How to protect the interpretation of the wave function against protective measurements

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A new type of procedures, called protective measurements, was proposed by Aharonov, Anandan, and Vaidman [Phys. Rev. A **97**, 4616 (1993); Found. Phys. **26**, 117 (1996)]. These authors argued that a protective measurement allows the determination of arbitrary observables of a single quantum system, and claimed that this favors a realistic interpretation of the quantum state. This paper proves that only observables that commute with the system's Hamiltonian can be measured protectively. It is argued that this restriction saves the coherence of alternative interpretations. [S1050-2947(99)01511-5]

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I. INTRODUCTION

Recent work by Aharonov and co-workers [1–3] introduced a new type of procedure in quantum mechanics, which they called ''protective measurements.'' In these procedures, one can measure, under certain conditions and for a specific set of states, the expectation value of an arbitrary observable of an individual system. Remarkably, such expectation values are obtained while avoiding the subsequent entanglement of the states of the system and the apparatus, even if the system was initially not in an eigenstate of the measured observable. In this respect, the protective measurement is very different from the more well-known von Neumann measurement procedure.

Aharonov and co-workers attributed this feature of protective measurements to a physical manifestation of the wave function of the system (Ref. [2], p. 4619). They claimed that by means of these measurements one can directly *observe* the wave function (or quantum state) of an individual system, and concluded from this that this quantum state should be given an ontological interpretation: if it is possible to observe the state of an individual system, it must correspond to a real property of this system.

This conclusion stands in sharp contrast to received opinion. To be sure, there is no consensus in the literature on the interpretation of the quantum state. But there does seem to be consensus that the state of an individual system is unobservable (i.e., not empirically accessible). In fact, it is only because of this generally shared opinion that so many different views on its meaning can peacefully coexist today. However, according to the above claims, we can decide the issue of the interpretation of the wave function by exploiting theoretical possibilities for measurement allowed by quantum theory itself. This poses a serious threat for many of these interpretations.

Several critical discussions of these exciting claims have already been published (See Refs. [4–11]). In this paper I will address an issue which, to my knowledge, has not previously been dealt with. I show that the conditions assumed in a protective measurement imply that the observable that is being measured commutes with the Hamiltonian of the system. While this limitation still allows the possibility of a unique determination of the quantum state within the specific set considered in a protective measurement, it undermines the claim that one sees its direct physical manifestation. I conclude that the threatened interpretations of the wave function can be saved in the face of protective measurements.

This paper is organized as follows. In Sec. II the theoretical background of protective measurements is reviewed. Section III discusses its consequences for the interpretation of quantum mechanics in more detail. In Sec. IV I prove the limitations on the observables that can be measured protectively. Section V provides an explanation of the mechanism of protective measurement that does not involve the manifestation of the wave function, and applies this to a thought experiment of Aharonov and co-workers. Section VI argues that alternative interpretations are not endangered by protective measurements, and discusses some possible objections to this conclusion.

II. PROTECTIVE MEASUREMENTS

The notion of a protective measurement is introduced by means of a concrete measurement model. Consider a system S in interaction with some measurement apparatus A and let $\mathcal{H}_S \otimes \mathcal{H}_A$ be their composite Hilbert space. Assume that the total Hamiltonian of this composite system is of the form

$$H_{\text{tot}}(t) = H_S + H_A + g(t) H_{\text{int}}, \tag{1}$$

where H_S and H_A denote the free Hamiltonians of the system and apparatus, respectively, and H_{int} is the interaction Hamiltonian. (As usual, H_S and H_A are shorthand for the operators $H_S \otimes \mathbb{1}$ and $\mathbb{1} \otimes H_A$.) Further, $g(t)$ is a switch function, which takes a constant value $1/\tau$ during a very long interval $[0,\tau]$, and vanishes smoothly and rapidly before and after this interval. We take

$$H_{\text{int}} = O \otimes P, \tag{2}$$

where O is an observable of the system and P is the canonical momentum conjugate to the pointer position of the apparatus.

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In general, the evolution generated by a time-dependent Hamiltonian such as Eq. (1) is given by

$$U = \mathcal{T} e^{-i\int H_{\rm tot}(t)dt}, \tag{3}$$

where $\mathcal{T}$ is the time-ordering operator. In the present case, the Hamiltonian commutes with itself during the period $[0,\tau]$ (i.e., $[H_{\rm tot}(t),H_{\rm tot}(t')]=0$ for $0 \leqslant t,t' \leqslant \tau$), so that time ordering is unimportant during this period. If we neglect the small interaction during the switching on and off periods, we can write

$$U = e^{-i[\tau(H_S+H_A)+O\otimes P]}. \tag{4}$$

We further need two specific assumptions about Hamiltonian (1). First, it is assumed that the system Hamiltonian is completely nondegenerate and discrete. Thus H_S has a complete orthonormal set of nondegenerate eigenvectors $\{|\phi_1\rangle,|\phi_2\rangle,\ldots\}$ in $\mathcal{H}_S$ such that

$$H_S|\phi_n\rangle = E_n|\phi_n\rangle. \tag{5}$$

Second, we will assume that the pointer momentum P commutes with the free Hamiltonian of the apparatus, i.e.,

$$[H_A,P]=0. \tag{6}$$

Now let the initial state be of the form

$$|\Psi\rangle_{\rm i} = |\phi_n\rangle|\chi\rangle, \tag{7}$$

where $|\chi\rangle$ is an arbitrary normalized state in $\mathcal{H}_A$. Evolution (4) will transform this into

$$|\Psi\rangle_{\rm f} = U|\Psi\rangle_{\rm i} = e^{-i[\tau(H_S+H_A)+O\otimes P]}|\phi_n\rangle|\chi\rangle. \tag{8}$$

At this point we note that since g is slowly varying and $|g(t)| \ll 1$ during the entire interval $[0,\tau]$, one can apply the adiabatic theorem and first-order perturbation theory. I will not go into the details of these approximation theorems, but merely note the result: for the special choice of Eq. (8), and in the limit $\tau\to\infty$, the following approximation for the final state is obtained:

$$|\Psi\rangle_{\rm f} \approx e^{-i\tau E_n}|\phi_n\rangle\, e^{-i[\tau H_A+\langle O\rangle_n P]}|\chi\rangle, \tag{9}$$

where

$$\langle O\rangle_n = \langle\phi_n|O|\phi_n\rangle. \tag{10}$$

(See Refs. [2,11] for more details.)

Result (9) has two important features. First, the apparatus state has been changed, not only by the free evolution $e^{-i\tau H_A}$, but also by the additional action of the operator $e^{-i\langle O\rangle_n P}$. Since $[H_A,P]=0$, this second operator corresponds to a shift, proportional to $\langle O\rangle_n$, in the pointer position variable Q that is canonically conjugate to P. Thus if $e^{-i\tau H_A}|\chi\rangle$ is a state such that the pointer position is reasonably well defined (i.e., the wave packet $\langle q|e^{-i\tau H_A}|\chi\rangle$ vanishes outside of an interval smaller than $\min_{n,m}|\langle O\rangle_n - \langle O\rangle_m|$) we can infer the value of $\langle O\rangle_n$ with a certainty by a reading of Q after the interaction is over. Thus the measurement procedure indeed yields the expectation value of the observable O of the system.

The second important feature is that there is no entanglement in the final state (9). That is to say, we can read the value of $\langle O\rangle_n$ from the pointer position without causing a collapse or reduction of the system-plus-apparatus state vector. In fact, the state vector of the system changes merely by a phase factor. Since state vectors differing by a phase factor represent the same state, this means that the state of the system remains completely unchanged in this procedure. Hence the name ''protective measurement.''

This is in sharp contrast to the usual von Neumann measurement scheme, where measurement of an arbitrary observable generically leads to entanglement. In this case the subsequent reading (or permanent registration) of the pointer observable leads to a disruption of the coherence of this entanglement, because of the projection postulate. In a protective procedure, however, the system is still available in its original state after the measurement. We can then repeat the procedure for other observables $O',O'',\ldots$, and determine their expectation values as well. Continuing in this manner, one eventually obtains sufficient data to determine the exact quantum state of the system uniquely (up to an overall phase factor). For example, if $\mathcal{H}_S$ is two-dimensional, three linearly independent observables will suffice. This is the basis for the conclusion of Aharonov and co-workers that one can observe the state by means of protective measurements on an individual system.

Note that this conclusion is obtained only under the condition that the system was initially in an eigenstate of H_S. Indeed, as pointed out by Aharonov and co-workers if the system is initially described by a superposition $|\psi\rangle = \Sigma_n c_n|\phi_n\rangle$ the protective measurement brings about the evolution

$$\sum_n c_n|\phi_n\rangle|\chi\rangle \to \sum_n c_n e^{-i\tau E_n}|\phi_n\rangle e^{-i(\tau H_A+\langle O\rangle_n P)}|\chi\rangle, \tag{11}$$

which results in an entangled superposition, just as in the von Neumann measurement.

However, they argued that in this case a protective measurement may still be feasible, by tailoring the Hamiltonian (e.g., by applying external fields) such that $|\psi\rangle$ becomes a nondegenerate eigenstate.

III. CONSEQUENCES FOR THE MEANING OF THE QUANTUM STATE

We have seen that by means of a protective measurement, the expectation values $\langle O\rangle_n$ can be obtained, for arbitrary O, for an individual system in an eigenstate $|\phi_n\rangle$ of the Hamiltonian H_S, without altering this state. By a sequence of protective measurements it is then possible to completely determine the quantum state of that system. What does this entail for the interpretation of the quantum state? Aharonov and Vaidman wrote, ''We have shown that stationary quantum states can be observed. This is our main argument for associating physical reality with the quantum state of a single particle.'' [1]. Indeed, there is immediate intuitive appeal for this conclusion. If the value $\langle O\rangle_n$ can be determined by inspection of a single particle, it seems natural to assume that the particle somehow ''knows'' this value, i.e., that it corresponds to a real attribute. When this holds for arbitrary observables O, all their expectation values represent real attributes. But this is equivalent to assuming that the quantum state itself is a real property of the particle.

Note, however, that this claim is established only for the eigenstates of H_S. That is to say, the protective measurement determines the state of a single system without disturbance only if it is given beforehand that this state belongs to the orthonormal family $\{|\phi_1\rangle,|\phi_2\rangle,\ldots\}$. One might object that under this restriction the achievement is not surprising. Indeed, the same claim could be made for a traditional von Neumann measurement of H_S. In that case, the measurement brings about the evolution

$$|\phi_n\rangle|\chi_0\rangle\rightarrow|\phi_n\rangle|\chi_n\rangle \tag{12}$$

for a special initial state $|\chi_0\rangle$, and with $|\chi_n\rangle$ denoting orthonormal pointer states. One can then identify $|\phi_n\rangle$ simply by reading off the label n from the pointer state.

Anticipating this type of objection, Aharonov, Anandan, and Vaidman emphasized [2,3] that in a protective measurement, one does not need knowledge of H_S. Thus, while in the above von Neumann measurement one can only reconstruct $|\phi_n\rangle$ from the observed value of n if the Hamiltonian (or the set of its eigenstates) is known, the protective measurement yields the data $\langle O\rangle_n,\langle O'\rangle_n$, etc. From this, one can reconstruct the form of $|\phi_n\rangle$ even when H_S or the precise form of its eigenvectors are unknown. Thus, even if the information obtained in a protective measurement is of the same kind as that in a von Neumann measurement, it is obtained under different conditions.

One may still doubt whether this rebuttal is convincing. Obviously, in a protective measurement, the reconstruction of the state from the experimental data requires knowledge of the exact form of the observables O,O', etc. This requirement seems completely analogous to the condition that the H_S is known in the case of the von Neumann measurement.

However, even if one concludes that the claim that it is possible to determine the state of an individual system, under the condition that this state is a member of some orthonormal set, is not by itself spectacular, the fact remains that a protective measurement achieves this result in a surprising manner. The procedure records the expectation value of an observable O in a single measurement, while the system is not necessarily in an eigenstate of O. This could never be achieved in a von Neumann measurement.

Another type of objection concerns the inference in the above argument from observability to physical reality of the state. Dickson [12] pointed out that this argument will not carry appeal for those who adopt an instrumentalist view, i.e., for those who regard the theory as merely a recipe for predicting experimental results; or for empiricists who may accept a theory when it is empirically adequate without feeling committed to believe any of its ontological claims.

However, one can also use the argument, not so much to infer the reality of the quantum state, but rather as a weapon to attack the internal coherence of other interpretations. Indeed, suppose some interpretation of quantum mechanics denies or qualifies the unconditional existence of the wave function of an individual system—as in fact most interpretations do. It would then seem most surprising, to say the least, if one could still determine its exact form by measurements on such an individual system. Hence, even if one doubts whether the analysis establishes the physical reality of the wave function, it can still be effective in establishing incoherence of alternative interpretations.

Let me mention two of these. The most obvious candidate in danger from the above conclusions is the ''ensemble'' or statistical interpretation. In this view, adopted by authors such as Einstein, Popper, Blochintsev and Ballentine, the quantum state describes not an individual system, but rather an ensemble of systems. The quantum-mechanical expectation values are then interpreted as averages over the members of the ensemble. Accordingly, one cannot determine the state of an individual system, simply because it is not a property of a single system. This view appears untenable in the light of the above claims.

Second, in the Copenhagen view, the quantum state is assumed to give a complete description of the individual system. But this description is, according to Bohr, ''symbolic'' and does not literally represent physical reality in the sense of a one-to-one correspondence. While the quantum state encodes complete information about the system, only part of this information is applicable in any given measurement context. Due to the principle of complementarity, one always has to collect experimental data from mutually exclusive measurement arrangements to obtain a full determination of the state.

To be more precise, a measurement context in which the nondegenerate observable A is measured is represented by the eigenbasis of the observable. In this basis we can expand the state, say $|\psi\rangle=\Sigma_i c_i|a_i\rangle$, and $|c_i|^2$ give the probabilities of finding the outcomes a_i. The phase relations between the coefficients c_i are not accessible in this context. To determine them, one needs to consider a measurement of some other observable B that does not commute with A. But the context defined by B is, according to the Copenhagen point of view, incompatible with the original one. Therefore, we can never obtain sufficient information to determine the quantum state of an individual system in a single context. By contrast, the series of protective measurements needed for a determination of the state are not mutually exclusive. Thus the claim of Aharonov and co-workers amounts to nothing less than a disproof of the principle of complementarity.

Remarkably, some other interpretations are no better off, even if they agree with the point of view that the wave function of an individual system represents a physically real entity. For example, in the Bohm interpretation, the modulus $R(x)$ of the wave function $\psi(x)=R(x)e^{iS(x)/\hbar}$ appears in the quantum potential $U(x)=(-\hbar^2/2m)(\Delta R/R)$ which represents an independently existing potential acting on the particle. In some versions of the Bohm interpretation, the phase $S(x)$ represents a real entity as well (the ''guidance field''). Nevertheless, it has been shown that these fields cannot be determined experimentally from the behavior of an individual particle (Ref. [7], pp. 369–378). Hence the Bohm interpretation is also committed to the conclusion that one cannot observe the wave function by inspection of an individual particle.

It follows that in order to avoid damage for the above interpretations, one should question the very starting point of the above argument, i.e., whether it has been sufficiently established that it is possible to perform a protective measurement for arbitrary observables. In Sec. IV we shall see that there are indeed severe restrictions on the performance of protective measurements.

IV. RESTRICTIONS ON THE OBSERVABLES THAT CAN BE MEASURED PROTECTIVELY

It is essential to note that for the purpose of the protective measurement the form of the evolution obtained in Eq. (9) should hold for all $|\phi_n\rangle$. Indeed, we are assuming that all we know about the initial state of the system is that it is one of the eigenstates of a nondegenerate Hamiltonian H_S, but not which one. It is the purpose of the procedure to determine this state. Therefore, one must guarantee that the desired form of the evolution holds for all $|\phi_n\rangle$, i.e., the approximation must be a good one for all these states. We exploit this to derive a simple but very restrictive property of the evolution.

Let us define an operator U_{app} that brings about the approximate evolution (9) exactly for all vectors of the form $|\phi_n\rangle|\chi\rangle$, i.e.,

$$U_{\mathrm{app}}:|\phi_n\rangle|\chi\rangle\rightarrow e^{-i\tau E_n}|\phi_n\rangle e^{-i(H_A\tau+\langle O\rangle_n P)}|\chi\rangle. \tag{13}$$

By linearity this extends to a unique definition of U_{app} as an operator on $\mathcal{H}_S\otimes\mathcal{H}_A$. One can also give an explicit expression for U_{app}. Let

$$\tilde{O}=\sum_n P_n O P_n \tag{14}$$

be an operator on $\mathcal{H}_S$, where $P_n=|\phi_n\rangle\langle\phi_n|$. It is easy to see that

$$U_{\mathrm{app}}=e^{-i(H_S+H_A)\tau-i\tilde{O}\otimes P}, \tag{15}$$

by checking that the right-hand side indeed produces transition (13) when acting on states of the form $|\phi_n\rangle|\chi\rangle$.

But then, since $[\tilde{O},H_S]=0$, it immediately follows that $[U_{\mathrm{app}},H_S]=0$, or, in other words,

$$U_{\mathrm{app}}^\dagger H_S U_{\mathrm{app}}=H_S. \tag{16}$$

This means that H_S is conserved under the evolution U_{app}.

This already suggests that the observable O appearing in the interaction Hamiltonian of a protective measurement must be subject to restrictions. Indeed, if the evolution operator U given by Eq. (8) contains an arbitrary self-adjoint operator O, one would not expect that, to a good approximation, U commutes with H_S. One would suspect that this is the case only if O commutes with H_S. However, since U_{app} is only an approximation of U, we have to be careful to spell this suspicion out.

To say that the approximation involved in Eq. (9) is good means that

$$\|(U-U_{\mathrm{app}})|\phi_n\rangle|\chi\rangle\|\rightarrow 0 \quad \text{if} \;\; \tau\rightarrow\infty. \tag{17}$$

As mentioned earlier, we assume this holds for for all n. Moreover, the approximation theorems apply for arbitrary $|\chi\rangle$. Thus Eq. (17) holds for all $|\phi_n\rangle$ and $|\chi\rangle$. This condition is then equivalent to

$$\lim_{\tau\rightarrow\infty}\|U-U_{\mathrm{app}}\|=0. \tag{18}$$

Together with Eq. (16), this implies[1]

$$\lim_{\tau\rightarrow\infty}\|U^\dagger H_S U-H_S\|=0. \tag{19}$$

Now consider the matrix element

$$\langle\phi_m|\langle\chi|(U^\dagger H_S U-H_S)|\phi_n\rangle|\chi\rangle. \tag{20}$$

Since, for any self-adjoint operator, the operator norm majorizes the absolute value of its matrix elements, we conclude from Eq. (19) that, as $\tau\rightarrow\infty$,

$$\langle\chi|\langle\phi_m|U^\dagger H_S U|\phi_n\rangle|\chi\rangle\rightarrow\langle\chi|\langle\phi_m|H_S|\phi_n\rangle|\chi\rangle=E_n\delta_{nm}. \tag{21}$$

Let $\{|p,\alpha\rangle\}$ be a complete orthonormal set of (improper) common eigenstates in $\mathcal{H}_A$ of both H_A and P:

$$P|p\rangle=p|p,\alpha\rangle, \quad H_A|p\rangle=E(p,\alpha)|p,\alpha\rangle. \tag{22}$$

Here the index α is used to allow for degeneracy in P and H_A. We expand the left-hand side of Eq. (21):

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[1]Because $\|U^\dagger H_S U-H_S\|=\|U^\dagger H_S U-U_{\mathrm{app}}^\dagger H_S U_{\mathrm{app}}\|=\|(U-U_{\mathrm{app}})^\dagger H_S U+U_{\mathrm{app}}^\dagger H_S(U-U_{\mathrm{app}})\|\leqslant 2\|U-U_{\mathrm{app}}\|\,\|H_S\|\rightarrow 0$, at least if H_S is bounded. However, if H_S is unbounded, the argument can be rerun, while replacing H_S with the set of its spectral projections.

$$\begin{aligned}\langle\chi|\langle\phi_m|U^\dagger H_S U|\phi_n\rangle|\chi\rangle &= \sum_{\alpha\beta}\int\int dp\,dp'\langle\chi|p',\beta\rangle\langle p',\beta|\langle\phi_m|U^\dagger H_S U|\phi_n\rangle|p,\alpha\rangle\langle p,\alpha|\chi\rangle\\ &= \sum_{\alpha\beta}\int\int dp\,dp'\langle\chi|p',\beta\rangle\langle p,\alpha|\chi\rangle\\ &\quad\times\langle p',\beta|\langle\phi_m|e^{i\{\tau[H_S+E(p',\beta)]+p'O\}}H_S e^{-i\{\tau[H_S+E(p,\alpha)]+pO\}}|\phi_n\rangle|p,\alpha\rangle\\ &= \sum_{\alpha\beta}\int\int dp\,dp'\langle\chi|p',\beta\rangle\langle p,\alpha|\chi\rangle\langle p',\beta|p,\alpha\rangle\\ &\quad\times e^{i\tau[E(p',\beta)-E(p,\alpha)]}\langle\phi_m|e^{i(\tau H_S+p'O)}H_S e^{-i(\tau H_S+pO)}|\phi_n\rangle\\ &= \sum_{\alpha}\int dp|\langle p,\alpha|\chi\rangle|^2\langle\phi_m|e^{i(\tau H_S+pO)}H_S e^{-i(\tau H_S+pO)}|\phi_n\rangle \end{aligned} \tag{23}$$

According to Eq. (21) this expression will approach zero if $m\neq n$, and E_n otherwise. But, since $|\chi\rangle$ is arbitrary, this happens only if for almost all values of p:

$$\langle\phi_m|e^{i(\tau H_S+pO)}H_S e^{-i(\tau H_S+pO)}|\phi_n\rangle\to E_n\delta_{mn},$$

or, equivalently,

$$e^{i\tau(E_m-E_n)}\langle\phi_m|e^{ipO}H_S e^{-ipO}|\phi_n\rangle\to E_n\delta_{mn}.$$

This means that for almost all $p\in\mathbb{R}$,

$$e^{ipO}H_S e^{-ipO}=H_S, \tag{24}$$

which implies

$$[O,H_S]=0. \tag{25}$$

Thus we conclude that U_{app} is a good approximation to U only if the observable O commutes with the system Hamiltonian.

Notice that we did not rely on the differential form of evolution (4). Had we done so, we would have immediately obtained the result

$$[H_S+H_A+g(t)O\otimes P,H_S]=g(t)[O,H_S]\otimes P\to 0, \tag{26}$$

by noting that the switch function g is of the order $g\approx\tau^{-1}$ so that commutator (26) vanishes automatically in the limit $\tau\to\infty$. Thus, this approach would not reveal a constraint on $[O,H_S]$.

V. AN ALTERNATIVE LOOK AT PROTECTIVE MEASUREMENTS

We have reached the conclusion that the assumptions involved in a protective measurement entail that the observable whose expectation value is obtained commutes with the Hamiltonian H_S of the system. This obviously presents a major restriction. In Copenhagen terms, it means that the information provided by a protective measurement is restricted to that belonging to a single measurement context only. Indeed, in view of this, one might even doubt whether the claim that the quantum state can be uniquely determined by means of protective measurement is valid at all. I shall argue here that this claim is still true, but at the same time that this need not be interpreted as evidence for the physical reality of the quantum state.

To see this, let us compare the approximative evolution (15) with the (almost) exact evolution (4). This shows that the approximations involved amount to the replacement of the original observable O by the sandwiched observable $\tilde{O}$. But this observable combines two interesting virtues: (i) it commutes with H_S, and (ii) its expectation value in any eigenstate $|\phi_n\rangle$ equals that of O: $\langle\tilde{O}\rangle_n=\langle O\rangle_n$. Thus the measurement of $\tilde{O}$, which is compatible with H_S, suffices to determine the value of $\langle O\rangle_n$.

Thus we can give an alternative explanation for what happens in a protective measurement, which does not appeal to the idea that an individual system carries information about its quantum state. The interaction between system and apparatus is produced by a very small interaction term, viz. $g(t)O\otimes P$, that works for a very long time. The smallness is responsible for the fact that $|\phi_n\rangle$ remains unchanged; the long time explains that nevertheless a nonvanishing effect of the interaction builds up in the state of the apparatus. However, the effect that builds up in the course of time is due only to the part of O (namely, $\tilde{O}$) that commutes with H_S. It is only this operator whose expectation value is revealed. The procedure is insensitive, however, to to the remainder $O-\tilde{O}$, i.e., the part of O that does not commute with H_S. In fact, this statement can indeed be immediately verified: if we replace $|\phi_n\rangle$ in the initial state with an arbitrary superposition of the form $\Sigma_n c_n|\phi_n\rangle$, the protective measurement brings about transition (11). Here a reading of the pointer variable invariably leads to a disruption of the coherence of the terms, and we are cut off from establishing the phase relations between the coefficients c_n. In Aharonov and coworkers' terminology, this is expressed by saying that superpositions of the eigenstates $|\phi_n\rangle$ are not protected in this particular procedure. But from the present point of view, the incapability of a protective measurement to reveal the phase relations in a superposition, i.e., the incapability of discriminating the superposition $\Sigma_n c_n|\phi_n\rangle$ from the corresponding mixture $\Sigma_n|c_n|^2|\phi_n\rangle\langle\phi_n|$, can also be interpreted by saying

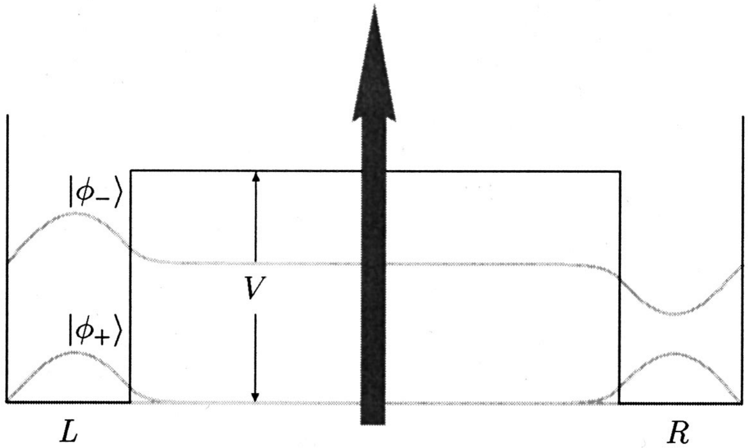

FIG. 1. A proton is in superposition $|\phi_+\rangle$ of two states localized in boxes L and R. In between the boxes there is an external constant potential V which lifts the degeneracy of $|\phi_+\rangle$ and $|\phi_-\rangle$. When the location of the proton is measured protectively, by sending an electron through the middle between the boxes, the electron will pass the boxes on a straight trajectory.

that one is actually measuring the observable $\tilde{O}$ rather than O.

In short, an alternative explanation for the surprising features of a protective measurement is that when one enforces the adiabatic conditions, i.e., the validity of approximation (9), the observable O is effectively replaced by $\tilde{O}$. This has no effect on its expectation value in the eigenstates $|\phi_n\rangle$ but a large effect on its commutation relation with H_S.

Let us try to illustrate these conclusions by means of an example. Perhaps one of the most striking examples discussed in Ref. [2] is that of a charged particle (say a proton), which is described by a superposition of two states localized in distant boxes L and R,

$$|\phi_+\rangle=\frac{1}{\sqrt{2}}(|\phi_L\rangle+|\phi_R\rangle), \tag{27}$$

where $|\phi_L\rangle$ and $|\phi_R\rangle$ are the ground states of the box potentials. The question is whether one can demonstrate that the proton is in this delocalized state.

If the two boxes are bordered by infinite potential walls, state (27) is degenerate with

$$|\phi_-\rangle=\frac{1}{\sqrt{2}}(|\phi_L\rangle-|\phi_R\rangle), \tag{28}$$

so that the analysis of Sec. II would not be applicable. But if one arranges that in the region between the two boxes the potential has a large but finite constant value V, the states $|\phi_L\rangle$ and $|\phi_R\rangle$ develop small tails this middle region, and one achieves that $|\phi_+\rangle$ and $|\phi_-\rangle$ are no longer degenerate (see Fig. 1).

Now suppose we measure the position of the proton, or somewhat more crudely, the observable:

$$O=-|\phi_L\rangle\langle\phi_L|+|\phi_R\rangle\langle\phi_R|. \tag{29}$$

This can be done be sending a charged test particle, e.g., an electron, straight through the middle between the boxes, perpendicular to the line joining the two boxes, and observing whether its trajectory deviates from a straight line. Aharonov and co-workers showed that if the procedure is that of a conventional von Neumann measurement, one will find a deflection of the electron to the left or right, with equal probability. Therefore, this procedure does not yield evidence that the proton is in the delocalized state $|\phi_+\rangle$.

However if the measurement is protective, the result is very different. The trajectory of the electron is now only sensitive to $\langle O\rangle_+=0$ and, therefore, it will continue through the boxes without deviation. This then seems a clear demonstration that the proton is really in a delocalized superposition. In the words of Ref. [2], "the interaction is as if half of [the particle] is in box [L] and the other is in box [R]" and "the protective measurement shows the manifestation of the wave function as an extended object."

How should one analyze this example from the point of view proposed above? In this view, the protective measurement does not measure O, but rather a related observable $\tilde{O}$. If, for simplicity, we restrict ourselves to the two-dimensional Hilbert space spanned by $|\phi_+\rangle$ and $|\phi_-\rangle$, an easy calculation shows that, in this example,

$$\tilde{O}=\sum_{j\in\{+,-\}}|\phi_j\rangle\langle\phi_j|O|\phi_j\rangle\langle\phi_j|=0. \tag{30}$$

This means the null result of the experiment should not surprise us: this particular protective measurement is incapable of yielding any other result.

This conclusion can be straightforwardly verified by considering the case where the procedure is carried out on a proton prepared in a localized state, say $|\phi_L\rangle$. Since this state is not protected in the procedure, one obtains the evolution

$$|\phi_L\rangle|\chi\rangle=\frac{1}{\sqrt{2}}(|\phi_+\rangle+|\phi_-\rangle)|\chi\rangle$$
$$\rightarrow\frac{1}{\sqrt{2}}(|\phi_+\rangle|\chi_0\rangle+|\phi_-\rangle|\chi_0'\rangle), \tag{31}$$

where $|\chi_0'\rangle$ is the final state of the electron in the case when the proton was initially in the state $|\phi_-\rangle$. Since $\langle O\rangle_+ =\langle O\rangle_-=0$, the electron travels a straight trajectory in the state $|\chi_0'\rangle$ as well as in $|\chi_0\rangle$.[2] Thus, the electron will indeed travel on a straight path, regardless of whether the proton is delocalized or not. Therefore, this experiment provides no evidence for the spatial delocalization of the proton.

At first sight, the conclusion that the electron is not deflected, even if the proton is localized, may seem counterintuitive because of the asymmetry of the Coulomb field produced in this case. But note that the adiabatic limit in this experiment involves letting the distance between the boxes, and the value of the potential in the middle region, go to infinity. Consequently, the electrostatic force on the electron, and hence the curvature of its trajectory also vanishes in this limit.

VI. CONCLUSION AND DISCUSSION

It has been shown here that for a system with a nondegenerate free Hamiltonian H_S, a protective measurement is only possible of observables O that commute with H_S. This is not in conflict with the claim that the measurement procedure is able to yield the expectation value of an arbitrary observable O. The explanation is simply that in the regime in which the conditions and approximations for the adiabatic theorem and first-order perturbation theory are valid, the procedure actually measures another observable $\tilde{O}$ which commutes with H_S but which, for the considered set of states, has the same expectation value as O. A similar conclusion was reached by Rovelli [8] by analysis of a concrete example.

In this explanation we do not need recourse to a manifestation of the wave function in the individual system. Rather, it is clear that in a protective measurement we are dealing with what from the Copenhagen point of view would be characterized as a single measurement context only: that of the Hamiltonian H_S. All the information obtained is in fact compatible with this context. Hence there is no threat to the complementarity principle.

Similarly, the ensemble interpretation of the wave function can be saved from incoherence. Assume that $|\phi_n\rangle$ describes an ensemble of similarly prepared systems. The ensemble is dispersionless for the Hamiltonian, and hence all members will produce identical outcomes when H_S is measured. The same holds for a measurement of $\tilde{O}$: since $\tilde{O}$ is a function of H_S, and $|\phi_n\rangle$ is its eigenstate, with

$$\tilde{O}|\phi_n\rangle=\langle O\rangle_n|\phi_n\rangle, \tag{32}$$

all the members of the ensemble will therefore reveal the same value $\langle O\rangle_n$ in the measurement of $\tilde{O}$. It is not necessary to conclude that, paradoxically, an individual system carries complete information about the quantum state, i.e., that it ''knows'' to which ensemble it belongs.

Finally, I want to discuss two possible objections to the present conclusions. First, an essential assumption I have used in Sec. IV is that the approximation (9) is valid for all states $|\phi_n\rangle$. However, one may object that this is too restrictive. A protective measurement might still be of interest, if the approximation is valid only for some subset, call it J, of $\{|\phi_1\rangle,|\phi_2\rangle,\ldots\}$.

In that case, the procedure would allow us to determine the state only when it is given that the initial state belongs to the subset J. However, since H_S is conserved in the subspace spanned by the set J, the final state lies in the same subspace. Thus, effectively, it then suffices to restrict our attention to a reduced Hilbert space, spanned by J. But in this reduced space we can make the same argument as above, because the approximation will now by valid for all eigenvectors of H_S in the reduced space. Hence this escape route will not bring about any essential change in our conclusions.

A second objection may be that I have not discussed the possibility of changing H_S between two protective measurements (e.g., by applying or varying some external fields). Indeed, one can imagine that a first protective measurement measures an observable O which commutes with H_S, and that then the Hamiltonian is changed to H_S' whereafter an observable O' such that $[O',H_S']=0$ is measured protectively, etc.

Thus, if $[H,H']\neq 0$, we might still be able to combine information from incompatible measurement contexts into one experiment. The problem with this proposal is of course that one must take care of what happens to the state of the system. If H_S is changed abruptly, the system will generally not be in an eigenstate of H_S' at the start of the second measurement. On the other hand, if H_S is changed quasistatically, so that the adiabatic theorem is applicable, one can arrange that the system's state will transform into an eigenstate of the new Hamiltonian.

A more careful analysis than that offered here is necessary to decide whether such a proposal would lead to a refutation of the complementarity principle or whether one can still maintain that this measurement defines a single but time-dependent context. In any case, this proposal would differ from that of Aharonov and co-workers in the sense that here not only the protectiveness *of* the measurements but also what is *in between* the measurements is essential.

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[2]There may be slight distinction between $|\chi_0\rangle$ and $|\chi_0'\rangle$ because of a different acceleration experienced by the electron, due to the different shape of the tails of $|\phi_\pm\rangle$ in the region between the boxes. However, in the adiabatic limit, this distinction will disappear.

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