}})^{11}(1d_{\frac{3}{2}})^7$ in the ^{38}K ground state. Already a 10 % intensity of this component can bring the theoretical values in agreement with the experimental ones.

6.2. THE β^+ DECAYS ^{38}Ca(g.s.) → ^{38}K(0.13 MeV) AND ^{38}K(0.13 MeV) → ^{38}Ar(g.s.)

In a shell-model calculation the three states involved only differ in M_T value. For these superallowed Fermi β^+ decays, the square of the Fermi matrix element has the value $|M_F|^2 = (T+M_T)(T-M_T+1) = 2$, independent of the configuration mixing of this state.

The ft values of $0^+ \to 0^+$ transitions in even-A nuclei with $T = 1$ can be used to investigate the influence of isospin mixing in the ground states. The calculations by Bohr *et al.* [18]) show a decrease of $|M_F|^2$ due to isospin impurity, but the effect is of the order of $\frac{1}{2}$ %, so that the experimental ft value has to be known with very high accuracy. From the β-ray end-point energy of 5038 ± 12 keV and the half life [11]) of 946 ± 5 ms an error of 1.3 % in the experimental ft value follows for the ^{38}K(0.13) decay. The experimental ft value for the ^{38}Ca(g.s.) decay [20]) has an error of 7 %. These errors are too large to allow a test of the isospin impurity of these states.

TABLE 5

Theoretical and experimental log ft values for allowed beta decay

Initial state				Final state				log ft	
nucleus,	E_x(MeV),	spin,	isospin	nucleus,	E_x(MeV),	spin,	isospin	exp.	theor.
^{38}Ca	0	0^+	1	^{38}K	0.13	0^+	1	3.49	3.49
^{38}Ca	0	0^+	1	^{38}K	0.45	(1^+)	0	>4.77	3.85
^{38}Ca	0	0^+	1	^{38}K	1.70	1^+	0	3.41	4.44
^{38}K	0	3^+	0	^{38}Ar	2.17	2^+	1	4.98	4.46
^{38}K	0	3^+	0	^{38}Ar	3.94	2^+	1	5.74	4.72
^{38}K	0.13	0^+	1	^{38}Ar	0	0^+	1	3.49	3.49
^{38}Cl	0	2^-	2	^{38}Ar	3.81	3^-	1	4.91	3.61

6.3. THE β^- DECAY ^{38}Cl(g.s.) → ^{38}Ar(3.81 MeV)

The experimental log ft value [19]) corresponds with $|M_{GT}|^2 = 0.055$, where M_{GT} is the Gamov-Teller matrix element. The wave function of the ^{38}Cl ground state can be written as $\psi(\text{gs}) = F(d^5_{\frac{3}{2}\frac{3}{2}}f_{\frac{7}{2}\frac{1}{2}})_{22} + P(d^5_{\frac{3}{2}\frac{3}{2}}p_{\frac{3}{2}\frac{1}{2}})_{22}$, where F and P denote amplitudes. With the values $F = +0.989$ and $P = -0.149$ taken from table 2 the resulting $|M_{GT}|^2$ is equal to 0.109. For $F = 1$, the result is $|M_{GT}|^2 = 0.122$ and for $F = 0.84$, $P = -0.55$ the result would be in agreement with experiment. Such a large admixture of $2p_{\frac{3}{2}}$, however, would be in contradiction with the spectroscopic factor for this state in the ^{37}Cl(d, p)^{38}Cl reaction (see sect. 7).

7. Spectroscopic factors for the ^{37}Cl(d, p)^{38}Cl reaction

Experimentally, this reaction has been studied by Rapaport and Buechner [21]). The experimental results for the lowest four levels are given in table 6. In the model of the preceding sections, the wave functions for the $T = 2$, $J^\pi = 2^-, 3^-, 4^-$, and

5^- levels are written as

$$\psi_{J^-,2} = F(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{J^-,2} + P(d^5_{\frac{3}{2}\frac{3}{2}} p_{\frac{3}{2}\frac{1}{2}})_{J^-,2}, \tag{3}$$

where F and P denote amplitudes.

Denoting with D the amplitude of the configuration $d^5_{\frac{3}{2}\frac{3}{2}}$ in the wave function for the $J^\pi = \frac{3}{2}^+$, $T = \frac{3}{2}$ ground state of ^{37}Cl, one obtains from the $2s_{\frac{1}{2}} - 1d_{\frac{3}{2}}$ shell-model calculations of refs. [8,9]), a value of $D^2 = 0.93$, while a recent calculation [22]) including $1d_{\frac{5}{2}}$ shell configurations yields $D^2 = 0.87$.

TABLE 6

Spectroscopic factors for the ^{37}Cl(d, p)^{38}Cl reaction

$E_x(^{38}\text{Cl})$ (MeV)	J^π	Experimental		Theoretical for $D^2 = 0.90$	
		$S(l=1)$	$S(l=3)$	$S(l=1)$	$S(l=3)$
0	2^-		0.58	0.02	0.88
0.67	5^-		0.78	0	0.90
0.76	3^-	0.09	0.59	0.14	0.76
1.31	4^-		0.70	0	0.90

Evaluation of the theoretical S factors gives the simple results,

$$S(l=3) = D^2F^2, \tag{4}$$

$$S(l=1) = D^2P^2. \tag{5}$$

For the F and P values from table 2 and for $D^2 = 0.90$ the results of eqs. (4) and (5) are given in table 6.

The theoretical $2p_{\frac{3}{2}}$ admixture in the $J^\pi = 3^-$ $T = 2$ state is about 50 % larger in intensity than the experimental value. The $2p_{\frac{3}{2}}$ admixture in the $J^\pi = 2^-$ $T = 2$ state is very small which is in agreement with experiment.

8. Discussion

The wave functions obtained with the MSDI in the model of the previous sections reproduce the main features of the electromagnetic decay, in particular for the $\Delta T = 1$ transitions. The experimentally observed $J \to J$ rule for these transitions is due to the presence of the configuration $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{J_f 1}$, which has intensities ranging from 20 to 60 % in the wave functions of the 4.59, 4.88, 5.66, 6.21 and 6.60 MeV levels.

These intensities, combined with the fact that the single-particle matrix element of the isovector part of the M1 operator for an $f_{\frac{7}{2}}$ orbit is an order of magnitude larger than for a $d_{\frac{3}{2}}$ orbit, cause the configuration $(d^5_{\frac{3}{2}\frac{3}{2}} f_{\frac{7}{2}\frac{1}{2}})_{J_f 1}$ to dominate the M1 transition.

Good agreement with the experimental data is obtained for the branching ratios from the 4.59 and 5.66 MeV levels with $J^\pi = 5^-$. The calculated mean life for the 5.66 MeV level is also in agreement with experiment; therefore it would be interesting to determine experimentally the lacking mean life of the 4.59 MeV level.

The poor agreement with experiment for the $\Delta T = 0$ transitions to or from $J^\pi = 3^-$ levels is probably due to the limited configuration space used. The wave function of the lowest $J^\pi = 3^-$ state in ^{40}Ca obtained in the calculation of ref. [25]) contains many components of relatively small amplitude, which indicates the necessity to take into account a large configuration space. The spectroscopic factor for the 2.17 MeV level in the ^{39}K(n, d)^{38}Ar reaction seems to indicate [26]) that the admixture of the component $(s^3d^7)_{21}$ in this wave function has to be an order of magnitude smaller than given in table 2. Pure $(s^4d^6)_{21}$ and $(s^3d^7)_{21}$ wave functions for the 2.17 and 3.94 MeV levels, respectively, would solve the large discrepancy between experiment and theory for the $3.94 \rightarrow 2.17$ MeV M1 transition. Also the E2 transitions from these levels to the ground state would be in better agreement with experiment. However, the recently observed allowed beta branch [20]) to the 3.94 MeV level in the ^{38}K$(\beta^+)^{38}$Ar decay would, in this case, be forbidden theoretically while the experimental $\log ft$ has the value 5.74.

For a detailed description of the positive-parity states in $A = 38$ nuclei, $d_{\frac{5}{2}}^{-1}$ and $(f_{\frac{7}{2}})^2$ configurations have to be taken in account.

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