THE THREE NUCLEON BOUND STATE WAVE FUNCTION AND THE COULOMB ENERGY OF ^{3}He

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A method previously employed to determine the binding energy of a three particle system using the Faddeev equations is extended to calculate the bound state wave function. It is applied to the three-nucleon bound state and as result the Coulomb energy and the S'-state probability are calculated using local central Yukawa type interactions.

In a recent paper [1] we described a simple method to compute exactly the binding energy of the ground state of a three particle system within the framework of the Faddeev equations. We used the method to study the triton with central Yukawa-type interactions in both the singlet and triplet channels of the two-particle sector. The effect of a repulsion in these channels was also considered. As a result we obtained a reasonable agreement with experiment.

However, up to now nothing has been said in this method about the corresponding three-particle bound-state wave function which can be used to compute among others, the Coulomb energy in first order perturbation theory and the S'-state probability. We shall now show that the method of ref. [1] is also well suited for the determination of the bound-state wave function.

Adopting the same notation as in ref. [1] the reduced Faddeev equations can formally be represented in the form

$$F(s) = F_0(s) - \lambda K(s) F(s) \tag{1}$$

Here the parameter λ has been added for convenience and it should be taken to be equal to one in order to get the Faddeev equations. As described in ref. [1] the solution of eq. (1) can be written at a given energy s as

$$F(s) = \sum_\alpha \frac{B_\alpha(s)}{\lambda_\alpha(s) - \lambda} + R(\lambda, s) \tag{2}$$

where $R(\lambda, s)$ is a meromorphic function of λ as a result of the compactness of the kernel K. Furthermore if we suppose that $\lambda_0(s)$ is the smallest eigenvalue of the corresponding homogeneous equation the binding energy s_0 of the ground state is given by the value of s for which $\lambda_0(s) = 1$. This value can simply be obtained as shown in ref. [1] from the condition

$$\lim_{n\to\infty} \frac{F_{n+1}(s_0)}{F_n(s_0)} = 1 \tag{3}$$

where F_n are the coefficients of the Neumann series solution of eq. (1).

Having found the binding energy s_0 we are now able to determine the corresponding bound-state wave function in the following way. Note from eq. (2) that near $s = s_0$ we may write for F

$$F(s) \approx \frac{B_0(s_0)}{(s - s_0)(\mathrm{d}\lambda_0(s)/\mathrm{d}s)_{s=s_0}} \tag{4}$$

As a result of this we shall get a pole in the full T-matrix at the porition $s = s_0$. The residue A of this pole can now simply be expressed in terms of the only unknown function B_0.

On other hand A can also be related in a well-known way to the corresponding three-particle bound state wave function ψ_t as follows

$$A = \text{const}\ (p^2 + q^2 - s_0)\, \langle p\, q\beta | \psi_t \rangle . \tag{5}$$

Here we have only exhibited the dependence on the relative momenta p and q and the label β which stands for the quantum numbers of spin and isospin. From eq. (5) we see that in order to determine ψ_t we have to calculate the residue A or equivalently the residue $B_0(s_0)$ in eq. (2). One easily convinces oneself that in view of eq. (2) B_0 can be found by

$$B_0(s_0) = \lim_{n\to\infty} F_n(s_0) . \tag{6}$$

So assuming that the set of F_n's converges rapidly enough which is actually so in the case con-

sidered , we have a practical way for computing the bound state wave function $\psi_t(p,q,\beta) \equiv \equiv \langle p\,q\,\beta|\psi_t\rangle$.

Following the analysis of Schiff [2] we make the following symmetry decomposition.

$$\psi_t(p,q,\beta) = \tag{7}$$
$$= \varphi_0(\beta)u(p,q) + [\varphi_1(\beta)v_2(p,q) - \varphi_2(\beta)v_1(p,q)]$$

where $\varphi_0(\beta)$ is completely anti-symmetric and $\varphi_1(\beta)$, $\varphi_2(\beta)$ are symmetric and anti-symmetric respectively for inter-change of the particles 2 and 3. The second term on the right hand side of eq. (7) corresponds to the state of mixed symmetry (S'-state) and the other one is the "dominant" S-state. The numerically obtained values of ψ_t and so u_1 v_2 and v_1, were fitted to functions with simple analytic structure as suggested in ref. [2].

Consider firstly the S-state. Fits were made with the Irving and Gaussian type wave functions

$$I(p,q) = A\,[1 + B(p^2 + q^2)]^{-\frac{7}{2}} \tag{8}$$
$$G(p,q) = A'\exp\,[-B'(p^2+q^2)]$$

where the first one is found to be the best one ($\chi^2 < 1\%$). The result for the Coulomb energy are given in table 1. The experimental value as well as those obtained by several authors using separable type potentials are also included in the table for comparison. For local potentials the only results available at this moment are those resulting from variational calculations [3] which suggest that there could be a charge asymmetry in the two-nucleon interaction because of the rather big discrepancy they found with the experimental value [4]. Our result does not allow already a definite conclusion concerning the charge dependence of nucleon forces but gives an indication that there may be no problem. As one can see from the results of table 1 the inclusion of a repulsion in the singlet interaction is of considerable importance not only for the binding energy but also for the Coulomb-energy, while introducing repulsion in the triplet interaction does not have any effect at all on the Coulomb-energy.

To compute the S'-state probability $P_{S'}$ we used wave functions of the Gaussian type. It turned out that they are not appropriate to obtain reasonable fits with it and as a consequence the corresponding S'-state probability contains a large uncertainty. The results of this calculation give $P_{S'} \approx 2\%$ which is roughly in agreement with other authors.

Table 1
^{3}H binding energies (MeV) and results for the Coulomb energy E_o (MeV).

$B(^3H)$	E_c	Ref.
11.1	0.79	5
9.1	0.68	5
8.7	0.73	6
12.1	0.85	1 and present calculation
8.4	0.72	1 and present calculation
8.3	0.72	1 and present calculation
8.48	0.764	Exp.

We conclude that the inclusion of a repulsive term in the singlet interaction is important in order to obtain reasonable agreement with experiment both for the binding energy and the ground state wave function.

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