Erratum: Low-dimensional Bose gases [Phys. Rev. A 66, 013615 (2002)]

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In our paper, we presented a modified Popov theory that is capable of calculating the quasicondensate density n_0 in arbitrary dimensions and at all temperatures. When a condensate exists, i.e., in three dimensions at sufficiently low temperatures and in two dimensions at zero temperature, the condensate density n_c is defined by the off-diagonal long-range order of the one-particle density matrix. In particular, in the modified Popov theory we have that

$$n_c = \lim_{\mathbf{x}\to\infty} n_0 e^{-1/2\langle[\hat{\chi}(\mathbf{x})-\hat{\chi}(\mathbf{0})]^2\rangle}, \tag{1}$$

where n_0 satisfies the equations

$$n = n_0 + \frac{1}{V}\sum_{\mathbf{k}}\left[\frac{\epsilon_{\mathbf{k}}-\hbar\omega_{\mathbf{k}}}{2\hbar\omega_{\mathbf{k}}} + \frac{n_0 T^{2\mathrm{B}}(-2\mu)}{2\epsilon_{\mathbf{k}}+2\mu} + \frac{\epsilon_{\mathbf{k}}}{\hbar\omega_{\mathbf{k}}}N(\hbar\omega_{\mathbf{k}})\right], \tag{2}$$

$$\mu = (2n-n_0)T^{2\mathrm{B}}(-2\mu), \tag{3}$$

and the phase fluctuations are determined by

$$\langle\hat{\chi}(\mathbf{x})\hat{\chi}(\mathbf{0})\rangle = \frac{T^{2\mathrm{B}}(-2\mu)}{V}\sum_{\mathbf{k}}\left[\frac{1}{2\hbar\omega_{\mathbf{k}}}[1+2N(\hbar\omega_{\mathbf{k}})] - \frac{1}{2\epsilon_{\mathbf{k}}+2\mu}\right]e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{4}$$

Here n is the total density of the gas, $T^{2\mathrm{B}}(-2\mu)$ is the two-body T matrix at energy -2μ, and μ is the chemical potential. Furthermore, $\epsilon_{\mathbf{k}} = \hbar^2k^2/2m$ and $\hbar\omega_{\mathbf{k}} = [\epsilon_{\mathbf{k}}^2 + 2n_0T^{2\mathrm{B}}(-2\mu)\epsilon_{\mathbf{k}}]^{1/2}$. In our paper we assumed that the exponent in Eq. (1) vanishes in the limit $\mathbf{x}\to\infty$ and thus found that at zero temperature the depletion of the condensate in two dimensions equals

$$\frac{n-n_0}{n} = \frac{1}{4\pi}(1-\ln 2)\frac{mT^{2\mathrm{B}}(-2\mu)}{\hbar^2}, \tag{5}$$

whereas in three dimensions it equals

$$\frac{n-n_0}{n} = \left(\frac{32}{3} - 2\sqrt{2}\pi\right)\sqrt{\frac{na^3}{\pi}}, \tag{6}$$

where a is the scattering length and we have used $T^{2\mathrm{B}}(-2\mu) = 4\pi a\hbar^2/m$. A careful analysis shows, however, that for a consistent calculation of the quantum depletion the above assumption is incorrect and we have to take the contribution from the phase fluctuations into account. In the limit $\mathbf{x}\to\infty$, we obtain in two and three dimensions

$$\langle[\hat{\chi}(\mathbf{x})-\hat{\chi}(\mathbf{0})]^2\rangle = \frac{1}{2\pi}(\ln 2)T^{2\mathrm{B}}(-2\mu) \tag{7}$$

and

$$\langle[\hat{\chi}(\mathbf{x})-\hat{\chi}(\mathbf{0})]^2\rangle = (2-\sqrt{2}\pi)\sqrt{\frac{na^3}{\pi}}, \tag{8}$$

respectively. Since we consider a weakly interacting Bose gas, the right-hand sides of Eqs. (7) and (8) are small and we can expand the exponents in Eq. (1) to first order. Combining this with Eqs. (5) and (6), we finally obtain

$$\frac{n-n_c}{n} = \frac{1}{4\pi}\frac{mT^{2\mathrm{B}}(-2\mu)}{\hbar^2}, \tag{9}$$

in two dimensions and

$$\frac{n-n_c}{n}=\frac{8}{3}\sqrt{\frac{na^3}{\pi}}\ , \tag{10}$$

in three dimensions. These results are in perfect agreement with the results obtained from Popov theory.

We would like to point out that none of the other results presented in our paper are affected by the above, since the theory is always applied in a regime where the gas parameter is small and the subtlety discussed here can be safely ignored.

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