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Spin ordering and quasiparticles in spin-triplet superconducting liquids

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Spin ordering and its effect on low-energy quasiparticles in a *p*-wave superconducting liquid are investigated. We study the properties of a two-dimensional (2D) *p*-wave superconducting liquid where the ground state is spin rotation invariant. In quantum spin disordered liquids, the low-energy quasiparticles are bound states of the bare Bogolubov–De Gennes (BdeG) quasiparticles and zero energy skyrmions, which are charge neutral bosons at the low-energy limit. Further more, spin collective excitations are fractionalized ones carrying a half-spin and obeying fermionic statistics. In thermally spin disordered limits, the quasiparticles are bound states of bare BdeG quasiparticles. The latter situation can be realized in some layered *p*-wave superconductors where the spin-orbit coupling is weak.

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Our fascination towards excitations carrying quantum numbers distinct from electrons in condensed-matter systems dated back at least 20 years ago.[1–4] It is now widely accepted that there are a variety of strongly interacting systems where quasiparticles carry only a fraction of quantum numbers an electron has. All known examples, though have been discovered in very diversified condensed-matter systems that bear no similarities at first sight, appear to share one remarkable common feature. It is topological excitations interacting with electrons one way or the other which make all sorts of exotic quasiparticles or collective excitations possible. This also lies in the heart of earlier examples discovered in field theories and mathematical physics.[5–7]

In this communication, we scrutinize the spin ordering and its influences on excitations, particularly, on the fractionalization of quasiparticles and collective spin excitations in 2D spin-triplet *p*-wave superconductors. Spin-triplet superconducting states are believed to exist in ^{3}He, many heavy-fermion superconductors and most recently layer perovskite Sr_2RuO_4 crystals.[8–11] We report the existence of a 2D two-dimensional spin-triplet *p*-wave superconducting state that is spin rotation invariant. This new state is characterized by a finite range spin correlation and $hc/4e$ vortices as the elementary topological excitations. The elementary quasiparticles are Bogolubov–De Gennes (BdeG) quasiparticles hosted in zero energy skyrmions. The spin collective excitations are shown to be fractionalized ones carrying a half-spin and obeying fermionic statistics, by contrast to the spin-wave excitations in spin ordered *p*-wave superconducting states (SOpSSs). We should mention that the general fractionalization pattern in some *p*-wave superconductors was recently classified in Ref. 12.

For a *p*-wave superconductor with an order parameter $\mathbf{d}(\mathbf{k})=\Delta_0(k_x+ik_y)\exp(i\chi)\mathbf{n}$, the Hamiltonian[8,9] in the Nambu space of $\Psi=(\psi^+,i\tau_2\psi)$ can be written as

$$H=\sigma_3\epsilon+\sum_{i=x,y}\sigma_i\{\partial_i,\hat{\Delta}\}_+,\tag{1}$$

where $\hat{\Delta}$ is defined as $\hat{\Delta}=\Delta_0\exp(i\sigma_3\chi)(\mathbf{n}\cdot\tau)$ and $\epsilon(\mathbf{k})=\hbar^2\mathbf{k}^2/2m-\epsilon_F$. We use σ as the Nambu space Pauli matrix and τ as the spin space one. We assume the spin-orbit interactions are weak and $\mathbf{n}$ is a unit vector in a sphere S^2. The internal space of the symmetry broken state is $\mathcal{R}=[S^1\times S^2]/Z_2$. The order parameter observes a discrete symmetry: $\hat{\Delta}(\mathbf{n},\chi)\rightarrow\hat{\Delta}(-\mathbf{n},\chi+\pi)$ and represents a quantum spin nematic *p*-wave superconducting state.

Spin-phase separation. To obtain an effective theory, we integrate over the fermionic degrees of freedom and make a gradient expansion.[13] At low temperatures, we report the result as

$$\mathcal{L}=\mathcal{S}_{ab}\mathbf{P}_{sa}\mathbf{P}_{sb}-\mathcal{T}_0\Phi^2+\frac{1}{8\pi}(\mathbf{E}^2+\mathbf{B}^2)+\mathcal{S}_{ab}^{\alpha\beta}\nabla_a\mathbf{n}_\alpha\nabla_b\mathbf{n}_\beta$$
$$-\mathcal{T}^{\alpha\beta}\partial_t\mathbf{n}_\alpha\partial_t\mathbf{n}_\beta+\frac{\mathcal{N}}{4\pi}\epsilon^{\mu\nu\lambda}\mathbf{A}_\mu\mathbf{F}_{\nu\lambda}.\tag{2}$$

$\mathbf{P}_s=\frac{1}{2}\nabla\chi+e\mathbf{A}^{em}$ and $\Phi=\frac{1}{2}\partial_t\chi+e\phi^{em}$ are the gauge invariant momentum and potentials, respectively. The superscript *em* is introduced to distinguish the usual electric magnetic vector potential $\mathbf{A}^{em}$ from the topological field $\mathbf{A}$ defined below in terms of $\mathbf{n}$. $\mathcal{T}_0\nu_0^{-1}$ is a unity at $T=0$ and varies smoothly as a function of the temperature, ν_0 is the averaged density of states at the fermi surface. $\mathbf{F}_{\mu\nu}=\frac{1}{2}\mathbf{n}\cdot\partial_\mu\mathbf{n}\times\partial_\nu\mathbf{n}$, and $\mathbf{A}$ is the vector potential of $\mathbf{F}_{\mu\nu}$. $\mathcal{S}_{ab}^{\alpha\beta}=\delta^{\alpha\beta}\mathcal{S}_{ab}$, $\mathcal{S}_{ab}=\rho_0/2m\tilde{\mathcal{S}}_{ab}$; and $\mathcal{T}^{\alpha\beta}=\delta^{\alpha\beta}\nu_0\tilde{\mathcal{T}}$. Finally, $\tilde{\mathcal{S}}_{ab}$, $\tilde{\mathcal{T}}$, and $\mathcal{N}$ are calculated as

$$\tilde{\mathcal{S}}_{ab}=\frac{2m}{\rho_0}\int\frac{d^2k}{(2\pi)^2}\mathbf{v}_a\mathbf{v}_bF(\mathbf{k}),\quad\tilde{\mathcal{T}}=\frac{1}{\nu_0}\int\frac{d^2k}{(2\pi)^2}F(\mathbf{k});$$

$$\mathcal{N}=\int\frac{d^2k}{4\pi}\epsilon^{\alpha\beta\gamma}\frac{\mathbf{M}_\alpha}{\Delta_0^2}\frac{\mathbf{M}_\beta}{\partial k_x}\frac{\mathbf{M}_\gamma}{\partial k_y}F(\mathbf{k});$$

$$F(\mathbf{k},T)=-\frac{\partial}{\partial\epsilon^2(\mathbf{k})}\frac{2\Delta_0^2(T)}{E(\mathbf{k})}\tan\left(\frac{E(\mathbf{k})}{2kT}\right).\tag{3}$$

ρ_0, m and ϵ_F are the density, mass, and Fermi energy respectively; $E(\mathbf{k})=\sqrt{\epsilon^2(\mathbf{k})+\Delta_0^2\mathbf{k}^2/k_F^2}$, and Δ_0 is the temperature dependent gap. At zero temperature, $\mathcal{N}$ is quantized to be unity in 2D. The director $\mathbf{M}$ is defined as $\mathbf{M}(\mathbf{k})$

$=[v_\Delta \mathbf{k}_x, v_\Delta \mathbf{k}_y, \epsilon(k)]$; $v_\Delta = \Delta_0/k_F$. When the director $\mathbf{n}$ is a planar vector confined in the $\theta=\pi/2$ equator, i.e., $\mathbf{n}=(\cos\phi, \sin\phi, 0)$, the spatial gradient terms coincide with previous results for the A phase of ^{3}He.[14,15] The action is valid when the frequency and the wave vector are smaller than $\Delta_0(T)$ and $\xi_0^{-1}(T)=\Delta_0(T)/v_F$ respectively.

Equation (2) suggests a few important properties of the quantum spin nematic p-wave superconductors. First of all, the dynamics of spin $\mathbf{n}$ and phase χ is completely decoupled at the low frequency limit (except the entanglement due to a Z_2 projection in the functional integral,[12] which we will not discuss in this paper). It reflects spin-phase separation in a p-wave superconductor. Second, for an isotropic Fermi surface that interests us in this article, $\mathcal{S}_{ab}=\delta_{ab}\rho_s(T)/2m$, and $\rho_s(T)$ is the temperature-dependent superfluid density that vanishes at the critical temperature T_c. So the spin and phase dynamics are characterized by an $O(3)$ σ-model (NLσM) and an xy model respectively. At the mean-field approximation, $\mathbf{n}=\mathbf{e}_z$ and χ is a constant. This corresponds to a conventional SOpSS. There are three Goldstone modes; two of them are spin waves $\delta\mathbf{n}=(1,\pm i,0)$ with a linear dispersion. In an isotropic case, $\tilde{\mathcal{S}}_{ab}=\delta_{ab}\tilde{\mathcal{S}}$; the spin-wave velocity is $v_s(T)=v_F\sqrt{\tilde{\mathcal{S}}/2\pi\tilde{\mathcal{T}}}$. And the last mode is the usual plasma wave, with a dispersion $\omega=\sqrt{2\pi e^2\rho_0 k/m}$ in 2D at $T=0$.

The topological term was previously derived in Refs. 16, 17. This term, orignating from the broken time-reversal and parity symmetries, determines the topological order in the fields $F_{\mu\nu}$ and defines the structure of quasiparticles. Implications of topological terms in other unconventional superconductors were also explored recently.[18–20] Here, we investigate the spin ordering, zero energy spin textures and quasiparticles based on Eq. (2). Let us emphasize that Eq. (2) is valid as far as the quasiparticles are gapped and the gradient expansion is possible; physically, it says all low-lying collective excitations below the BCS energy gap are correctly described by the action.

Spin ordering. Because of an extra branch of Goldstone modes in the spin sector, the spin order is more fragile than the phase order in the problem. In 2D, this provides a unique possibility of spin disordered p-wave superconducting states (SDpSSs) where the S^2-symmetry is restored and only the $U(1)$-symmetry is broken. Such a state that is rotation invariant in nature differs from the conventional SOpSSs where the S^2 symmetry is broken and there is a long-range order in $\mathbf{n}$.

The finite temperature phase diagrams of the $O(3)$ nonlinear σ model (NLσM) were previously analyzed in great details.[21] In the current situation, just as the superfluid velocity $\rho_s(T)$, all coefficients in the action, $\mathcal{S}_{ab}^{\alpha\beta}$, $\mathcal{T}_{ab}$, $\mathcal{S}_{ab}$, $\mathcal{T}$ depend on temperatures because of quasiparticle excitations. Taking this into account, we arrive at the following results in 2D.

When $\Delta_0 \ll \epsilon_F$, the spin order is established at zero temperature and the correlation length,

$$\xi_2=\frac{v_s(T)}{\Delta_s(T)}, \quad \Delta_s=T\exp\left(-\frac{2\pi[\rho_s(T)/2m-\Gamma]}{T}\right) \tag{4}$$

is finite only at finite temperatures (in a saddle-point approximation). Here $\Gamma\sim\Delta_0(T)$. For most of p-wave superconductors, the gap energy is about 1 K and the Fermi energy of order 1 eV, the superconductors are spin ordered at zero temperature. However, the liquids are spin disordered (in the absence of spin-orbit coupling) at any finite temperature as shown in Eq. (4). On the other hand, when Δ_0 is much larger than the Fermi energy, the spin long-range order could be spoiled by quantum fluctuations and the rotation invariance is preserved.[22] We will be concerned with both situations, *thermal* and *quantum* 2D SDpSSs in the following discussion.

Zero energy skyrmions. One of the most important feature of the *quantum* SDpSS is the existence of topological order and consequently topological stable zero energy skyrmions in the absence of spin stiffness. In $(2+1)$ space $\mathbf{x}=(\tau,\mathbf{r})$, it is convenient to introduce a field, $\mathbf{H}_\eta=\frac{1}{2}\epsilon^{\eta\mu\nu}\mathbf{F}_{\mu\nu}$. $\mathbf{H}_\tau=\mathbf{F}_{xy}$ represents $U(1)$ magnetic fields along z direction, $\mathbf{H}_x=\mathbf{F}_{y\tau}$ and $\mathbf{H}_y=\mathbf{F}_{\tau x}$ are the x,y components of the electric field. To facilitate a calculation at finite temperatures, the perimeter along τ direction L_τ is taken to be finite, i.e., $L_\tau=(kT)^{-1}$. Consider a rotating skyrmion terminated at the origin in a $S^2\times S^1$ space $\mathbf{n}(\rho,\phi)=[\sin\theta(\rho)\cos(\tilde{\phi}), \sin\theta(\rho)\sin(\tilde{\phi}), \cos\theta(\rho)]$ where

$$\tilde{\phi}=Q_m\phi-\gamma(\tau),$$

$$\theta(\rho,\tau)=2\arccos\frac{\rho}{\sqrt{\rho^2+v_s^2\tau^2}}\Theta(\tau),$$

$$\gamma(\tau+L_\tau)-\gamma(\tau)=N2\pi. \tag{5}$$

One can confirm that $\nabla\cdot\mathbf{H}=Q_m 2\pi\delta(\tau)\delta(\mathbf{r})$, corresponding to a space-time monopole of charge Q_m in $2+1d$. As ρ,τ approach infinity, $\mathbf{H}(\rho,\tau)$ becomes vanishingly small. The action of this Euclidean space monopole event is finite ($a\sim 1$),

$$\mathcal{S}_{m.}=\frac{a\Delta_0}{16\pi\Delta_s}+i\gamma_B, \gamma_B=\frac{Q_m\mathcal{N}}{4}[\gamma(L_\tau)-\gamma(\tau_0)]. \tag{6}$$

However, it has a Berry's phase due to the topological term, which characterizes a rotation of the skyrmion during its duration. $\gamma_B(0)$ obviously depends on the temporal coordinate at which the skyrmion is terminated, leading to destructive interferences between monopoles centered at different τ_0 with different rotation angles γ_B.

As a result, the fluctuations of space-time monopole events per unit volumn are ($c\sim 1$)

$$\langle Q_m^2\rangle=\delta(\mathcal{N})\frac{\Delta_0}{c\xi_0^2}\exp\left(-\frac{a\Delta_0}{16\pi\Delta_s}\right). \tag{7}$$

Equation (7) shows that at any finite $\mathcal{N}$ all monopole events are suppressed due to destructive interferences. It also implies that for $\mathcal{N}\neq 0$ the ground state has an infinitefold degeneracy compared with that of $\mathcal{N}=0$.

There are at least two important intraconnected consequences of the destructive interferences. First, Eq. (7) indicates the conservation of the skyrmion charges at $\mathcal{N}\neq 0$ in a *quantum* SDpSS, that is in the absence of the spin rigidity. If we define $c_w[\{\mathbf{n}(\mathbf{r})\}]=1/(2\pi)\int dxdy\mathbf{H}_z$ as the total number of skyrmions living on the 2D sheet, in the presence of space-time monopoles Q_m at $\{\mathbf{r}^m,\tau_m\}$,

$$\frac{\partial c_w(\tau)}{\partial \tau}=\sum Q_m\delta(\tau-\tau_m). \tag{8}$$

A space-time monopole essentially connects a trivial vacuum to a skyrmion configuration and causes a change in the topological charge c_w by one unit. At $\mathcal{N}=1$, following Eqs. (8) and (7), we conclude that a skyrmion whose energy could vanish in the absence of the spin stiffness, is a well-defined topological configuration in a *quantum* SDpSS. This remarkable feature which does not exist at $\mathcal{N}=0$ is also a consequence of a zero energy fermionic mode hosted by instantons.

Second, the suppression of monopole events leads to very distinct behaviors of fields $\mathbf{F}_{\mu\nu}$ in SDpSSs. The Wilson-loop integral defined as $\mathcal{W}_{U(1)}=\langle\mathcal{P}\exp(i\oint\mathbf{A}\cdot d\mathbf{r})\rangle$ has different asymptotical behaviors in the large loop limit in the presence or absence of topological order in c_w. When the topological charge c_w is conserved at any finite $\mathcal{N}$, $\mathcal{W}_{U(1)}=\exp(-L_cC_1)$ (L_c is the perimeter of the Wilson loop) and the gauge fields are deconfining. This is true for a *quantum* SDpSS at zero and finite temperatures as far as L_τ is longer than the duration of space-time monopoles. However, in a *thermal* SDpSS, c_w is unconserved and the gauge fields are confining (except around the quantum critical point which I will not discuss here).

Quasiparticles. We now employ the generalized Bogolubov–De Gennes equation to study the properties of quasiparticles in SDpSSs. In the presence of a topological configuration of $\mathbf{n}(\mathbf{r})$, it is convenient to introduce a gauge transformation $\Psi\rightarrow U_s(\mathbf{n})U_c(\chi)\Psi$ and work in a rotated representation; then one obtains a new Hamiltonian

$$H=\sigma_3\epsilon(i\hat{\nabla})+v_\Delta\sum_{i=1,2}\{\sigma_i\tau_3,i\hat{\nabla}_i\}_+. \tag{9}$$

Here $v_\Delta=\Delta_0/k_F$, $i\hat{\nabla}=i\nabla-\mathbf{A}_c-\mathbf{A}_s$ is a covariant derivative. We have defined $U_s^{-1}\mathbf{d}\cdot\tau U_s=\tau_3$, $U_c^{-1}\sigma_i\exp(i\sigma_3\chi)U_c=\sigma_i$. The vector potentials are defined in terms of the $U(1)$ rotation U_c and $SU(2)$ rotation U_s as $\mathbf{A}_{c\mu}=iU_c^{-1}\partial_\mu U_c=\sigma_3(\mathbf{A}^{em}_\mu+\frac{1}{2}\partial_\mu\chi)$, $\mathbf{A}_{s\mu}=iU_s^{-1}\partial_\mu U_s=\tau_\alpha\cdot\mathbf{W}^\alpha_\mu$ ($\mu=0,1,2$, stands for coordinates in $1+2$ dimension space). An explicit calculation also shows that $\mathbf{W}^3_\mu=\mathbf{A}_\mu$. At last, the corresponding Lagrangian density is

$$\mathcal{L}_{BdeG}=\Psi^+[\hat{\partial}_\tau-\mathcal{H}(\mathbf{A}^{em}_\mu,\mathbf{A}_{s\mu})]\Psi, \tag{10}$$

and $\widehat{\partial_\tau}=\partial_\tau-\sigma_3A^{em}_0-\boldsymbol{\tau}\cdot\mathbf{A}_{s0}$.

Following Eq. (9), besides a usual electric magnetic charge defined with respect to $\mathbf{A}_{e.m.}$ fields, a BdeG quasiparticle also carries a unit $U(1)$ charge with respect to $\mathbf{A}_\mu$ fields and is minimally coupled with $\mathbf{F}_{\mu\nu}$. The energy of a BdeG particle is determined by the Wilson-loop integral of $\mathbf{A}$. In *quantum* SDpSSs, the Wilson-loop integral decays exponentially as a function of the perimeter of the loop. The interactions between BdeG quasiparticles mediated by the topological fields $\mathbf{F}_{\mu\nu}$ are rather weak and the BdeG quasiparticle energy is finite. But most importantly, in this case, skyrmions themselves carry $U(1)$ charges with respect to the fields $\mathbf{A}_\mu$. This is indicated in Eq. (2) if we introduce the skyrmion density-current density as $4\pi\mathbf{j}_\mu=\mathcal{N}\epsilon_{\mu\nu\eta}\partial_\nu\mathbf{A}_\eta$ and express the topological term in a form of minimal coupling $\mathbf{j}_\mu\mathbf{A}_\mu$. By minimizing the action of $\mathcal{L}+\mathcal{L}_{BdeG}$ with respect to $\mathbf{A}_0$, $\mathbf{A}^{em}$ and taking $\mathcal{N}=1$ at low-temperature limit, we do obtain a saddle-point equation $4\pi\langle\Psi^+\tau_3\Psi\rangle=\mathbf{e}_z\nabla\times\mathbf{A}$, $\langle\Psi^+\sigma_3\Psi\rangle=0$. This indicates that a skyrmion configuration carries a half-spin but no charge. In other words, a spin $\frac{1}{2}$ but chargeless BdeG quasiparticle is hosted by, or confined with a skyrmion, with the confinement mediated by the spin fluctuations.

To examine the BdeG quasiparticles dressed with spin textures, we consider a skyrmion in polar coordinates (ρ,ϕ). The director has a spatial distribution as $\mathbf{n}(\rho,\phi)=[\sin\theta(\rho)\sin\phi,\sin\theta(\rho)\cos\phi,\cos\theta(\rho)]$; $\theta(\rho)$ is a smooth function of ρ, the asymptotics of which is $\theta(\rho=0)=0$ and $\theta(\rho\rightarrow\infty)=\pi$. The corresponding $\mathbf{A}$ field can be chosen as

$$\mathbf{A}=\frac{1-\cos\theta(\rho)}{2\rho\sin\theta}\mathbf{e}_\phi,\nabla\times\mathbf{A}=\frac{\sin\theta(\rho)}{2\rho}\frac{\partial\theta(\rho)}{\partial\rho}\mathbf{e}_z. \tag{11}$$

The $SU(2)$ field $\mathbf{W}^\alpha$ at $\rho\rightarrow\infty$ can be shown to take a simple form; $\mathbf{W}^3_i=\mathbf{A}_i$ ($i=1,2$), $\mathbf{W}^3_0=0$, and $\mathbf{W}^1_\mu=\mathbf{W}^2_\mu=0$.

A BdeG quasiparticle remains gapped in a texture. However, following Eq. (11) when a spin-1/2 BdeG particle moves in a closed circle of radius ρ in a skyrmion defect, it acquires a Berry's phase of $\pi[1-\cos\theta(\rho)]$ which approaches 2π at an infinity ρ. Consequently, under the interchange of coordinates, the two-body wave function of composite quasiparticles acquires an additional π phase because of hosting skyrmions, and $\Psi(\mathbf{r}_1,\mathbf{r}_2)=\Psi(\mathbf{r}_2,\mathbf{r}_1)$, which also follows the linking number theorem for skyrmions.[23] These composite quasiparticles are therefore Bosons. We also observe the BdeG quasiparticles are charge neutral at $\epsilon(\mathbf{k})=0$ with respect to an *em* field; they also carry zero $U(1)$ charges so to minimize the interaction between composite excitations. The life time of the quasiparticles is limited by the life time of zero energy skyrmions; for the *quantum* disordered case, the zero energy skyrmions are stable even at low temperatures.

In *thermal* SDpSSs, the suppression of space-time monopoles is incomplete since L_τ is longer than the monopoles' duration Δ_s^{-1}. The gauge field then is confining and the BdeG quasiparticles form bound states, with zero or one total spin. This unexpected feature should be observed in future experiments on some layered *p*-wave superconductors.

Collective spin excitations. The nature of the collective spin excitations in an SDpSS can be explored in a spinor representation of Eq. (2). By introducing $\eta^+\tau\eta=\mathbf{n}$, $\eta=(\eta_1,\eta_2)^T$ and $\eta^+\eta=1$, we obtain for η the following Lagrangian in SDpSSs:

$$\mathcal{L}_\eta = \frac{1}{2f^2}|(i\partial_\mu - \mathbf{A}_\mu)\eta|^2 + \frac{\Delta_s(T)}{\Delta_0(T)}\eta^+\eta + \frac{\mathcal{N}}{4\pi}\epsilon^{\mu\nu\lambda}\mathbf{A}_\mu\mathbf{F}_{\nu\lambda}. \tag{12}$$

And η is a bosonic field carrying a unit charge with respect to **A** fields and spin 1/2. In Eq. (12), $2f^2 = 2m\Delta_0/\sqrt{\tilde{\mathcal{S}}\tilde{\mathcal{I}}}\rho_0$; we have introduced the following rescaling: $t \rightarrow t\xi_0/v_s$, $\mathbf{r} \rightarrow \mathbf{r}\xi_0$.

In quantum SDpSSs, an η quantum is bound with a skyrmion such that the bound state becomes a fermion.[24] Each spin one spin wave excitation which is an elementary excitation in an SOpSS, is fractionalized into two elementary fermionic spinors hosted in skyrmions in quantum SDpSSs. Each spinor-skyrmion composite is a spin-1/2 excitation carrying no $U(1)$ charge, by contrast to a bare η excitation. In the thermal SDpSSs , the spin collective excitations are spin-wave ones with spin one.

($\hbar c/4e$) *vortices*. For the sake of completeness, I am also listing some properties of vortices. The linear defects in a symmetry broken state with an internal space $\mathcal{R} = [S^1 \times S^2]/Z_2$ have been recently discussed extensively in the context of Bose-Einstein condensates of ^{23}Na.[13] In SOpSSs, the linear defects are superpositions of $hc/4e$ vortices and π disclinations because of the Z_2 symmetries in the problem. And a bare $hc/4e$ vortex is forbidden because of the catastrophe of a cut. In SDpSSs, however, $hc/4e$ vortices can exist by their own right and are elementary excitations.

The SDpSS discussed here has the following order parameters: $\langle\hat{\Delta}\rangle = 0$, $Tr\langle\hat{\Delta}\hat{\Delta}\rangle \neq 0$, $\langle\exp(i\chi)\rangle \neq 0$. The existence of SC^* with $\langle\hat{\Delta}\rangle = 0$, $Tr\langle\hat{\Delta}\hat{\Delta}\rangle \neq 0$, $\langle\exp(i\chi)\rangle = 0$, and other fractionalized states examined in Ref. 12 appears to be beyond the model studied here. Physically, the SDpSS has Josephson oscillations of $2eV$ frequency while in SC^* the frequency is $4eV$.

In the presence of spin-orbital couplings, the mean-field solution indicates that the director of **n** points along $\pm\mathbf{e}_z$ direction and the internal space is $[Z_2 \times S^1]/Z_2$. However, at an energy scale higher than the spin-orbit coupling ones, **n** would be free to rotate on a two sphere. The spin order-disorder transition still takes place at a finite temperature below the superconductor-metal transition temperature T_c when the spin-orbit scattering rate is much smaller than $\Delta_0(0)$. As the spin is disordered, the above discussions on the spin textures and BdeG quasiparticles are still valid.

In conclusion, we also would like to remark that some aspects of the BdeG quasiparticles in spin disordered superconductors considered here reminisce the chiral-bag defect model for the nucleon.[25,26] The presence of spin-1/2 bosonic chargeless BdeG excitations in a quantum SDpSS is an example of Fermi number fractionalization; it belongs to the same class phenomenon as the midgap quasiparticles hosted in domain wall excitations in one-dimension polyacetylene[1] and the statistical transmutation proposed in some magnetic models.[24,27–29]

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