Magnetic string contribution to hadron dynamics in QCD

Yu. A. Simonov
Institute of Theoretical and Experimental Physics, Moscow, Russia

J. A. Tjon
Institute for Theoretical Physics, University of Utrecht, 3584 CC Utrecht, The Netherlands and KVI, University of Groningen, 9747 AA Groningen, The Netherlands

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The dynamics of a light quark in the field of a static source (heavy-light meson) is studied using the nonlinear Dirac equation, derived recently. Special attention is paid to the contribution of the magnetic correlators and it is found that it yields a significant increase of string tension at intermediate distances. The spectrum of heavy-light mesons is computed taking into account this contribution and compared to experimental and lattice data.

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I. INTRODUCTION

The nature of the QCD string between static charges (the static string) was studied extensively both analytically [1–3] and on the lattice [4–6]. It was shown in these papers that the static string is predominantly electric (the connected probing plaquette is used for the analysis) and the electric field is directed along the string axis.

In terms of the field correlator formalism (FCM) [7,8] the static string is made of the correlators of electric fields only, and a recent analysis in terms of Casimir scaling [9,10] shows that up to 1% accuracy the string is formed by the bilocal correlator of electric fields $D_{\parallel}^{E}(x,y) \equiv \langle (g^2/N_c) \mathrm{tr}\, E_{\parallel}(x) \Phi(x,y) E_{\parallel}(y) \rangle$ [where $\Phi(x,y)$ is the parallel transporter and $E_{\parallel}(x)$ is the component of the electric field parallel to the string axis]. Thus the confinement dynamics for heavy (static) quarks is defined by $D_{\parallel}^{E}(x,y)$ with this accuracy.

This fact was used to construct the effective Lagrangian for light quarks in the field of the static charge (the heavy-light quark-antiquark system) [11,12]. The analysis made in [11] has shown that using this Lagrangian one can derive in the large N_c limit the nonlinear and nonlocal Dirac equation for the light quark Green's function,

$$(-i\hat{\partial} - im - i\hat{M})S = \hat{1} \tag{1}$$

where the kernel $\hat{M} = M(x,y)$ is proportional to the bilocal (Gaussian) field correlator,

$$\left\langle \frac{g^2}{N_c} \mathrm{tr}\, F_{\mu\nu}(x) \Phi(x,y) F_{\lambda\sigma}(y) \right\rangle \equiv D_{\mu\nu,\lambda\sigma}(x,y),$$

and to the Green's function S.

It was shown in [11,12] that the scalar confinement occurs at large distances in the self-consistent solution of Eq. (1), signaling chiral symmetry breaking (CSB). The subsequent analysis in [13] has supported this conclusion and the spectrum of the heavy-light meson states was calculated together with first estimates of the chiral condensate.

In [13] it was assumed that the electric correlators are dominant in the string and that the magnetic part could be neglected. On the other hand, the analysis of the heavy quark mass case [i.e., of Eq. (1) with the replacement of $M(D,S)$ by $M(D,S_0)$ where S_0 is the free Green's function for the heavy mass m], done in [11], Appendix 5, and in [14,15], has shown that the magnetic correlators can also significantly contribute (at least in the regime when $mT_g \ll 1$, where T_g is the slope of $D_{\mu\nu,\lambda\sigma}$). It is a purpose of the present paper to study systematically the role of magnetic correlators for the light quark mass case, $m \ll \sqrt{\sigma}$, and to make a quantitative analysis in this case. As a by-product of our analysis the case of a heavy quark mass is reconsidered and some refinements of previous results are obtained.

II. MAGNETIC FIELD CONTRIBUTION TO THE CONFINING STRING

We study in this section the solution of Eq. (1) with the help of the relativistic WKB approach, similarly as in [11]. The kernel $\hat{M}$ (where only the bilocal field correlator is kept and the Gaussian form for it is assumed) can be written in the form

$$iM(h,\mathbf{x},\mathbf{y}) = \gamma_\mu S(x,y) \gamma_\nu J_{\mu\nu}(x,y), \tag{2}$$

where

$$J_{\mu\nu}(x,y) = \exp(-h^2/4T_g^2) J_{\mu\nu}(\mathbf{x},\mathbf{y}), \quad h = x_4 - y_4, \tag{3}$$

and

$$J_{44} \equiv J^{(E)}(\mathbf{x},\mathbf{y}) = \mathbf{xy} f_E(\mathbf{x},\mathbf{y}) \frac{\sigma}{2\pi T_g^2}, \tag{4}$$

$$J_{ik} \equiv J_{ik}^{(M)} = (\mathbf{xy}\delta_{ik} - y_i x_k) f_M(\mathbf{x},\mathbf{y}) \frac{\sigma}{2\pi T_g^2}. \tag{5}$$

Finally,

$$f_{E(M)}(\mathbf{x},\mathbf{y})=\int_0^1 ds\int_0^1 dt(st)^\alpha \exp\left(-\frac{(\mathbf{x}s-\mathbf{y}t)^2}{4T_g^2}\right), \quad (6)$$

where $\alpha=0$ for f_E and 1 for f_M.

In what follows only asymptotic values of $f_{E,M}$ will be of importance, with $|\mathbf{x}-\mathbf{y}|\ll|\mathbf{x}|,|\mathbf{y}|\gg T_g$, in which case one has

$$f_E(\mathbf{x},\mathbf{x})=3f_M(\mathbf{x},\mathbf{x})\cong\frac{2\sqrt{\pi}T_g}{|\mathbf{x}|}. \quad (7)$$

It should be noted at this point that subscripts and superscripts E and M refer to the correlators of color-electric and color-magnetic fields, respectively. Because of the structure of Eqs. (5), (6) for $\mathbf{x}\cong\mathbf{y}\to\infty$, the kernel is dominated by the color-electric field contribution. On the basis of this, in [13] the magnetic part of M was disregarded. However, in what follows we will show that for light quarks the magnetic part plays an important role at intermediate distances and as a consequence it can affect strongly the lower lying states of the heavy-light systems.

The kernel M, Eq. (2), contains the light quark Green's function S, which is a self-consistent solution of Eq. (1). Following Ref. [11] we can determine S assuming that M in the first approximation is instantaneous. In this case S has a spectral decomposition in terms of eigenvalues ε_n and eigenfunctions ψ_n,

$$\begin{aligned}S(h,\mathbf{x},\mathbf{y})&=i\left\{\sum_{n+} e^{-\varepsilon_n h}\theta(h)\psi_n(\mathbf{x})\psi_n^+(\mathbf{y})\right.\\&\left.-\sum_{n-} e^{-\varepsilon_n h}\theta(-h)\psi_n(\mathbf{x})\psi_n^+(\mathbf{y})\right\}\gamma_4\\&\equiv i\{\theta(h)S^{(+)}-\theta(-h)S^{(-)}\}\gamma_4\end{aligned} \quad (8)$$

and $\varepsilon_{n^+}=\varepsilon_n^{(+)}$ and $\varepsilon_{n^-}=-\varepsilon_n^{(-)}$ where Σ_{n+} and Σ_{n-} refer to sums over positive and negative eigenvalues, respectively.

Insertion of Eq. (8) into Eq. (2) then yields, for M,

$$\begin{aligned}M(h,\mathbf{x},\mathbf{y})&=-i\gamma_\mu S\gamma_\nu J_{\mu\nu}\\&=\theta(h)[S^{(+)}\gamma_4 J^{(E)}-J_{ik}^{(M)}\gamma_i S^{(+)}\gamma_4\gamma_k]\\&\quad-\theta(-h)[S^{(-)}\gamma_4 J^{(E)}-J_{ik}^{(M)}\gamma_i S^{(-)}\gamma_4\gamma_k].\end{aligned} \quad (9)$$

To find the properties of $\hat{M}$ we replace S by S_{lin} in Eq. (2), obtaining in this way M_{lin}, where S_{lin} is the quark Green's function of the linear Dirac equation (1) with the kernel $\hat{M}$ replaced by $\sigma|\mathbf{x}|\delta^{(4)}(x-y)$ (i.e., the static Dirac equation with linear potential σx). We now demonstrate that M_{lin} indeed tends to $\sigma|\mathbf{x}|$ at large distances, and thus yields the *a posteriori* proof that the large distance dynamics of the heavy-light quark system is governed by the linear static local confining potential. At the same time, in the framework of the same formalism, it will be shown that at an intermediate distance a region appears, where the dynamics is again local in time and static but with a larger string tension due to the contribution of magnetic terms. For $S^{(\pm)}$ the spherical spinor expansion has the form

$$S^{(\pm)}(h,\mathbf{x},\mathbf{y})=\sum_{(n^\pm)} e^{-\varepsilon_n h}\psi_n(\mathbf{x})\psi_n^+(\mathbf{y})=\sum_{n^\pm}\frac{e^{-\varepsilon_n h}}{xy}\begin{pmatrix}G_n(x)G_n^*(y)\Omega_{jlm}\Omega_{jlm}^*, & -iG_n(x)F_n^*(y)\Omega_{jlm}\Omega_{jl'm}^*\\ iF_n(x)G_n^*(y)\Omega_{jl'm}\Omega_{jlm}^*, & F_n(x)F_n^*(y)\Omega_{jl'm}\Omega_{jl'm}^*\end{pmatrix}. \quad (10)$$

Similarly, as in Ref. [11], we may carry out the summation over partial waves in Eq. (10) using the WKB approximation for the solutions G_n,F_n. As exploited in [11,21] the results for $S^{(-)}$ can simply be obtained from $S^{(+)}$ using the symmetry of ε_{n^+} and ε_{n^-} solutions, namely, $\varepsilon_n^{(+)}=\varepsilon_n^{(-)}\equiv\varepsilon_n$ and $(\varepsilon_n,G_n,F_n,\kappa)\leftrightarrow(-\varepsilon_n,F_n,G_n,-\kappa)$. We quote the final result of the WKB analysis for $S^{(\pm)}$, when $|\mathbf{x}-\mathbf{y}|\ll|\mathbf{x}|$:

$$S^{(\pm)}=\frac{\sigma e^{-\lambda}}{4\pi y}\delta(1-\cos\theta)\begin{pmatrix}\Delta_1\pm\Delta_0, & X\\ \bar{X} & \Delta_1\mp\Delta_0\end{pmatrix}, \quad (11)$$

where $\lambda=(m+\sigma x)|h|$. The matrix elements $X,\bar{X}$ contribute a nongrowing part to M and will be of no interest to us in what follows, while Δ_1,Δ_0 are defined as

$$\begin{aligned}&\frac{2}{\pi}\int_1^\infty d\tau e^{-\lambda(\tau-1)}\frac{\cos(a\sqrt{\tau^2-1})}{\sqrt{\tau^2-1}}\\&=\frac{2}{\pi}K_0(\sqrt{\lambda^2+a^2})e^\lambda\equiv\Delta_0(a)\end{aligned} \quad (12)$$

and

$$\begin{aligned}&\frac{2}{\pi}\int_1^\infty\frac{\tau d\tau e^{-\lambda(\tau-1)}}{\sqrt{\tau^2-1}}\cos(a\sqrt{\tau^2-1})\\&=\frac{2}{\pi}e^\lambda\frac{\lambda K_1(\sqrt{\lambda^2+a^2})}{\sqrt{\lambda^2+a^2}}\equiv\Delta_1(a).\end{aligned} \quad (13)$$

Here $a\cong(\sigma x+m)|x-y|$ for $\sigma x^2\gg1$ and $|\mathbf{x}-\mathbf{y}|\ll x$. From expressions (12) and (13) we see that Δ_0,Δ_1 are normalized as

$$\int_0^\infty \Delta_0(a)da=\int_0^\infty \Delta_1(a)da=1 \tag{14}$$

and hence diagonal elements of $S^{(\pm)}$ behave as smeared δ functions:

$$\int S^{(\pm)}(h,\mathbf{x},\mathbf{y})d^3y=\frac{1}{2}e^{-\lambda}\begin{pmatrix}1\pm1 & \\ & 1\mp1\end{pmatrix}. \tag{15}$$

Indeed for large σx the functions Δ_0,Δ_1 decrease exponentially fast when $|x-y|$ increases. Consider now Eq. (1),

$$\begin{aligned}(-i\gamma_\mu\partial_\mu-im)S(h,\mathbf{x},\mathbf{y})\\ -i\int e^{(h-h')^2/4T_g^2}dh'e^{-(\sigma x+m)|h-h'|}\\ \times\left\{\begin{pmatrix}\theta(h-h') & \\ & \theta(h'-h)\end{pmatrix}J^{(E)}(\mathbf{x},\mathbf{x})\right.\\ \left.+\begin{pmatrix}\theta(h'-h) & \\ & \theta(h-h')\end{pmatrix}J_{ik}^{(M)}\gamma_i\gamma_k\right\}S(h',\mathbf{x},\mathbf{y})\\ =\delta^{(4)}(x-y).\end{aligned} \tag{16}$$

In Eq. (16) the integration $\int M(\mathbf{x},\mathbf{z})S(\mathbf{z},\mathbf{y})d^3z$ has been carried out using Eq. (15). The equation obtained is a Dirac equation with a time-dependent interaction, localized in configurational space. We may now take into account that at large $x\gg T_g$ we have

$$J^{(E)}(\mathbf{x},\mathbf{x})=\frac{\sigma x}{\sqrt{\pi}T_g},\quad J_{ik}^{(M)}\gamma_i\gamma_k=\frac{2}{3}\frac{\sigma x}{\sqrt{\pi}T_g}. \tag{17}$$

There exist two regions in $\mathbf{x}$, where Eq. (16) can be simplified further. Considering the integration over dh' in Eq. (16), in the situation when $\sigma x^2\gg1$ and $T_g\to0$, we have the two possibilities

$$\text{(i)}\quad (m+\sigma x)T_g\ll1, \tag{18}$$

$$\text{(ii)}\quad (m+\sigma x)T_g\gg1. \tag{19}$$

In the first case, i.e., when Eq. (18) holds, the leading contribution to the integral over dh' comes from the region $|h'-h|\lesssim 2T_g$ because of the first factor in the integrand of Eq. (16). Since the remaining factors vary smoothly over this region, provided Eq. (18) is satisfied, we may replace $h'=h$ in these factors, including $S(h',\mathbf{x},\mathbf{y})$. In so doing we get

$$\left(-i\gamma_\mu\partial_\mu-im-\frac{i5\sigma}{3}x\right)S=\delta^{(4)}(x-y). \tag{20}$$

Note that corrections to the interaction have the form of a series in powers of $[1/(m+\sigma x)T_g]$, and can be neglected in the first approximation. For this case both color-electric and color-magnetic terms contribute to the kernel M.

Let us now turn to the second case, Eq. (19). Since $S(h',\mathbf{x},\mathbf{y})$ varies as $\exp[-(\sigma x+m)|h'|]$ [cf. Eq. (11)] and there is the factor $\exp[-(\sigma x+m)|h-h'|]$ in Eq. (16), it is essential that the above factors integrated out with $\theta(h-h')$ or $\theta(h'-h)$ in the expression (16). For the term with $\theta(h-h')$ we get $\exp[-(\sigma x+m)h]$ and the bounds of the integral are defined by T_g, whereas for the term with $\theta(h'-h)$ the integration over dh' yields a factor $1/(\sigma x+m)$, so that this contribution does not grow for large $\mathbf{x}$. As a consequence the color-magnetic term can be neglected in the case (19).

Consequently writing the $S(h,\mathbf{x},\mathbf{y})$ in the form

$$S(h,\mathbf{x},\mathbf{y})=ie^{-(\sigma x+m)|h|}g(\mathbf{x},\mathbf{y})\begin{pmatrix}\theta(h) & \\ & \theta(-h)\end{pmatrix}, \tag{21}$$

where $g(\mathbf{x},\mathbf{y})\approx\tilde{\delta}^{(3)}(\mathbf{x}-\mathbf{y})$ is a smeared δ function, one obtains an equation

$$(-i\gamma_\mu\partial_\mu-im-i\sigma x)S(h,\mathbf{x},\mathbf{y})=\delta^{(4)}(x-y), \tag{22}$$

where the entire interaction σx is due to the electric term $J^{(E)}(\mathbf{x},\mathbf{x})$.

In this way we have confirmed *a posteriori* that the solution S of Eq. (1) has at large distances the form of the Green's function for the linear potential, as given by Eq. (22), and hence our choice of $S=S_{lin}$ as the first approximation for the kernel M, Eq. (2), is justified.

Let us now discuss the regimes (18) and (19) in more detail. Consider first the case of a heavy quark mass,

$$mT_g\gg1. \tag{23}$$

In this case one automatically obtains the regime (19) and hence the linear potential as in Eq. (22). Since $T_g\sim1\ \mathrm{GeV}^{-1}$, only top and bottom quarks satisfy Eq. (23), while the charmed quark mass lies at the boundary. In the limit $m\to\infty$ the Green's function S_{lin}, Eqs. (8),(11), becomes the standard heavy-quark expression

$$\begin{aligned}S_{lin}\to S_0=\frac{ie^{-m|h|}}{2}\delta^{(3)}(\mathbf{x}-\mathbf{y})\{\theta(h)(1+\gamma_4)\\ +\theta(-h)(1-\gamma_4)\},\end{aligned} \tag{24}$$

which agrees with Refs. [11], [14], and [15]. However, one should not interpret this as corresponding to an admixture of scalar and vector confining pieces, i.e., $V_{mix}=\sigma x(\frac{5}{6}+\frac{1}{6}\gamma_4)$, since as was shown explicitly in [11] and also here, in view of the symmetry of the spectrum, in the limiting case $mT_g\gg1$ the potential has to be a pure scalar $V_{scalar}=\sigma x$, Eq. (22).

We now turn to the case $(m+\sigma x)T_g\ll1$. It is clear that this condition is valid only for a restricted region of x. However, for light quarks the self-consistent solution of Eq. (1) with the replacement (24) in the kernel $\hat{M}$ is not a good approximation for calculating the spectrum of the lower lying states, since Eq. (23) is violated. This conclusion agrees with the one found in [15]. For low mass states, where the effective region of interaction x_{eff} satisfies

$$(m+\sigma x_{eff})T_g\ll1, \tag{25}$$

TABLE I. Ground state energy eigenvalues $\varepsilon_n(j,l)$ in GeV, $n=0$, for two values of α_s and $\sigma=0.18$ GeV^2, $T_g=0.23$ fm, $m=0$.

	$\frac{1}{2}$,0	$\frac{1}{2}$,1	$\frac{3}{2}$,1	$\frac{3}{2}$,2	$\frac{5}{2}$,2	$\frac{5}{2}$,3	$\frac{7}{2}$,3
$\alpha_s=0$	0.520	0.817	0.732	0.934	0.911	1.147	1.070
$\alpha_s=0.3$	0.360	0.665	0.620	0.885	0.818	1.057	0.987

one should use Eq. (16) with the approximation valid for the regime $mT_g \ll 1$. It is a reasonable starting approximation for the whole region of x as long as we are interested in the spectrum, satisfying condition (25). It should be noted that in that case the controversy discussed in Ref. [15] for the replacement (24) does not take place. Moreover, the conclusion reached in Ref. [14], that the effective interaction has the form $V=\frac{5}{3}\sigma x$, applies only to the states for which x_{eff} satisfies Eq. (25), whereas for higher excited states inevitably another regime, Eq. (19), starts to apply with $V_{eff}=\sigma x$ instead of $\frac{5}{3}\sigma x$.

The region of validity of the magnetic string tension is important from the physical point of view, since the additional $\frac{2}{3}\sigma x$ originates from magnetic field correlators. From our analysis we clearly see that at asymptotic large $\mathbf{x}$, i.e., for very long (and therefore heavy) strings, the confining mechanism is purely electric. Moreover, the field contents are independent of the quark masses at the end of the string. It is only at intermediate regions that the magnetic contribution may play an important role. In particular, we have found that instead of regimes $mT_g \gg 1, mT_g \ll 1$ investigated in Refs. [11] and [14,15] one has the two regimes (18) and (19) where the total mass of the string plus quark mass enters, and the resulting confining force is linear, but with different strength. For heavy quarks with $mT_g \gg 1$ the regime (18) is essentially absent and we may safely use the color-electric confinement mechanism.

From the phenomenological point of view the lowest states of light quarks with the property (15) feel string tension $\frac{5}{3}\sigma$ and this may be important for the resulting masses of heavy-light systems, such as D, D_s, B, B_s, as will be demonstrated in the next section (a similar remark about D_s and B_s was made earlier in [14]).

Moreover, this increase of string tension resolves substantially (at least for the lowest levels) the descrepancy found for the Regge slopes of light mesons using relativistic quasipotential equations for particles with spin; cf. [17]. In particular, it was found in Ref. [16] that a larger string tension of $\sigma=0.33$ GeV^2 than the usually accepted value of 0.18 GeV^2 was needed to fit the experimental spectrum of the light mesons. This should be contrasted with the prediction $\alpha_1=1/8\sigma$ for the Regge slope, given by $J=\alpha_0+\alpha_1 M^2$, in the case of the spinless Salpeter equation [18], which is close to the nonrelativistic prediction. A similar result is found in the case of our nonlocal kernel for the light-heavy quark system. Considering the Regge trajectories as obtained in Ref. [13] and Table I of this paper we find for the case of the light-heavy quark system a value of $\alpha_1 \cong 1/\sigma$. This is to be compared with the spinless Salpeter prediction for this system, given by $\alpha_1=1/4\sigma$. The effect of the covariant treatment of the spin can already be seen when we use the linear Dirac equation with $V(x)=\sigma x+c_0$ for heavy-light mesons. In this case we get a slope of $1/2\sigma$ for $c_0=0$. Most of the difference between our results and those of the linear Dirac equation can be attributed to having at large distance an effective negative constant term c_0 in the linear potential present in the case of the nonlocal kernel [13], which leads to the shifting of the bound state masses. It should be noted that there is another mechanism [19,20] to decrease the Regge slope. Considering a rotating string [19] the Regge slope gets somewhat closer to the nonrelativistic result. One finds a value of $1/\pi\sigma$ in the case of the light-heavy quark system, corresponding to a physical half-string.

III. SPECTRUM OF HEAVY-LIGHT MESONS

The analysis of the previous section suggests that the lowest bound state solutions of Eq. (1) can be determined from the approximate instantaneous nonlocal Dirac equation of the form

$$(\boldsymbol{\alpha}\mathbf{p}+\beta m)\psi_n(\mathbf{x})+\beta\int \tilde{M}(\mathbf{x},\mathbf{z})\psi_n(\mathbf{z})d^3\mathbf{z}=\varepsilon_n\psi_n(\mathbf{x}). \tag{26}$$

Using the WKB solution for the Green's function S we find that for small T_g the kernel $\tilde{M}$ can be approximated by

$$\tilde{M}(\mathbf{x},\mathbf{z})=\Bigg[\sqrt{\pi}T_g J^{(E)}(\mathbf{x},\mathbf{z}) + \int dh'\,\theta(h')e^{-[(h')^2/4T_g^2]}e^{-2(\sigma x+m)h'}J_{ik}^{(M)}(\mathbf{x},\mathbf{z})\gamma_i\gamma_k\Bigg] \times\tilde{\delta}(\mathbf{x},\mathbf{z}), \tag{27}$$

where $\tilde{\delta}(\mathbf{x},\mathbf{z})$ is defined as

$$\tilde{\delta}(\mathbf{x},\mathbf{z})=\frac{\sigma}{4\pi z}\delta(1-\cos\theta_{xz})[\tilde{\Delta}_0(a)+\tilde{\Delta}_1(a)], \tag{28}$$

with $\tilde{\Delta}_0(a)$ and $\tilde{\Delta}_1(a)$ denoting the limiting values of $\Delta_0(a)$ and $\Delta_1(a)$, respectively, when $\lambda\equiv\sigma x h\sim\sigma x T_g$ tends to zero. For low lying states of the light quark system the spatial regions of interest are expected to satisfy the condition (18), i.e., $(m+\sigma x)T_g \ll 1$. Hence we get

$$\tilde{M}(\mathbf{x},\mathbf{z})=\sqrt{\pi}T_g\{J^{(E)}(\mathbf{x},\mathbf{z})+J_{ik}^{(M)}(\mathbf{x},\mathbf{z})\gamma_i\gamma_k\}\tilde{\delta}(\mathbf{x},\mathbf{z}). \tag{29}$$

It is seen in Eq. (28) that at large x and z the function $\tilde{\delta}(\mathbf{x},\mathbf{z})$ tends to $\delta^{(3)}(\mathbf{x}-\mathbf{z})$ and from Eq. (17) one immediately obtains

$$\tilde{M}(\mathbf{x},\mathbf{z})\cong\frac{5}{3}\sigma x\,\delta^{(3)}(\mathbf{x}-\mathbf{z}). \tag{30}$$

In what follows we shall solve Eq. (26) in the two cases (i) when $\tilde{M}(\mathbf{x},\mathbf{z})$ is replaced by its large distance limit (30) and (ii) when $\tilde{M}(\mathbf{x},\mathbf{z})$ is approximated by

TABLE II. The same as in Table I but for $m=0.15$ GeV.

	$\frac{1}{2}$,0	$\frac{1}{2}$,1	$\frac{3}{2}$,1	$\frac{3}{2}$,2	$\frac{5}{2}$,2	$\frac{5}{2}$,3	$\frac{7}{2}$,3
$\alpha_s=0$	0.623	0.898	0.832	1.078	1.010	1.233	1.168
$\alpha_s=0.3$	0.445	0.741	0.712	0.965	0.911	1.140	1.081

$$\widetilde{M}(\mathbf{x},\mathbf{z})=\widetilde{M}_1(\mathbf{x},\mathbf{z})=\sqrt{\pi}T_g\frac{5}{3}J^E(\mathbf{x},\mathbf{z})\frac{\sigma\delta(1-\cos\theta_{xz})}{\pi^2\sqrt{xz}}K_0(a). \tag{31}$$

The latter approximation is justified in the situation when $|\mathbf{x}-\mathbf{z}|\ll x$, which follows from the exponential damping of $K_0(a)$ when $|a|$ grows.

One can exploit the computations from [13] to obtain the eigenvalues $\varepsilon_n(j,l)$ of Eq. (26) with the kernel (31) for the lowest states, given in Table I for the case of the vanishing quark mass, $m=0$, in Table II for the mass $m=0.15$, and in Table III for $m=0.20$ GeV.

In what follows we shall first concentrate on the *s*- and *p*-state eigenvalues and compare them to the results of the QCD sum rules and lattice calculations. In the language of heavy quark effective theory one has the following expansion [22–25] for the heavy-light meson mass m_H:

$$m_h=m_Q\left[1+\frac{\overline{\Lambda}}{m_Q}+\frac{1}{2m_{Q^2}^2}(\lambda_1+d_H\lambda_2)+O\left(\frac{1}{m_q^3}\right)\right], \tag{32}$$

where $\overline{\Lambda}(n,j,l)=\varepsilon_n(j,l)$. Using $\sigma=0.18$ GeV^2, the solution of Eq. (26) with the kernel (31), yields the *S*-wave eigenvalue $\overline{\Lambda}(0,\frac{1}{2},0)\equiv\overline{\Lambda}_S$:

$$\overline{\Lambda}_S=0.520\ \text{GeV}\quad\text{for}\quad\alpha_s=0,$$

$$\overline{\Lambda}_S=0.360\ \text{GeV}\quad\text{for}\quad\alpha_s=0.3, \tag{33}$$

which are about a factor of $\sqrt{\frac{3}{5}}$ smaller in the absence of the magnetic contribution. The predicted values (33) should be compared with the results of the QCD heavy flavor sum rules [22–25], $\overline{\Lambda}_S=0.57\pm0.07$ GeV, and the result of the analysis from semileptonic *B* decays [26], $\overline{\Lambda}_S=0.39\pm0.11$ GeV. A similar value was obtained recently from the QCD sum rules [27]:

$$\overline{\Lambda}_S=0.45\pm0.15\ \text{GeV}. \tag{34}$$

One can see a reasonable agreement of our results with the latest sum rule calculations (34). Note here that we have taken into account the color Coulomb interaction to all orders, whereas in the QCD sum rules only the leading order term is retained; therefore one may expect that higher orders will decrease somewhat the value (34).

TABLE III. The same as in Table I but for $m=0.20$ GeV.

	$\frac{1}{2}$,0	$\frac{1}{2}$,1	$\frac{3}{2}$,1	$\frac{3}{2}$,2	$\frac{5}{2}$,2	$\frac{5}{2}$,3	$\frac{7}{2}$,3
$\alpha_s=0$	0.659	0.927	0.867	1.107	1.044	1.263	1.202
$\alpha_s=0.3$	0.476	0.769	0.744	0.994	0.944	1.169	1.114

TABLE IV. Energy eigenvalues $\overline{\Lambda}_S$ of the heavy-light system in the static heavy quark approximation obtained in different approaches.

Refs.	Method	$\overline{\Lambda}_S$ (GeV)
[24]	QCD sum rules	0.5
[25]	QCD sum rules	0.4–0.5
[27]	QCD sum rules	0.45±0.15
[26]	Experiment	0.39±0.11
[13]	Nonlin. Dirac	0.287
this work	Nonlin.+magnetic	0.360

In a similar way one may compute energy eigenvalues for the strange heavy-light mesons. From Tables II and III we see that with $\alpha=0.3$ we have

$$\overline{\Lambda}_S^{(s)}=0.445\ \text{GeV}\ \ \text{for}\ \ m=0.15\ \text{GeV},$$

$$\overline{\Lambda}_S^{(s)}=0.476\ \text{GeV}\ \ \text{for}\ \ m=0.20\ \text{GeV}. \tag{35}$$

These numbers can be compared to the values from the experimental B_s and D_s masses. We find

$$\Delta M_s^{(B)}=M_{B_s}-M_B\cong 90\ \text{MeV},$$

$$\Delta M_s^{(D)}=M_{D_s}-M_D\cong 100\ \text{MeV}. \tag{36}$$

Similar values are found from the spectrum of heavy-light mesons, computed recently on the lattice [28] for strange mesons. From Eq. (35) we see that the experimental data are close to our predicted value of $\Delta M_s=\Lambda_S^{(s)}-\Lambda_S=85$ MeV for $m=0.15$ GeV. The various available data on Λ_S are summarized in Table IV.

We turn now to orbital and radial excitations. For the states with $l=1$ and $j=\frac{3}{2}$ and $\frac{1}{2}$ the mass splitting is due to the spin-orbit interaction inherent in the Dirac equation. Denoting these energies as $\varepsilon_n(j,1)\equiv\overline{\Lambda}_P(j)$ we find, for the nonstrange quark (in GeV),

$$\overline{\Lambda}_P\left(\frac{1}{2}\right)=0.817,\quad \overline{\Lambda}_P\left(\frac{3}{2}\right)=0.732\quad(\alpha_s=0), \tag{37}$$

$$\overline{\Lambda}_P\left(\frac{1}{2}\right)=0.665,\quad \overline{\Lambda}_P\left(\frac{3}{2}\right)=0.620\quad(\alpha_s=0.3). \tag{38}$$

Similarly for strange mesons with a strange quark mass $m=0.15$ GeV, we obtain

$$\overline{\Lambda}_P^{(s)}\left(\frac{1}{2}\right)=0.898,\quad \overline{\Lambda}_P^{(s)}\left(\frac{3}{2}\right)=0.832\quad(\alpha_s=0), \tag{39}$$

$$\overline{\Lambda}_P^{(s)}\left(\frac{1}{2}\right)=0.741,\quad \overline{\Lambda}_P\left(\frac{3}{2}\right)=0.712\quad(\alpha_s=0.3). \tag{40}$$

TABLE V. The same as in Table IV but for $\bar{\Lambda}_P - \bar{\Lambda}_s$.

Refs.	Method	$\bar{\Lambda}_P - \bar{\Lambda}_S$ (GeV)
[27]	QCD sum rules	0.55 ± 0.35
[31]	QCD string	0.40
[28]	Lattice	0.47
PDG	Experiment	0.383
This work	Nonlin.+magnetic	0.305 $[\bar{\Lambda}_P(\frac{1}{2}) - \bar{\Lambda}_S]$

These calculations can be compared with the results of lattice calculations in [28], with experiment and the recent QCD sum rule calculations [27]. The latter yield, for $m=0$,

$$\bar{\Lambda}_P = (1 \pm 0.2) \ \ \text{GeV}. \tag{41}$$

This value is somewhat higher than the results (37) and (38). Lattice calculations in [28] give for the difference $M(B_J^*) - M(B) \simeq \bar{\Lambda}_P - \bar{\Lambda}_S \approx 456$ MeV, which should be compared with our results, $\Delta\bar{\Lambda} \equiv \bar{\Lambda}_P(\frac{1}{2}) - \bar{\Lambda}_S \simeq 305$ MeV for $\alpha_s = 0.3$, and with experiment, $M(B_J^*) - M(B) \simeq 338$ MeV. Here $M(B) = \frac{3}{4}M_B(1^-) + \frac{1}{4}M_B(0^-)$. In addition there is a calculation of heavy-light mesons in the framework of the QCD string approach [29], where the only input is current quark masses (m_n, m_d, m_s), string tension σ, and α_s. These results have been obtained in [30] and recently in [31] for real B, B_s, D, D_s mesons and are easily computed for the limiting case of $m_q \to \infty$, which yields values listed in Table V. The rather low value found for $\Delta\bar{\Lambda}$ suggests that we still miss some strength in the orbital excitation in the present work. In Table V the results of different approaches to $\bar{\Lambda}_P$ and $\bar{\Lambda}_P^{(s)}$ are collected.

Radial excitations are readily obtained from solving Eqs. (26), (31) and yield, for the $n=1$ state,

$$\varepsilon_1\left(\frac{1}{2},0\right) = 0.951 \quad (\alpha_s = 0), \quad 0.805 \quad (\alpha_s = 0.3) \tag{42}$$

and, for the strange meson with $m = 150$ MeV,

$$\varepsilon_1^{(s)}\left(\frac{1}{2},0\right) = 1.036 \quad (\alpha_s = 0), \quad 0.880 \quad (\alpha_s = 0.3), \tag{43}$$

while for the radial excitation with $l=1$ one obtains

TABLE VI. The same as in Table IV but for the radial excitation $\Lambda_S' - \Lambda_S$.

Refs.	Method	$\Lambda_S' - \Lambda_S$ (GeV)
[32]	experiment, $M_{B*} - M_B$	0.581
[28]	Lattice, $M_{B*} - M_B$	0.602
[31]	QCD string	0.564
This work	Nonlin.+magn.	0.631

$$\varepsilon_1(j,1) = 1.140 \quad (\alpha_s = 0), \quad 0.997 \quad (\alpha_s = 0.3), \tag{44}$$

$$\varepsilon_1^{(s)}(j,1) = 1.221 \quad (\alpha_s = 0), \quad 1.076 \quad (\alpha_s = 0.3). \tag{45}$$

These values are compared in Table VI with the results of the QCD string approach and recent lattice calculations [28].

IV. SUMMARY AND CONCLUSIONS

The main results of the present paper are of both theoretical and phenomenological scope. On the theoretical side it is shown in Sec. II that there exist two possible dynamical regimes for the quark at the end of the Dirac string, namely, Eqs. (18) and (19). For the case of a heavy quark with $mT_g \ll 1$ only one regime (19) is available and results are the same as discussed in [11,14,15]; i.e., the color-magnetic contribution to the string is suppressed in this case and one has at large x a local Dirac equation with linear potential.

For the case of a light quark there is the possibility of another regime (18), where the color-magnetic field also contributes. It is shown that this regime operates at intermediate distances and yields a static Dirac equation with increased string tension. It is demonstrated also that the use in this case in the kernel of the nonlinear Dirac equation of the full quark propagator S (or its WKB approximation S_{WKB}) leads to consistent results, while the use of the heavy mass propagator S_0 leads to the inconsistencies shown in [15]. On the phenomenological side the regime (18) yields energy eigenvalues which are in better agreement with other calculations and experiment, as demonstrated in Tables IV–VI, as compared to our previous results [13] where the color-magnetic contribution has been neglected.

The study of the role of color-magnetic fields in the dynamics of light quarks is at its beginning and the first results call for more detailed investigations of the transition between regimes (18) and (19) and other applications, e.g., to mesons and baryons consisting of light and heavy quarks.

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