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STRONG AND WEAK INTERACTIONS IN A SIMPLE FIELD-THEORETICAL MODEL

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Synopsis
An exactly renormalizable model of quantum fields, introduced earlier by Th. W. Ruijgrok and the present author, is considered for large but finite cut-off. It gives rise to strong and weak interaction effects. In the limit of infinite cut-off the weak interactions vanish and the strong interactions obey a certain symmetry law. This symmetry is violated by the weak interactions. It is emphasized that none of these properties is explicitly present in the hamiltonian of the theory, the form of which could hardly be guessed from the behaviour of the system at energies well below the cut-off.

1. *Introduction.* A simplified model of field theory has been introduced some time ago by Th. W. Ruijgrok and the present author [1]). It forms a non-trivial but still exactly renormalizable extension of the well known Lee model [2]) and it has been used by Ruijgrok for renormalization studies, especially for constructing a renormalized form of the Schrödinger equation for the field system itself [3]). We want to describe here a completely different aspect of the same model, absent in the special case considered by Lee. Taken for very large but finite cut-off K our model gives rise to observable interaction effects of two different types, the effects of one type being very much weaker than those of the other. The former, which we shall call the weak interaction effects, vanish in the limiting case of an infinite cut-off, whereas the latter, to be termed strong interaction effects, remain present when $K \to \infty$. In this limit, *i.e.* when only the strong interaction effects are present, the system strictly obeys a certain symmetry law. This invariance is no longer valid, however, for K large but finite, i.e. in presence of the weak interactions. In other words, the strong interactions obey a symmetry law which the weak interactions violate.

Although the ingredients of our model do not resemble any of the existing particles, the occurrence of strong and weak interactions and of an approximate symmetry law is of course strikingly similar to a most remarkable qualitative property of the known elementary particles. In the many attempts at a theoretical discussion of the latter it is customary to postulate lagrangians containing as main interaction terms expressions describing the strong interactions, with their proper symmetries, the weak interactions being relegated to additional small terms which violate the symmetries of the main terms. To put it more briefly, one supposes that the distinction between strong and weak interactions and the accompanying approximate symmetries are already explicitly present in the basic field equations, and that these features can therefore be used in guessing the basic equations. It is remarkable that such is not the case for our model. The hamiltonian involves only what one would call strong interaction terms, but these are very different from the observable strong interaction effects and do not possess their symmetry. At least at energies well below the cut-off, the observable effects are thus quite different from what the basic field equations would suggest and, in particular, standard perturbation theory is a very poor guide at detecting even the qualitative properties of the system. Oversimplified and unrealistic as it is, our model may give a warning that proper descriptions of strong and weak interactions and of approximate symmetries may have to be found in another direction than is commonly attempted, and these remarkable properties of elementary particles may be closely tied up with the effective cut-offs that unknown high energy phenomena will perhaps introduce in our still incomplete field theories.

2. *Hamiltonian of the model.* The model is characterized by a heavy or V-particle field and a light or θ-particle field. The θ-particles are bosons of mass μ, without internal degree of freedom. The V-particles are fermions with two possible internal states, V_1 and V_2 *). Their renormalized mass m is considered infinite, which amounts to assuming the inequalities

$$m \gg \mu, \quad m \gg K, \tag{2.1}$$

where K is the cut-off momentum for the θ–V interaction. This interaction is described by terms in the hamiltonian which correspond to the following elementary transitions

$$V_1 \rightleftarrows V_2 + \theta, \quad V_2 \rightleftarrows V_1 + \theta \tag{2.2}$$

with unrenormalized coupling constants $g_1{}^0$, $g_2{}^0$ respectively. Following the conventional notation also used in Ruijgrok's papers, we have for the complete hamiltonian

$$\left.\begin{aligned} &H = H_0 + H_1,\\ &H_0 = \textstyle\sum_{\boldsymbol{p}} m[\psi_1{}^*(\boldsymbol{p})\psi_1(\boldsymbol{p}) + \psi_2{}^*(\boldsymbol{p})\psi_2(\boldsymbol{p})] + \sum_{\boldsymbol{k}} \omega(k)a^*(\boldsymbol{k})a(\boldsymbol{k}),\\ &H_1 = -\textstyle\sum_{\boldsymbol{k}}\omega(k)X(k)[a(\boldsymbol{k})\{g_1{}^0 \sum_{\boldsymbol{p}} \psi_1{}^*(\boldsymbol{p}+\boldsymbol{k})\psi_2(\boldsymbol{p}) + g_2{}^0 \sum_{\boldsymbol{p}} \psi_2{}^*(\boldsymbol{p}+\boldsymbol{k})\psi_1(\boldsymbol{p})\}\\ &+ \text{herm. conj.}] - \textstyle\sum_{\boldsymbol{p}} [\delta m_1\psi_1{}^*(\boldsymbol{p})\psi_1(\boldsymbol{p}) + \delta m_2\psi_2{}^*(\boldsymbol{p})\psi_2(\boldsymbol{p})]. \end{aligned}\right\} \tag{2.3}$$

$X(k)$ stands for $(2v)^{-1/2}\,\omega^{-3/2}\,(k)f(k)$ with v the quantization volume and $f(k)$

*) The number of internal states could be taken larger than two [1][3].

the cut-off factor, e.g. $f(k) = K^2(k^2 + K^2)^{-1}$. The mass renormalizations δm_1, δm_2 are given by (I.2.8) *). Units are used such that $\hbar = c = 1$. The unrenormalized coupling constants $g_1{}^0$, $g_2{}^0$ are dimensionless. We assume them to be positive, of order 1 or larger, and we assume $g_1{}^0 > g_2{}^0$, $g_1{}^0 - g_2{}^0$ of the same order as $g_2{}^0$, for example $g_1{}^0 \simeq 2g_2{}^0$.

We will discuss this model for $K \gg \mu$, first considering the interaction effects in the limiting case $\mu/K \to 0$, and treating next the corrections for small but non-vanishing μ/K. The former constitute what we shall call the strong interaction effects. The latter are the weak interactions and the accompanying small modifications of the strong interactions. The next section deals with the strong interactions (limit $\mu/K = 0$), the case of non-vanishing μ/K being treated in section 4. The papers already published [1]) [3]) contain all the information needed for our discussion. The reader is referred to them for many derivations.

3. *Strong interaction effects.* The renormalized coupling constants g_1, g_2 are positive and given by [1]) [3])

$$\frac{g_1}{g} = \frac{g}{g_2} = \left[\frac{(g_1{}^0 + g_2{}^0)\exp(g^2L) + (g_1{}^0 - g_2{}^0)\exp(-g^2L)}{(g_1{}^0 + g_2{}^0)\exp(g^2L) + (g_2{}^0 - g_1{}^0)\exp(-g^2L)}\right]^{1/2} \tag{3.1}$$

with

$$g = (g_1{}^0 g_2{}^0)^{1/2} > 0, \quad L = \textstyle\sum_k X^2(k). \tag{3.2}$$

For $\mu/K \ll 1$, the only case we are interested in, L is approximately given by

$$L = (2\pi)^{-2}\log(K/\mu). \tag{3.3}$$

In the limit $\mu/K = 0$, L becomes infinite and

$$g_1 = g_2 = g. \tag{3.4}$$

The physical consequences of the interaction for $\mu/K = 0$, i.e. what we call the strong interaction effects, can be determined exactly. They are namely equivalent to those which follow if one replaces in the hamiltonian $H = H_0 + H_1$ both unrenormalized coupling constants $g_1{}^0$, $g_2{}^0$ by g, and the new hamiltonian H' thus obtained can easily be brought to diagonal form. The first of these facts is established by noting that H' does not give rise to charge renormalization: putting $g_1{}^0 = g_2{}^0 = g$ in equation (3.1) indeed gives $g_1 = g_2 = g$. Since in a renormalizable theory with infinite cut-off all renormalized propagators and vertex functions are finite functions of the renormalized coupling constants and the momenta involved **), these functions and the S-matrix will be the same for the hamiltonians H and H',

*) This symbol refers to equation (2.8) in the first of Ruijgrok's papers [3]). The latter will be referred to hereafter as I, II, III.

**) To second order these functions are given by (I.7.2), (I.7.3). A factor $(m - p^0)$ is missing in the last term of (I.7.2).

provided $\mu/K = 0$. The physical effects are therefore the same, although of course the actual structure of the dressed particles in terms of bare particles is different.

The diagonalization of H' is closely related to the symmetry law obeyed by the strong interactions. If $g_1{}^0 = g_2{}^0 = g$ the hamiltonian (2.3) is symmetrical in the internal states of the V-particle, *i.e.* invariant for the interchange of V_1 and V_2. It is therefore natural to express H' in terms of the "even" and "odd" internal states (see I p. 10)

$$V_+ = 2^{-1/2}(V_1 + V_2), \quad V_- = 2^{-1/2}(V_1 - V_2). \tag{3.5}$$

The coupling scheme (2.2) then reduces to

$$V_+ \rightleftarrows V_+ + \theta, \quad V_- \rightleftarrows V_- + \theta \tag{3.6}$$

with coupling constants g, $-g$ respectively. The field-theoretical problem thus obtained is essentially identical to the often discussed scalar theory with static sources and the hamiltonian can be completely diagonalized by a simple canonical transformation. The results are a Yukawa interaction between (physical) V-particles and a complete decoupling of the θ- and V-fields, i.e. no interaction between θ-particles and physical V-particles. For two physical V- particles, one V localized at a point $\boldsymbol{x}$ of space and one V' localized at $\boldsymbol{x}'$, one obtains an eigenstate of H' if the particles are in even or in odd states. The energy values for the four possible choices of internal states are, in obvious notation,

$$E(V_+V_+') = E(V_-V_-') = -U(r), \tag{3.7}$$

$$E(V_+V_-') = E(V_-V_+') = U(r), \tag{3.8}$$

$$U(r) = (g^2/4\pi r)\exp(-\mu r), \quad r = |\boldsymbol{x} - \boldsymbol{x}'|. \tag{3.9}$$

Other eigenstates of H' would be obtained by taking the combinations

$$V_+V_+' + V_-V_-' = V_1V_1' + V_2V_2', \quad V_+V_+' - V_-V_-' = V_1V_2' + V_2V_1', \tag{3.10}$$

$$V_+V_-' + V_-V_+' = V_1V_1' - V_2V_2', \quad V_-V_+' - V_+V_-' = V_1V_2' - V_2V_1'. \tag{3.11}$$

The even combinations (3.10) have energy $-U(r)$, the odd ones (3.11) have energy $U(r)$.

This completes the discussion of the "strong" interactions. To summarize, they consist of simple Yukawa interactions between V-particles, they leave V- and θ-particles uncoupled and they are invariant for $V_1 - V_2$ interchange (a symmetry property one might perhaps compare to charge symmetry, i.e. symmetry for neutron-proton interchange, in nuclear physics).

4. *Weak interaction effects.* Going over to the case of large but finite cut-off, we have for the renormalized coupling constants, from (3.1) and (3.3),

$$\left.\begin{aligned} g_1 &= g + \gamma \\ g_2 &= g - \gamma \end{aligned}\right\} \text{ with } \gamma = g \frac{g_1^0 - g_2^0}{g_1^0 + g_2^0} \left(\frac{\mu}{K}\right)^{g^2/2\pi^2}. \tag{4.1}$$

Obviously $\gamma \ll g$ under our assumption $\mu \ll K$. All effects whose strength is determined by γ are therefore called weak interaction effects. They are of two types, a coupling between θ- and physical V-particles, giving rise to scattering, and a modification of the $V-V$ interaction. Both types violate the symmetry for V_1-V_2 interchange obeyed by the strong interactions.

The weak interaction effects are cut-off dependent and cannot be calculated in closed form. Their qualitative properties are revealed, however, by explicitly renormalized perturbation formulae valid to low order in g_1, g_2. Such formulae have been derived in I, II and III. Thus the equation following (I.7.3) gives the S-matrix element for $\theta-V$ scattering. It is of order $g\gamma$ and is of opposite sign for the two internal states V_1, V_2 of the V-particle. This implies that, when expressed in terms of the "even" and "odd" states $V_\pm$ defined in (3.5), the scattering does not conserve the internal state of the heavy particle. To lowest order one actually has

$$V_+ + \theta \rightleftarrows V_- + \theta.$$

The symmetry law obeyed by the strong interactions is obviously violated in this weak process. As for the effect of the weak interaction on the $V-V$ coupling, it is exhibited in III where an improved form of perturbation theory has been used to find the stationary states of two V-particles in terms of physical particle states and renormalized coupling constants. These stationary states differ from (3.10), (3.11) by admixtures of order $g^2\gamma$ and the admixtures violate the parity of the state for V_1-V_2 interchange. Thus the admixtures to the even states (3.10) are odd to lowest order, and vice-versa for (3.11). As for the energy values they involve different corrections for all four states (3.10), (3.11). Explicit formulae need not be given here.

In conclusion, our model is seen to possess the qualitative features described in the introduction. These features, strong and weak interactions and approximate symmetry, are not manifestly present in the hamiltonian (2.3). Actually they hold only in the region of energies $E \ll K$, and a completely different physical situation would be found for collision processes with energies of the order of the cut-off. Still, if one only knows the behaviour of the system for $E \ll K$, one would be tempted to describe it by a hamiltonian containing the strong and weak interactions separately and having the V_1-V_2 symmetry in the strong interaction term. Needless to say, this guess would be quite misleading although its erroneous nature might manifest itself only in the high energy region. We of course do not know whether a comparable situation will present itself for the fundamental particles in nature, but we thought it worthwhile to show on a simple example the theoretical possibility of such a circumstance.

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