Confinement and the analytic structure of the one-body propagator in scalar QED

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We investigate the behavior of the one-body propagator in SQED. The self-energy is calculated using three different methods: (i) the simple bubble summation, (ii) the Dyson-Schwinger equation, and (iii) the Feynman-Schwinger representation. The Feynman-Schwinger representation allows an *exact* analytical result. It is shown that, while the exact result produces a real mass pole for all couplings, the bubble sum and the Dyson-Schwinger approach in the rainbow approximation lead to complex mass poles beyond a certain critical coupling. The model exhibits confinement, yet the exact solution still has one-body propagators with *real* mass poles.

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I. INTRODUCTION

The nature and implications of particle confinement remain one of the mysteries of QCD. It is clear that confinement implies that quarks and antiquarks cannot be separated from each other at large distances (as demonstrated by lattice calculations [1,2]). An essential consequence of this is that a bound state cannot decay into its constituent quarks even if the decay is kinematically allowed. Such a decay will certainly be prevented if the dressed quark propagators cannot have any real mass poles. This possibility has been investigated, and often implicitly assumed, within the context of Dyson-Schwinger equations [3–8]. However, this condition is not necessary; an alternative point of view is that confinement is not due to the lack of mass poles but through the exchange interaction between the constituents forming the bound state [9–11]. In Ref. [11], the authors have shown that a relativistic generalization of the nonrelativistic linear interaction leads to a bound state vertex function that vanishes when both particles are on shell. According to this result, the correct nonrelativistic limit favors the vanishing of the vertex function when both particles are on-shell, rather than the lack of physical mass poles.

Clearly the structure of the one-body propagator deserves a closer look and a more rigorous understanding is needed to clarify what the Dyson-Schwinger results mean. In this article we study the one-body propagator in the context of massive scalar QED in 0+1 dimensions [12]. The simplicity of this toy model field theory allows one to obtain an analytical solution for the dressed mass by using the Feynman-Schwinger representation (FSR) [12–16]. The FSR is an approach based on Euclidean path integrals similar to lattice gauge theory. In this approach the path integrals over quantum fields are integrated out at the expense of introducing path integrals over the trajectories of the particles. The FSR approach sums up all possible interactions including the ones with ''quark'' loops. Therefore the FSR approach provides us the means to test and understand how much of the physics is included in the Dyson-Schwinger equation with the rainbow approximation. The rainbow approximation corresponds to using bare interaction vertices and a bare exchange field propagator. At the other extreme, one may consider the dressed mass as obtained by a simple bubble summation. This method sums fewer diagrams than the other two. In Fig. 1 the typical diagrams involved in all three approaches are displayed. In the next section we briefly discuss how the dressed mass is obtained in each one of the three methods mentioned above, and how the results compare with each other.

II. SCALAR QED

Massive scalar QED in 0+1 dimensions is a simple interaction that enables one to obtain a fully analytical result for the dressed and bound state masses within the FSR approach.

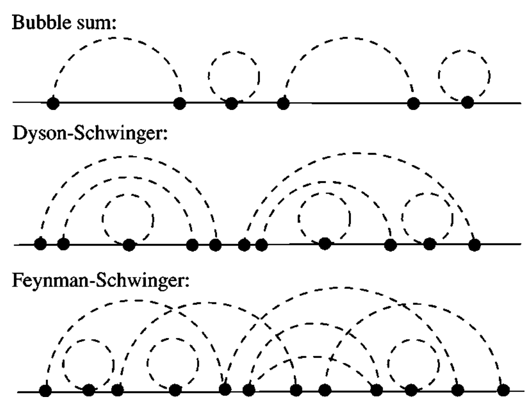

FIG. 1. Various interactions included in each approach are shown. The Feynman-Schwinger approach includes all diagrams. In one-dimension the contribution of diagrams with loops of charged particles identically vanishes [Eq. (2.14)].

In this section we compare the self-energy result obtained by three different approaches; namely, the simple bubble sum, the Dyson-Schwinger equation, and the Feynman-Schwinger representation. The Minkowski metric expression for the scalar QED (SQED) Lagrangian in Feynman gauge is given by

$$\mathcal{L}_{\rm SQED} = -m^2\chi^2 - \tfrac{1}{4}F^2 + \tfrac{1}{2}\ \mu^2 A^2 - \tfrac{1}{2}(\partial A)^2 + (\partial_\mu - ieA_\mu)\chi^*(\partial^\mu + ieA^\mu)\chi, \tag{2.1}$$

where A represents the gauge field of mass μ and χ is the charged field of mass m. The field tensor F is zero in $0+1$ dimensions, and the dynamics is described by the gauge fixing term $(\partial A)^2$. The presence of a mass term for the exchange field breaks the gauge invariance. Here the mass term was introduced in order to avoid infrared singularities which are present in $0+1$ dimension. For dimensions larger than $n=2$ the infrared singularity does not exist and therefore the limit $\mu \to 0$ can be safely taken to restore the gauge invariance.

A. The bubble sum

The bubble sum is the simplest subset of all diagrams contributing to the self-energy. The Euclidean expression for self-energy in $0+1$ dimensions is given by

$$\Sigma(p) = -e^2\int_{-\infty}^{\infty}\frac{dk}{2\pi}\frac{(2p-k)^2}{(k^2+\mu^2)[(p-k)^2+m^2]} + e^2\int_{\infty}^{\infty}\frac{dk}{2\pi}\frac{1}{k^2+\mu^2}. \tag{2.2}$$

The dressed propagator corresponding to this self-energy is

$$\Delta_d(p) = \frac{1}{p^2+m^2+\Sigma_E(p)}. \tag{2.3}$$

The dependence of M on the coupling strength e can be obtained from the solution of the on-shell condition

$$M = \sqrt{m^2+\Sigma_E(iM)}, \tag{2.4}$$

which must be real if the dressed mass is to be stable. Therefore, for massive SQED, the equation determining the dressed mass takes the following form:

$$M^2 = m^2 + \frac{e^2}{2}\times\left[\frac{(\mu-2M)^2}{\mu[m^2-(\mu-M)^2]} + \frac{(m-M)^2}{m[\mu^2-(m+M)^2]} + \frac{1}{\mu}\right]. \tag{2.5}$$

B. The Dyson-Schwinger equation

The Dyson-Schwinger equation is usually solved in the rainbow approximation. This is due to the fact that a completely self-consistent determination of the interaction vertex is impossible. The one-body Dyson-Schwinger equation in the rainbow approximation is given by

$$m^2(p) = m^2 - e^2\int_{-\infty}^{\infty}\frac{dk}{2\pi}\frac{(2p-k)^2}{(k^2+\mu^2)[(p-k)^2+m^2(k)]} + e^2\int_{\infty}^{\infty}\frac{dk}{2\pi}\frac{1}{k^2+\mu^2}. \tag{2.6}$$

The structure of this equation is very similar to the earlier bubble sum expression Eq. (2.2). The main difference is the momentum dependence of the dressed mass. The coordinate space form of the dressed propagator is

$$\Delta_d(t) = \int_{-\infty}^{\infty}\frac{dp}{2\pi}\left(\frac{e^{ipt}}{p^2+m^2(p)}\right) \simeq N e^{-Mt}. \tag{2.7}$$

Therefore the ground state mass pole of the one-body propagator can be extracted using

$$M = -\lim_{T\to\infty}\frac{d}{dT}\log[\Delta_d(t)]. \tag{2.8}$$

C. The Feynman-Schwinger representation

In the FSR approach the field theoretical path integral expression for the one-body propagator is transformed into a quantum mechanical path integral over trajectories of the particles [13,15]. The FSR expression for the one body propagator is given by

$$G(0,T) = \int ds\int(\mathcal{D}z)_{0T}\,\exp[iK[z,s]-V[z]], \tag{2.9}$$

where

$$K[z,s] = (m^2+i\epsilon)s - \frac{1}{4s}\int_0^1 d\tau\,\dot{z}^2(\tau), \tag{2.10}$$

$$V[z] = \frac{e^2}{2}\int_0^1 d\tau\,\dot{z}(\tau)\int_0^1 d\tau'\,\dot{z}(\tau')\times\Delta[z(\tau)-z(\tau')], \tag{2.11}$$

$$\Delta(z) = \int\frac{dp}{2\pi}\frac{e^{ipx}}{p^2+\mu^2} = \frac{e^{-\mu|z|}}{2\mu}, \tag{2.12}$$

where $\Delta(z)$ is the interaction kernel, and the boundary conditions are chosen to be $z(0)=0$, and $z(1)=T$. $K[z,s]$ represents the mass term and the kinetic term, and $V[z,s]$ is the interaction term. Because of the simplicity of working in one dimension, the integral of the self-interaction Eq. (2.11) can be done analytically:

$$V[z] = \frac{e^2T}{2\mu^2}\left[1 - \frac{1-e^{-\mu T}}{\mu T}\right]. \tag{2.13}$$

In higher dimensions the result of this integral depends on the trajectory of the particle. However in one dimension all trajectories contribute equally, which is what makes the one-dimensional calculation analytically doable. In addition, the contribution of loops, which has been omitted in Eq. (2.9), can be shown to identically vanish in one dimension. The typical loop contribution in one dimension has the following form:

$$\int_0^1 d\tau\, \dot{z}(\tau) \oint d\tau'\, \dot{z}(\tau')\Delta[z(\tau)-z(\tau')]$$
$$= \int_0^T dz \left(\int_{z_i}^{z_f} dz' + \int_{z_f}^{z_i} dz' \right) \Delta(z-z') = 0. \tag{2.14}$$

Therefore in one dimension matter loops do not contribute and the FSR results provided here are exact. Next, the path integral over z can be evaluated after a discretization in proper time. Since the only path dependence in the propagator is in the kinetic term, the path integral over z involves Gaussian integrals which can be performed easily by using the following discretization:

$$(\mathcal{D})_{0T} \rightarrow (N/4\pi s)^{N/2} \Pi_{i=1}^{N-1} \int dz_i\,. \tag{2.15}$$

The s integral can be evaluated by the saddle point method giving

$$G(0,T) = N \exp\left[-mT - e^2 \frac{T}{2\mu^2} + \frac{e^2}{2\mu^3}(1-e^{-\mu T}) \right]. \tag{2.16}$$

This is an exact result for large times T. The dressed mass can easily be obtained by taking the logarithmic derivative of this expression. Therefore, the one-body dressed mass for SQED in 0+1 dimension according to the FSR formalism is given by

$$M = m + \frac{e^2}{2\mu^2}. \tag{2.17}$$

Having outlined the calculation of the dressed mass in three different approaches, we next compare the results obtained by these methods.

III. DISCUSSIONS AND CONCLUSIONS

The kind of diagrams included in each method discussed above is displayed in Fig. 1. The main difference between the Dyson-Schwinger and the Feynman-Schwinger diagrams is the crossed diagrams. These diagrams involve photon lines that cross each other. The FSR approach also includes all possible four-point interaction contributions while the rainbow DSE only includes the tadpole type four-point interactions. In principle all four-point interactions can also be incorporated into the simple bubble sum and the rainbow DSE.

In Fig. 2 we display all dressed mass results. The bubble summation develops a complex mass pole beyond a critical coupling $e^2_{\text{crit}} = 0.4$ (GeV)^3. At the critical point a ''collision'' takes place with another real solution of Eq. (2.5), leading to two complex conjugated solutions with increasing e^2. This happens at $M = 1.45$ GeV. It is interesting to note that the result obtained from the Dyson-Schwinger equation displays a similar characteristic. At low coupling strengths the rainbow DS and the bubble results are very close and they converge to the exact result given by the Feynman-Schwinger approach. As the coupling strength is increased the DS result maintains a closer distance to the bubble result rather than the FSR result. Similar to the bubble result the DS result develops a complex mass pole at a critical coupling of $e^2_{\text{crit}} = 0.49$ (GeV)^3.

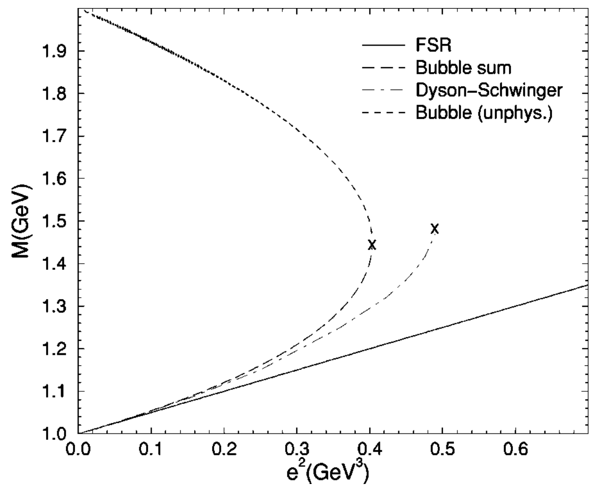

FIG. 2. The function $M(e^2)$ calculated by the FSR approach, the Dyson-Schwinger equation, and the bubble summation for values of $m = \mu = 1$ GeV. According to the bubble sum there is a critical point at $e^2_{\text{crit}} = 0.4$ (GeV)^3 beyond which the dressed mass becomes complex. A similar result happens for the DSE. The FSR result is real for all couplings.

There are two important observations to be made from these results. (i) The dynamical generation of complex mass poles in the rainbow DS and bubble approaches is not an indication of confinement (see also Ref. [17]). These complex masses occur at large couplings, when it might appear that some sort of confining phase transition has taken place, but since the exact FSR answer shows no such behavior we are forced to conclude that these complex poles occur simply because the subset of the possible interaction diagrams included in these approaches is insufficient to qualitatively reproduce the correct result. (ii) The nature of the rainbow DS result is closer to the bubble sum than the exact FSR result.

Further insight follows from examination of the masses of two-body bound states. The simplicity of SQED in 0+1 dimensions also allows one to get an analytical result for the two-body bound state mass. The total result is

$$M_b = \left(m + \frac{e^2}{2\mu^2} \right) + \left(m + \frac{e^2}{2\mu^2} \right) - \frac{e^2}{\mu^2} = 2m, \tag{3.1}$$

where the first two terms are the dressed one-body contributions and the last term is the contribution from the exchange interaction [12]. Hence there is *only one* two-body bound state, and the *continuum spectrum does not exist*. In light of the fact that we are working in only 1 time dimension the lack of a continuum is not surprising, since the particles cannot move. In 0+1 time dimensions one would naturally expect confinement simply because there is no room for quarks to break free. However it is still interesting to note that confinement, which is unavoidable in 0+1 dimension, is a basic property of the four-point function and it does not imply the lack of physical mass poles in the one body propagator. This feature is similar to that used in Refs. [10,11]. Moreover, in QCD it would clearly also be more appropriate to discuss confinement in the color-white sectors, e.g., for the $q\bar{q}$ system, instead of for the single quark propagator.

The dynamical mass generation and binding contained in Eq. (3.1) is quantitatively similar to that of the generation of massless Goldstone bosons in QCD. In particular it is known that the pion mass m_π is proportional to the current quark mass m_u:

$$m_\pi^2 = m_u \frac{\langle \bar{\Psi}\Psi \rangle}{f_\pi}, \tag{3.2}$$

where $\langle \bar{\Psi}\Psi \rangle$ is the quark condensate and f_π is the pion decay constant. This is similar to the result found in Eq. (3.1). In SQED the positive shifts of one-body masses are exactly compensated by the negative binding energy created by the exchange interaction. Therefore the total bound state mass is exactly equal to the sum of *bare* masses, and the bound state mass vanishes as the current particle mass vanishes. However in scalar QED particles do not carry spin. Therefore the similarity to the dynamical chiral symmetry breaking of QCD is only accidental.

Finally, the FSR formalism allows us to make the following observation about the significance of the vertex dressings of the interaction: If one starts with dressed masses given in Eq. (2.17) and uses only the exchange interaction to calculate the bound state masses, the resultant bound state mass would have been the same. This means that the vertex contributions do not change the bound state energy. This type of prediction underlines the potential usefulness of the FSR calculations. In principle, in addition to being a rigorous and powerful tool for calculation of the nonperturbative propagators, the FSR approach can also provide much needed information about the role of various vertices and propagators. This information would be useful as input in other nonperturbative approaches such as the Dyson-Schwinger equations.

This paper focused on a simple toy model, namely, the SQED in 0+1 dimensions. Through this simple model we have been able to compare various nonperturbative methods. It would be interesting to see whether insights provided by this simple model could be extended to higher dimensions. We have in particular shown that the exact solution is confining and yet the one-body propagator has real mass poles. Whether this scenario is realized in 3+1 dimensions is an interesting and important question.

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