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APPLICATION OF THE TABAKIN INTERACTION TO THE sd SHELL NUCLEI WITH $36 \leqq A \leqq 39$

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Abstract: The Tabakin interaction is applied in a shell-model calculation of sd shell nuclei with mass numbers $36 \leqq A \leqq 39$. If a slightly modified interaction is used, satisfactory agreement with experiment can be obtained, otherwise the agreement for ^{38}K is poor. Nuclear moments, electromagnetic transition rates and spectroscopic factors are calculated and compared to the predictions of the modified surface delta interaction. In general, a reasonable agreement with experiment is obtained from both interactions except for the $T = 0$ levels in ^{38}K calculated from the modified surface delta interaction.

1. Introduction

Numerous shell-model calculations have been performed in the lower end of the sd shell with both phenomenological [1–3]) and so-called realistic [4–8]) forces. The upper end of the sd shell has been explored [6]) less extensively in such calculations partly because of lack of experimental information. Recent experiments have yielded several interesting new data, therefore a detailed theoretical investigation now seems worthwhile.

One of the interesting features in the $A = 38$ nuclei is the occurrence of two low-lying $J^\pi = 1^+$ $T = 0$ states in ^{38}K, which probably can be described quite well in terms of shell-model states. In a recent study [9]) of the β^+ decay of ^{38}Ca, it was found that the ground state predominantly decays to the second $J^\pi = 1^+$ level with the rather small logft value of 3.41. In a previous shell-model calculation [10]) performed in the $(2s_{\frac{1}{2}}1d_{\frac{3}{2}})^{-2}$ space, the strongest decay was predicted to proceed instead to the lowest $J^\pi = 1^+$ level. Furthermore, it appears that the observed retardation of the β^+ decay of the $J^\pi = 3^+$ ground state of ^{38}K to the first excited $J^\pi = 2^+$ level is ^{38}Ar (log $ft = 5.0$) cannot satisfactorily be explained in terms of $(2s_{\frac{1}{2}}1d_{\frac{3}{2}})^{-2}$ configurations.

It seems unlikely that admixtures of $1f_{\frac{7}{2}}^2(2s1d)^{-4}$ components can completely account for these discrepancies. A more likely explanation lies in break-up of the $1d_{\frac{5}{2}}$ shell, even though the $1d_{\frac{3}{2}}^{-1} - 1d_{\frac{5}{2}}^{-1}$ splitting observed in ^{39}K is quite large (about 7 MeV).

In this paper, we present a shell-model calculation for $A = 36$–39 nuclei using the full sd-shell configuration space. The residual shell-model interaction is derived from the Tabakin [11]) interaction. The Tabakin interaction is a sum of separable potentials

which fit the S-, P- and D-wave phase shifts for free nucleon-nucleon scattering fairly well. Among the soft-core potentials designed for Hartree-Fock and shell-model calculations, it is one of the most successful potentials. The Tabakin interaction has been applied in many shell-model calculations [8,12,13]). In an sd shell-model study of ^{18}O and ^{18}F, good agreement with experiment was found [8]) when the Tabakin scattering interaction was converted to an effective shell-model interaction by applying second-order corrections to take into account effects from rather high-energy nucleon-nucleon correlations and effects from one-hole–one-particle core excitations. The ^{18}O and ^{18}F energy spectra obtained from this Tabakin interaction were also very close to those obtained from effective interactions based on the Hamada-Johnston potential [4,5]).

The soft-core Tabakin interaction can be converted quite easily to an interaction suitable for shell-model calculations, but the hard-core Hamada-Johnston potential requires the solution of the G-matrix equation.

In the course of the present investigation, the use of the Tabakin interaction led to some undesired features in the shell-model results. For the $A = 38$ nuclei the agreement with experiment for some $T = 0$ levels turned out to be rather unsatisfactory. Furthermore it was found that, if the single-hole energies in $A = 39$ nuclei were calculated from the single-particle energies as observed in ^{17}O, an unreasonably large $1d_{\frac{3}{2}}^{-1} - 1d_{\frac{5}{2}}^{-1}$ splitting was obtained (about 11.5 MeV). A similarly large spin-orbit splitting was previously reported in a Hartree-Fock calculation with the Tabakin interaction by Kerman *et al.* [14]). Perhaps the Tabakin interaction is faulty, and thus partly responsible for these deviations. If one compares the two-body 2s1d shell matrix elements of the Tabakin interaction with the corresponding G-matrix elements of realistic hard-core potentials [8,15]), one finds in general a remarkable similarity. However, a few $T = 0$ matrix elements differ appreciably. The most conspicuous differences are given by

$$\langle 1d_{\frac{3}{2}}1d_{\frac{3}{2}};\ 30|V|1d_{\frac{3}{2}}1d_{\frac{3}{2}};\ 30\rangle = -1.92 \text{ MeV}, -2.43 \text{ MeV},$$

$$\langle 1d_{\frac{3}{2}}1d_{\frac{5}{2}};\ 10|V|1d_{\frac{3}{2}}1d_{\frac{5}{2}};\ 10\rangle = \ \ 0.15 \text{ MeV}, -0.22 \text{ MeV},$$

$$\langle 1d_{\frac{3}{2}}2s_{\frac{1}{2}};\ 10|V|1d_{\frac{3}{2}}2s_{\frac{1}{2}};\ 10\rangle = -1.74 \text{ MeV}, -3.02 \text{ MeV},$$

for the Tabakin [8]) and the Hamada-Johnston [5]) interactions, respectively (calculated for $A = 18$ nuclei without core polarization taken into account); these discrepancies persist after inclusion of core polarization corrections. These matrix elements are important in the calculation of $T = 0$ levels in the upper end of the 2s1d shell. A closer inspection of the various contributions to these two-body matrix elements suggested that these differences arise mainly from differences in the $^3D_1$ reduced integrals (matrix elements in the relative-coordinate frame). The latter are roughly four times more repulsive in the Tabakin case than in the Hamada-Johnston case. The values for the Tabakin case are probably not as reliable, since the Tabakin interaction does not fit the $^3S_1$–$^3D_1$ phase shifts very well as can be seen from Taba-

kin's original paper [11]). Recently Elliott *et al.* [16]) have obtained a set of two-body matrix elements in the relative-coordinate frame directly from nucleon-nucleon scattering phase shifts. The $^3D_1$ matrix elements thus obtained are also more than twice as small as the Tabakin matrix elements. To investigate the effect of a modification of the $^3D_1$ reduced integrals on the shell-model results, two similar calculations have been performed, i.e. one with matrix elements obtained from the original Tabakin potential and a second one with matrix elements obtained from $^3D_1$ reduced integrals that were 50 % of the original value. For most low-lying levels the effect of this modification appeared to be small; however, for the $T = 0$ levels in ^{38}K the agreement with experiment appreciably improved. The large calculated $1d_{\frac{3}{2}}^{-1} - 1d_{\frac{5}{2}}^{-1}$ splitting also decreased to a more reasonable value (about 9.4 MeV).

The wave functions have been used for the calculation of nuclear moments, transition rates, log *ft* values and spectroscopic factors. In order to give some idea about the sensitivity of these quantities to the residual interaction, we shall compare the predictions of the Tabakin interaction with those obtained recently by Glaudemans [17]) in the same configuration space with the modified surface delta interaction [18]) (MSDI). The latter interaction with four adjustable parameters has been employed rather successfully as a residual interaction in several shell-model calculations[10,13,18]) of energy spectra.

In sect. 2, an outline is given of the calculation of the two-body matrix elements of the Tabakin interaction. In sect. 3, the calculation of the energy spectra is described. In sect. 4, nuclear moments, transition rates and spectroscopic factors are given, and a comparison is made with the results of the MSDI. Concluding remarks are made in sect. 5.

2. Calculation of shell-model matrix elements

The shell-model matrix elements to be evaluated are $\langle ab; JT|V|cd; JT\rangle$. Here $|ab; JT\rangle$ denotes a normalized antisymmetric state of two particles in single-particle states a and b coupled to angular momentum J and isospin T. The calculation of the effective shell-model matrix elements of the Tabakin interaction has been described [8]) in detail elsewhere. Harmonic-oscillator wave functions are used for the single-particle wave functions. With the use of the Brody-Moshinsky transformation brackets, all matrix elements are expressed in terms of matrix elements in the relative coordinate frame $\langle nlSJT|V|n'l'SJT\rangle$. The latter integrals can be evaluated from the radial functions appearing in the Tabakin interaction. For the harmonic-oscillator parameter a value $\nu = m\omega/2\hbar = 0.143$ fm^{-2} has been employed. For reasons mentioned in sect. 1, the calculations have been performed also for a modified interaction such that the diagonal $^3D_1$ reduced integrals were diminished by 50 %, while the $^3D_1 - {}^3S_1$ reduced integrals were unchanged. This can be achieved by reducing the strength of the radial function $h_{\alpha l}(r)$ of the $^3D_1$ potential by approximately a factor $\sqrt{2}$.

Because of the rather small model space used in the shell-model diagonalization, the matrix elements should be renormalized for the configurations omitted from the

model space. It is convenient to distinguish between two types of renormalization effects.

The importance of second-order Goldstone corrections in nuclear matter has already been demonstrated [11]) by Tabakin. Shell-model calculations have shown [8]) that inclusion of second-order Born corrections, which represent the effect of the excitation of two nucleons to high energies, improves substantially the agreement with experiment.

These corrections can be taken into account in good approximation by replacing the matrix elements in the relative coordinate system, $\langle nlSJT|V|n'l'SJT\rangle$, by

$$\langle nlSJT|V|n'l'SJT\rangle + \sum_{\beta} \frac{\langle nlSJT|V|\beta\rangle\langle\beta|V|n'l'SJT\rangle}{E_0 - E_\beta}.$$

The evaluation of this expression has been performed as described in ref. [8]). In this evaluation, the intermediate two-particle states $|\beta\rangle$ are represented by plane waves, and the angle-averaged Pauli operator is used. The Fermi momentum is taken as $k_F = 1.3\ \mathrm{fm}^{-1}$, and the energy of the unperturbed bound-state pairs as $E_0 = -20$ MeV.

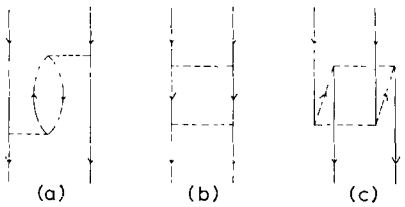

Fig. 1. Second-order corrections.

Beyond the second-order Born corrections, there are renormalization corrections as symbolized by the diagrams shown in fig. 1. These are the hole analogues of the particle diagrams considered by Kuo and Brown [4]) and by Kuo [5]). The particle analogue to diagram 1a has been shown by Kuo and Brown [4]) to improve agreement with experiment substantially.

The three-hole–one-particle corrections of diagram 1a are given by

$$\langle c^{-1}d^{-1};\ JT\left|\frac{V_{3\mathrm{h}\,1\mathrm{p}}}{\Delta E}\right|a^{-1}b^{-1};\ JT\rangle = (1+E(a,b))(1+E(c,d))$$
$$\times((1+\delta_{ab})(1+\delta_{cd}))^{-\frac{1}{2}} \sum_{phJ'T'} \frac{1}{\Delta E}(2J'+1)(2T'+1) \begin{Bmatrix} j_a & j_b & J \\ j_d & j_c & J' \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & T \\ \frac{1}{2} & \frac{1}{2} & T' \end{Bmatrix}$$
$$\times(-1)^{J+T+1+j_c+j_d} F(dbhp;\ J'T')F(achp;\ J'T'),$$

where the particle-hole matrix elements F are given by

$$F(abcd; JT) = -((1+\delta_{aa})(1+\delta_{bc}))^{\frac{1}{2}} \times \sum_{J'T'} (2J'+1)(2T'+1) \begin{Bmatrix} j_a & j_b & J \\ j_c & j_d & J' \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & T \\ \frac{1}{2} & \frac{1}{2} & T' \end{Bmatrix} \langle adJ'T'|V|cbJ'T' \rangle.$$

The exchange operator $E(a, b)$ is defined such that

$$E(a, b)\Phi(a, b, c, d; J, T) = (-1)^{j_a+j_b+J+T}\Phi(b, a, c, d; J, T).$$

The contribution of diagram 1b is given by

$$\langle c^{-1}d^{-1}; JT \left| \frac{V_{2h}}{\Delta E} \right| a^{-1}b^{-1}; JT \rangle = \tfrac{1}{2} \sum_{h_1h_2} \frac{\langle h_1h_2; JT|V|ab; JT \rangle \langle cd; JT|V|h_1h_2; JT \rangle}{\Delta E}.$$

For diagram 1a, the intermediate hole states are taken in the 1p (and 2s1d) shells and the particle states in the 2p1f (and 3s2d1g) shells. For diagram 1b, the intermediate hole states are taken in the 1p shell. All energy denominators are approximated by $\Delta E = -2\hbar\omega$. The contribution of diagram 1c has not been taken into account, because there is some overlap of these corrections with the second-order Born corrections. Moreover, it was found that the inclusion of diagram 1c would make the interaction too strong in the $J = 0$ $T = 1$ states.

3. Calculation of energy levels

The energies of even-parity states in $A = 36$–38 nuclei are obtained from a diagonalization of the residual interaction in the $(2s1d)^{-n}(n = 4, 3, 2)$ space with ^{40}Ca being taken as the reference state. It is assumed that the same renormalized two-body forces can be used for $A = 38, 37, 36$ nuclei, i.e. that in these nuclei there is no appreciable perturbation from effective three-hole and more-hole forces. The effect of three-body forces to the binding energies has been shown [19]) to be negligible for the Ca isotopes.

The energy matrices were constructed with the help of tapes produced by the Oak Ridge-Rochester shell-model computer programs [20]). (Each many-body matrix element in our $A = 36-38$ calculations is expressible as a linear combination of the two-body matrix elements and the single-particle energies. The Oak Ridge-Rochester tapes supplied the numerical coefficients for these linear combinations.)

3.1. CHOICE OF THE SINGLE-HOLE ENERGIES

Most naturally the single-particle energies can be derived from the experimental single-hole energies from ^{39}K/Ca. However, the position of the $1d_{\frac{3}{2}}$ hole state is not well determined experimentally. Pick-up experiments [21]) on ^{40}Ca indicate that the $1d_{\frac{3}{2}}^{-1}$ strength in $A = 39$ nuclei is fractionated over a large number of components with centre of gravity lying more than 6.6 MeV above the $1d_{\frac{3}{2}}^{-1}$ state. The

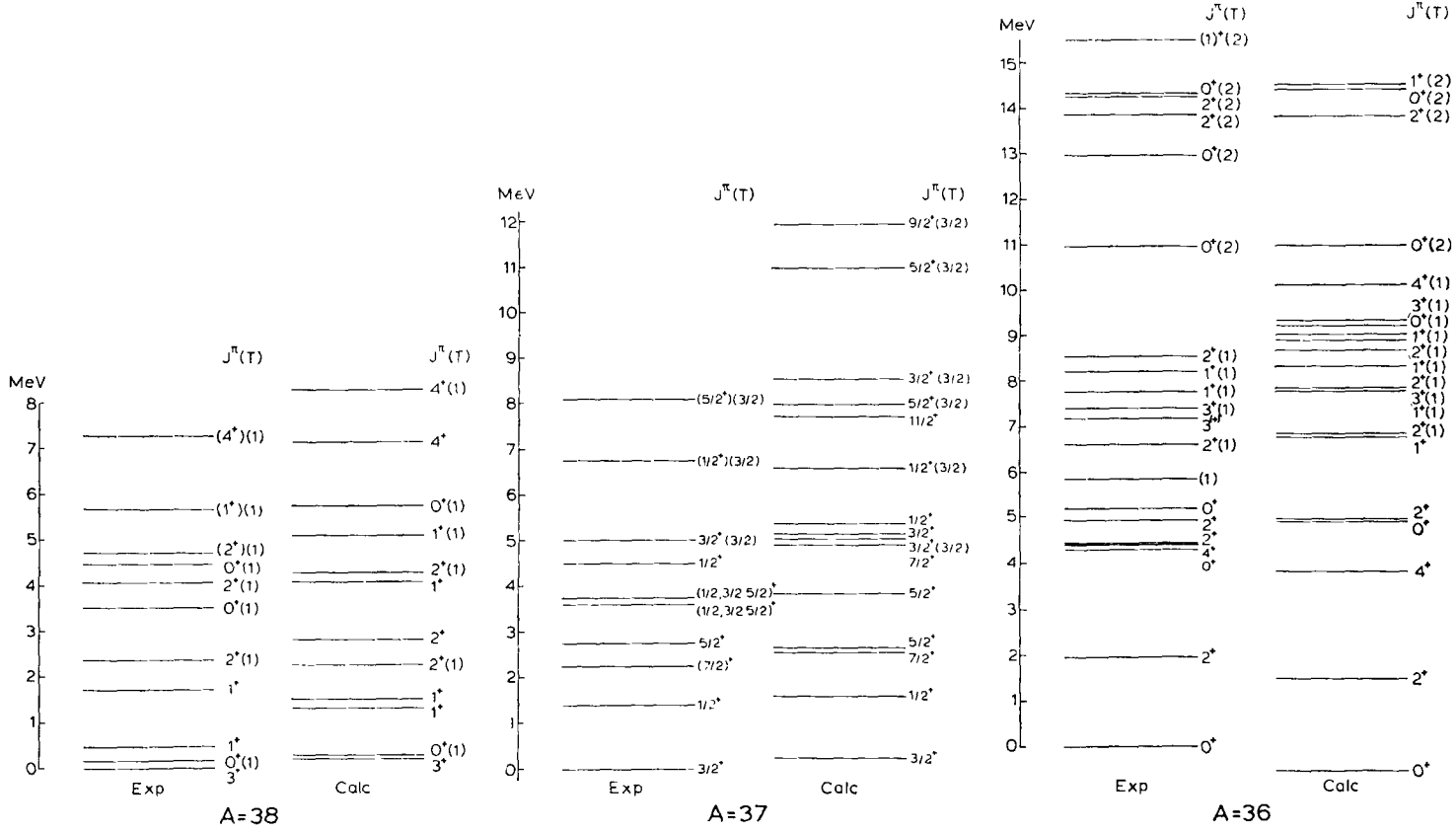

Fig. 2. Experimental and calculated energy spectra for $A = 38$, 37 and 36. Only even-parity levels are drawn. If the same state is experimentally observed in nuclei with different T_z, the averaged excitation energy has been given. The majority of the experimental data are from ref. [22]).

position of the lower $J = 1$ $T = 0$ states in ^{38}K is rather sensitive to the choice of the $1d_{\frac{5}{2}}^{-1}$ energy, because the matrix element $\langle 1d_{\frac{3}{2}}1d_{\frac{5}{2}}; 10|V|1d_{\frac{3}{2}}1d_{\frac{5}{2}}; 10\rangle$ is strongly negative, and thus the $1d_{\frac{3}{2}}^{-1}1d_{\frac{5}{2}}^{-1}$ configuration can substantially mix with the $1d_{\frac{3}{2}}^{-2}$ configuration. The separation energy of a $1d_{\frac{3}{2}}$ neutron with respect to the ^{40}Ca core has been taken as -15.60 MeV. The $1d_{\frac{3}{2}}^{-1}-2s_{\frac{1}{2}}^{-1}$ and $1d_{\frac{3}{2}}^{-1}-1d_{\frac{5}{2}}^{-1}$ energy splittings have been taken 2.6 MeV and 7.6 MeV, respectively, for the calculation of energy levels in $A = 38$ nuclei. For the $A = 36$ and 37 nuclei, the $1d_{\frac{3}{2}}^{-1}-2s_{\frac{1}{2}}^{-1}$ and $1d_{\frac{3}{2}}^{-1}-1d_{\frac{5}{2}}^{-1}$ splittings were slightly increased, which especially improved the relative spacing of the $J^\pi = \frac{3}{2}^+$, $\frac{1}{2}^+$ and $\frac{5}{2}^+$, $T = \frac{1}{2}$ and $\frac{3}{2}$ levels in $A = 37$ nuclei. The calculations for $A = 36$ and $A = 37$ were performed with $E(1d_{\frac{3}{2}}^{-1})-E(2s_{\frac{1}{2}}^{-1}) = 3.2$ MeV and $E(1d_{\frac{3}{2}}^{-1})-E(1d_{\frac{5}{2}}^{-1}) = 8.2$ MeV.

3.2. CALCULATIONS

The energy levels calculated with the modified Tabakin interaction are shown in fig. 2.

It can be seen that a reasonable agreement with experiment is obtained both for the absolute binding energies (corrected for Coulomb energies) and for excitation energies. None of the first excited $J^\pi = 0^+$ levels in ^{36}S, ^{36}Ar and ^{38}Ar can be reproduced by the present calculation. It is most reasonable to assume that these states mainly possess a $(2s1d)^{-n}(2p1f)^2$ character. This assumption is consistent with calculations by Erné [22]) in the case of ^{38}Ar. However, for $A = 36$ nuclei Erné's calculation yielded $J^\pi = 0^+$ states at energies near the energies of the second excited $J^\pi = 0^+$ state.

4. Calculation of transition rates and *ft* values

With the wave functions obtained in the present calculation, we have calculated nuclear moments, electromagnetic transition rates, *ft* values and spectroscopic factors. They are displayed in tables 1–5. For comparison, the results obtained [17]) with the modified surface delta interaction (MSDI) are also given.

TABLE 1

Nuclear dipole moments in n.m.

Nucleus	J	T	exp [a])	Tabakin	MSDI
^{39}K	$\frac{3}{2}$	$\frac{1}{2}$	0.39	0.13	0.13
^{38}K	3	0	1.37	1.24	1.27
^{37}K	$\frac{3}{2}$	$\frac{1}{2}$	0.204 [b])	−0.21	0.13
^{37}Ar	$\frac{3}{2}$	$\frac{1}{2}$	0.95±0.20	1.49	1.15
^{37}Cl	$\frac{3}{2}$	$\frac{3}{2}$	0.68	0.56	0.41
^{36}Cl	2	1	1.29	1.60	1.21

[a]) Ref. [24]). [b]) Ref. [25]).

In the MSDI case, the values of the four interaction parameters and the three single-hole energies are determined from a least-squares fit to 24 levels in $A =$

36–39 nuclei; the following values of the parameters were obtained [17]):

$$A_0 = \ 0.24 \text{ MeV}, \quad A_1 = 0.92 \text{ MeV},$$

$$B_0 = -2.13 \text{ MeV}, \quad B_1 = 0.49 \text{ MeV},$$

$$E(1d_{\frac{5}{2}}^{-1}) = -23.12 \text{ MeV}, \quad E(2s_{\frac{1}{2}}^{-1}) = -18.12 \text{ MeV}, \quad E(1d_{\frac{3}{2}}^{-1}) = -15.62 \text{ MeV}.$$

4.1. THE $A = 38$ NUCLEI

In the $A = 38$ nuclei two β^+ decay processes are observed which justify a closer examination because their lifetimes cannot be explained in terms of a $1d_{\frac{3}{2}}^{-2}$ configuration. The β^+ decay of the $J^\pi = 3^+$ ground state of ^{38}K to the $J^\pi = 2^+$ level in ^{38}Ar at 2.17 MeV has a log ft value of 5.0, to be compared with a value of 4.3 for a pure $1d_{\frac{3}{2}}^{-2} \rightarrow 1d_{\frac{3}{2}}^{-2}$ transition. In pick-up experiments [26,27]) on ^{39}K, only a very small $l = 0$ admixture in the $J^\pi = 2^+$ $T = 1$ state has been found. Since the $1d \rightarrow 2s$ transition is l-forbidden, small $1d_{\frac{3}{2}}^{-1}2s_{\frac{1}{2}}^{-1}$ admixtures to the wave function of the $J^\pi = 2^+$ state will not produce a large enough reduction of the Gamow-Teller matrix element. The cancellation of contributions to the Gamow-Teller matrix elements could be due to $1d_{\frac{3}{2}}^{-1}\,1d_{\frac{5}{2}}^{-1}$ admixtures in the wave functions. It turns out indeed, that the calculated 2.1 % $1d_{\frac{3}{2}}^{-1}1d_{\frac{5}{2}}^{-1}$ admixture in the $J^\pi = 3^+$ wave function reduces the Gamow-Teller matrix elements by approximately 50 %. (It is striking that the effect of an equally large $1d_{\frac{3}{2}}^{-1}d_{\frac{5}{2}}^{-1}$ admixture in the $J^\pi = 2^+$ state can almost be ignored; this is caused by the magnitude of the recoupling coefficients involved.) The log ft value is thus increased to 4.76. In the MSDI case, the $1d_{\frac{3}{2}}^{-1}1d_{\frac{5}{2}}^{-1}$ component in the $J^\pi = 3^+$ state almost vanishes (< 0.1 %); therefore there is no cancellation in the Gamow-Teller matrix element.

Recently the β^+ decay of the ^{38}Ca ground state has been studied [9]). A strong branch to the 1.70 MeV $J^\pi = 1^+$ level in ^{38}K has been observed (log $ft = 3.41$), whereas for the decay to the 0.45 MeV $J^\pi = 1^+$ level a lower limit for the log ft value has been found (log $ft > 4.77$). In the following, we assume that both $J^\pi = 1^+$ states possess $(2s1d)^{-2}$ character; this seems to be not unreasonable because sd shell-model calculations [10,17]) predict two $J^\pi = 1^+$ states below 2 MeV and because no other $J = 1$ state has been observed below 2 MeV. Since the energy differences of the $1d_{\frac{3}{2}}^{-2}$, $2s_{\frac{1}{2}}^{-1}1d_{\frac{3}{2}}^{-1}$, $2s_{\frac{1}{2}}^{-2}$ and $1d_{\frac{3}{2}}^{-1}1d_{\frac{5}{2}}^{-1}$ $J^\pi = 1^+$ $T = 0$ states before diagonalization are smaller than 2.5 MeV and large off-diagonal matrix elements are involved, the wave functions after diagonalization become complicated mixtures of these four two-hole states. As the amplitudes are critically dependent on the two-body matrix elements, the β-transition rates to these states provide a valuable check of the calculated wave functions. If the original Tabakin matrix elements are used, the β-decay is found to proceed predominantly to the lowest $J^\pi = 1^+$ state. The use of the interaction with the modified $^3D_1$ reduced integrals essentially interchanges the nature of the lowest two $J^\pi = 1^+$ wave functions, such that a far more satisfactory agreement is obtained with experiment, as can be seen from table 2.

TABLE 2
log *ft* values

Nuclei	J_i	T_i	J_f	T_f	E_{xf} (MeV)	exp	log *ft*	
							Tabakin	MSDI
$^{39}Ca \rightarrow {}^{39}K$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$		3.6	3.52	3.52
$^{38}Ca \rightarrow {}^{38}K$	0	1	1	0	0.45	>4.77 [a])	5.58	3.61
$^{38}Ca \rightarrow {}^{38}K$	0	1	1	0	1.70	3.41±0.09 [a])	3.25	4.97
$^{38}Ca \rightarrow {}^{38}K$	0	1	0	1	0.13	3.49	3.49	3.49
$^{38}K \rightarrow {}^{38}Ar$	0	1	0	1	0			
$^{38}K \rightarrow {}^{38}Ar$	3	0	2	1	2.17	5.0	4.75	4.34
$^{37}K \rightarrow {}^{37}Ar$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	0	3.6	3.55	3.62
$^{37}K \rightarrow {}^{37}Ar$	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	2.80	3.8	3.49	4.14
$^{37}K \rightarrow {}^{37}Ar$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1.41	>5.7	7.61	5.17
$^{37}Ca \rightarrow {}^{37}K$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	5.02	3.3 ±0.3	3.29	3.29
$^{37}Ar \rightarrow {}^{37}Cl$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	0	5.0	5.12	4.49
$^{36}K \rightarrow {}^{36}Ar$	2	1	2	0	1.97	4.5	4.85	4.36
$^{36}K \rightarrow {}^{36}Ar$	2	1	2	1	6.61	3.6	3.49	3.49

All experimental data are from ref. [22]) unless indicated otherwise.
[a]) Ref. [9]).

In the MSDI case, the situation is quite different. All $J^\pi = 1^+$ states are almost pure, the lowest state being predominantly (97 %) $1d_{\frac{3}{2}}^{-2}$, the second state predominantly (98 %) $1d_{\frac{3}{2}}^{-1}2s_{\frac{1}{2}}^{-1}$. The resulting log *ft* values do not agree with experiment.

TABLE 3
Calculated mean lifetimes of $J^\pi = 1^+$ states in ^{38}K

J_iT_i	E_{xi}^{exp} MeV	J_fT_f	Reduced transition rates		Mean lifetimes (s)	
			Tabakin	MSDI	Tabakin	MSDI
10	0.45	01	B(M1)/n.m.^{2} 0.17	0.14	1.0×10^{-11}	1.2×10^{-11}
		30	B(E2)/e^2fm^4 8.5	4.3		
10	1.70	01	B(M1)/n.m.^{2} 1.70	0.013	8.6×10^{-15}	9.0×10^{-13}
		10	B(M1)/n.m.^{2} 0.002	0.0002		
		10	B(E2)/e^2fm^4 5.5	1.2		
		30	B(E2)/e^2fm^4 0.06	6.0		

The predicted mean lives for γ-decay of the $J^\pi = 1^+$ states are given in table 3. A measurement of these life times would be a welcome additional check on the wave functions.

The lowest two $J^\pi = 2^+$ levels in ^{38}Ar experimentally show γ-decay to the $J^\pi = 0^+$ ground state. The measured mean lifetimes [10]) are 600±40 fs for the 2.17 MeV level and 105±16 fs for the 3.94 MeV level corresponding to E2 transitions with B(E2) = 28 e^2fm^4 and 8.2 e^2fm^4, respectively. With free proton and neutron charges

and harmonic-oscillator wave functions, the calculated reduced transition probabilities are $B(E2) = 13.0\ e^2fm^4$ and $3.3\ e^2fm^4$, respectively. One cannot expect that the E2 transition rates are well reproduced by the model wave functions because of their collective character. The effects of two types of additional configuration mixing are expected to be important; (i) core polarization due to one-particle–one-hole excitations and (ii) admixtures of deformed $(2s1d)^{-4}(1f2p)^2$ components. The former effect has been recently investigated [30]). In first-order perturbation theory effective single-proton charge operators were evaluated. With these effective charges, due to core polarization, the reduced transition probabilites become $B(E2) = 20\ e^2fm^4$ and $4.8\ e^2fm^4$, respectively. The remaining discrepancies are probably due to the omission of $(2s1d)^{-4}(2p1f)^2$ configuration mixing effects. A recent investigation [26]) of the $^{39}K(d, {}^3He)^{38}Ar$ reaction strongly indicates that the second excited $J^\pi = 2^+$ state is not very well described by $(2s1d)^{-2}$ configurations. The same reaction yielded also information about other states in ^{38}Ar. It is interesting that no population of the first excited $J^\pi = 0^+$ state was observed. This state is believed [23]) to possess predominantly $1d_{\frac{3}{2}}^{-4}1f_{\frac{7}{2}}^2$ character, which indicates that the ^{39}K ground state has only small $(1d_{\frac{3}{2}})^{-3}1f_{\frac{7}{2}}^2$ components. Therefore pick-up experiments on ^{39}K will provide valuable information on $A = 38$ nuclei. It can be seen from table 4 that the agreement of the calculated spectroscopic factors with experiment in the $T = 1$ case is very good. However, the theoretical values do not appear to depend very strongly on the effective interaction employed.

TABLE 4

Spectroscopic factors for pick-up reactions leading to ^{38}K

State			exp		Tabakin			MSDI		
J	T	E_x (MeV)	$l=0$	$l=2$	$l=0$,	$l=2$ ($j=\frac{3}{2}$)	$l=2$ ($j=\frac{5}{2}$)	$l=0$	$l=2$ ($j=\frac{3}{2}$)	$l=2$ ($j=\frac{5}{2}$)
3	0	0	0	1.24	0	1.72	0.02	0	1.75	0
1	0	0.45	< 0.06	0.32	0.17	0.27	0.04	0.01	0.73	0
1	0	1.70	< 0.08	0.51	0.02	0.38	0.10	0.36	0.02	0
2	0				0.53	0	0.07	0.62	0	0
0	1	0	0	0.74	0	0.68	0	0	0.68	0
2	1	2.17	0	3.75	0.10	3.41	0.02	0.17	3.34	0.02
2	1	(4.59)	1.8	0.3	1.66	0.27	0.01	1.65	0.37	0.02
1	1	(5.55)	1.17	0	1.12	0	0	1.12	0	0

Experimental data are from refs. [26 27]).

Information about $T = 0$ levels should come from the $^{39}K(^3He, \alpha)^{38}K$ reaction [27]). Unfortunately, in this case the spectroscopic factors extracted from experiment contain rather large uncertainties. Less than 10 % of the total $l = 0$ strength has been observed in the lowest two $J = 1$ states. The present calculation with the Tabakin interaction predicts approximately 50 % $l = 0$ strength, mainly concentrated in the lowest $J = 1$ state.

4.2. THE $A = 37$ NUCLEI

Experimental magnetic moments and *ft* values provide a possible test of the wave functions of the $J^\pi = \frac{3}{2}^+$, $T = \frac{1}{2}$ and $\frac{3}{2}$ ground states of ^{37}Ar/^{37}K and ^{37}Cl, respectively. The differences between the experimental magnetic moments of ^{37}Ar and ^{37}K and the Schmidt values are much smaller than the calculation with the Tabakin interaction predicts (see table 1). However, the sum of the predicted values for the $T_z = \frac{1}{2}$ and $T_z = -\frac{1}{2}$ cases is almost equal to the corresponding sum of the Schmidt values; this is also true for the MSDI. Thus it appears that the isoscalar part of the μ-operator is insensitive to configuration admixing effects. In a recent calculation [30]) of second-order corrections to the magnetic moments of ^{39}Ca and ^{39}K, it was also found that the corrections obtained for the isoscalar part of the operator are very small. Analysis of the experimental magnetic moments of mirror nuclei indicates [34]) that the isoscalar part is indeed insensitive for configuration mixing.

For the calculation of the β-decay of the ^{37}K ground state to the $J^\pi = \frac{5}{2}^+$ level at 2.80 MeV in ^{37}Ar, it was assumed that the latter state corresponds to the lowest calculated $J^\pi = \frac{5}{2}^+$ level. (The level [22]) observed at 2.22 MeV (spin assignment $J^\pi = \frac{5}{2}^+$ or $\frac{7}{2}^+$) very likely possesses a spin value $J^\pi = \frac{7}{2}^+$, since this state is not populated in the ^{36}Ar(d, p)^{37}Ar reaction or in the β-decay of ^{37}K.) The Tabakin interaction and the MSDI yield two quite different wave functions for this $J^\pi = \frac{5}{2}^+$ level; in the Tabakin case, it is a complicated mixture of several components (with 25 % $1d_{\frac{3}{2}}^{-2}(01)1d_{\frac{5}{2}}^{-1}$), whereas in the MSDI, it is an almost pure (91 %) $1d_{\frac{3}{2}}^{-3}$ configuration. The resulting *ft* value is seen to be too small in the Tabakin case, but it is too large in the MSDI case (see table 2).

The electric quadrupole moment of ^{37}Cl (experimental value -6.3 ± 0.2 $e$fm^2) is calculated to be $Q = -5.4$ $e$fm^2 (Tabakin) and $Q = -5.3$ $e$fm^2 (MSDI).

4.3. THE $A = 36$ NUCLEI

Low-lying levels in ^{36}Cl are excited in neutron pick-up experiments on ^{37}Cl. The spectroscopic factors are displayed in table 5. As the experimental results strongly

TABLE 5

Spectroscopic factors for reaction [a]) ^{37}Cl(^{3}He, α)^{36}Cl and reaction [b]) ^{37}Cl(d, t)^{36}Cl

State J	T	E_x (MeV)	exp [a]) $l=0$	exp [a]) $l=2$	exp [b]) $l=0$	exp [b]) $l=2$	Tabakin $l=0$	Tabakin $l=2$ ($j=\frac{3}{2}$)	Tabakin $l=2$ ($j=\frac{5}{2}$)	MSDI $l=0$	MSDI $l=2$ ($j=\frac{3}{2}$)	MSDI $l=2$ ($j=\frac{5}{2}$)
2	1	0		1.30	0	1.30	0	0.84	0.01	0	0.97	0
3	1	0.79		1.91	0	2.0	0	1.20	0.02	0	1.39	0
1	1	1.17		0.44	0.03	0.5	0.03	0.13	0.01	0.05	0.50	0
1	1	1.60	0.22		0.07	0.5	0.07	0.10	0	0.01	0	0
2	1	1.95	0.50		0.2	0.5	0.05	0	0.03	0.04	0.02	0
0	2	4.33		1.75			0	0.92	0	0	0.96	0

[a]) Ref. [28]). [b]) Ref. [29]).

disagree for some of the levels, a detailed comparison seems rather premature. It can be seen that in general the calculated spectroscopic factors are smaller than the experimental ones (in particular for the Tabakin interaction). This may partly be caused by the normalization procedure employed for the determination of the experimental absolute spectroscopic factors. The difference between the calculated (1.60 n.m.) and experimental (1.29 n.m.) values of the magnetic dipole moment of ^{36}Cl (compared with a value of 0.85 n.m. for a pure $1d_{\frac{3}{2}}^{-4}$ configuration) indicates that the Tabakin interaction causes too strong a configuration mixing for this state.

The reduced transition probability $B(E2) = 10.7\ e^2\text{fm}^4$ for the $2^+ \rightarrow 0^+$ transition in ^{36}Ar was obtained with the Tabakin interaction for free neutron and proton charges. The observed [31]) value is given by $B(E2) = 60 \pm 15\ e^2\text{fm}^4$. The MSDI yields only a slightly smaller value $B(E2) = 8.3\ e^2\text{fm}^4$ for this transition, though the wave functions are much purer than the Tabakin wave functions.

5. Discussion

Although the Tabakin interaction and the MSDI lead to very similar energy spectra, it is seen that the application of the wave functions sometimes yields different results in the two cases. After comparison of the wave functions a satisfactory agreement is found in the cases of identical nucleons, i.e. if only $T = 1$ two-body matrix elements are involved. As soon as also $T = 0$ two-body matrix elements enter, the Tabakin wave functions show a much stronger configuration mixing than the MSDI wave functions. It is to be expected that these differences are largely due to the tensor part of the Tabakin interaction.

The retardation of the $J = 0$, $T = 1$ to $J = 1$, $T = 0$ β-decay rate in $A = 38$ nuclei resembles the drastic retardation of the β-decay observed in $A = 14$ nuclei between states with the same spin and isospin. The extremely long lifetime for the β-decay of the ^{14}C ground state ($\log ft = 9.03$) to the ^{14}N ground state cannot be explained in terms of the $1p^{-2}$ model space with a conventional two-body interaction without a tensor force. It was shown [32]) by Jancovici and Talmi that a two-body interaction with a tensor force could make the Gamow-Teller matrix element to vanish. Using the Hamada-Johnston potential as a residual interaction in the $1p^{-2}$ space, Zamick [33]) obtained indeed a large retardation of the β-decay rate of ^{14}C. For the β^+ transition from the ground state of ^{38}Ca to the first excited $J^\pi = 1^+$ state of ^{38}K the situation is more complicated. In this case a small Gamow-Teller matrix element can result either from a large $2s_{\frac{1}{2}}^{-1}1d_{\frac{3}{2}}^{-1}$ component in the wave functions of the $J^\pi = 1^+$ state or from a cancellation between the various terms contributing to the Gamow-Teller matrix element. In the Tabakin case both of these mechanisms help to reduce the Gamow-Teller matrix element; in the MSDI case neither of these mechanisms is effective.

As the modification of the $^3D_1$ part of the Tabakin potential substantially improves the agreement of the calculated results with the experimental data, it would be worthwhile to reconsider the unsatisfactory fit of the Tabakin force parameters for the $^3D_1 - {}^3S_1$ states to the experimental phase shifts.

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