Spontaneous squeezing of a vortex in an optical lattice

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We study the equilibrium states of a vortex in a Bose-Einstein condensate in a one-dimensional optical lattice. We find that quantum effects can be important and that it is even possible for the vortex to be strongly squeezed, which reflects itself in a different quantum-mechanical uncertainty of the vortex position in two orthogonal directions. The latter is observable by measuring the atomic density after an expansion of the Bose-Einstein condensate in the lattice.

I. INTRODUCTION

Vortices play a crucial role in explaining the rotational and dissipative properties of superfluids. In the superfluid systems studied until now, such as liquid helium [1], superconductors [2], and Bose-Einstein condensates [3–11], a vortex line behaves as a classical object. However, placing a Bose-Einstein condensate in an optical lattice [12–16] leads to an unprecedented control over the system parameters, which has already enabled experimental studies on quantum phase transitions [15,17,18], superfluidity [12,13], and number squeezing [14]. Furthermore, a Bose-Einstein condensate in a one-dimensional optical lattice has a layered structure similar to the cuprate superconductors and is a promising way to achieve the quantum Hall regime in a Bose-Einstein condensed gas [19]. This suggests that a careful examination of the quantum properties of a vortex line in a one-dimensional optical lattice is warranted.

A characteristic feature of all superfluids is their ability to support persistent currents. Quantized vortices play a crucial role in understanding the decay of these currents, and also the superfluid's response to rotation or to an external magnetic field in the case of a superconductor. In the pioneering papers, Fetter developed the quantum theory of vortices in liquid helium [20] and so-called type-II superconductors [21]. In these systems, quantum fluctuations turned out to play such a small role that they are experimentally inaccessible. Here we show, however, that the truly quantum-mechanical behavior of the macroscopic vortex occurs in a Bose-Einstein condensate in a one-dimensional optical lattice. This is due to the reduced dimensionality as well as to the reduced number of atoms in every well of the optical lattice. Remarkably, it turns out that the vortex can spontaneously become strongly squeezed, which is reflected in the quantum-mechanical probability distribution of the vortex position. Unlike coherent states, which are described by the Gross-Pitaevskii theory, squeezed states are highly nonclassical.

II. BOSE-HUBBARD HAMILTONIAN

We consider a Bose-Einstein condensate in a one-dimensional optical lattice with lattice spacing $\lambda/2$, where λ

is the wavelength of the laser beams creating the standing wave of the lattice. The optical lattice splits the Bose-Einstein condensate into a stack of N_s weakly coupled pancake-shaped condensates, each containing N atoms. For concreteness we always use ^{87}Rb atoms in the following. The depth of the lattice V_L can be easily changed and controls the tunneling of an atom from one pancake-shaped condensate to the next, i.e., the strength of the interlayer Josephson coupling. The condensate also experiences a harmonic trapping potential in the radial direction with an oscillator frequency ω_r. The longitudinal trapping potential along the direction of the laser beams is assumed to be so weak that it can be neglected. While the lattice is taken to be deep enough to allow us to use a tight-binding approximation and to include only the weak nearest-neighbor Josephson coupling, it is also taken to be shallow enough to support a superfluid state as opposed to the Mott-insulator state [18].

Furthermore, we consider a vortex line that pierces at the position (x_n, y_n) through each layer of the stack labeled by the index n. In each layer, the density of the pancake-shaped condensate varies as a function of the radial distance. As a result, the vortex experiences an effective potential that depends on its distance from the origin [11]. Without rotation of the gas this potential is approximately an inverted parabola. Hence, the vortex is energetically unstable and will tend to spiral out of the system. However, the vortex is stabilized if we rotate the Bose-Einstein condensate with a rotation frequency Ω that is larger than a critical frequency Ω_c. In the rest of this paper we consider the regime where the vortex is energetically stable and $\Omega > \Omega_c$.

In the tight-binding approximation the attraction between the nearest-neighbor parts of the vortex turns out to be harmonic with respect to their separation. We denote the typical strength of this attraction by J_V. Physically, the attractive interaction is due to the energy cost for phase differences between the two layers. The theory of the resulting coupled harmonic oscillators can be quantized by introducing the bosonic annihilation operators for the eigenmodes of the vortex line [22–24]. These modes are the Kelvin modes (or kelvons), and correspond physically to a wiggling of the vortex line with a wavelength $2\pi/k$. The momentum-space operators $\hat{v}_k$ are related to the coordinate-space kelvon operators $\hat{v}_n$, and to the vortex positions through

$$\hat{x}_n = \frac{R}{2\sqrt{N}}(\hat{v}_n^\dagger + \hat{v}_n) = \frac{R}{2\sqrt{NN_s}}\sum_k e^{ikn\lambda/2}(\hat{v}_{-k}^\dagger + \hat{v}_k) \qquad (1)$$

and

$$\hat{y}_n = \frac{iR}{2\sqrt{N}}(\hat{v}_n^\dagger - \hat{v}_n), \qquad (2)$$

where R is the typical radial size of each pancake-shaped condensate. In this way we obtain the dispersion relation $\hbar\omega_K(k) = \hbar\omega_K(0) + J_V[1 - \cos(k\lambda/2)]$ of the kelvons [24]. The energetic stability of the vortex is reflected in a positive value of the dispersion at zero momentum. Physically, the frequency at zero momentum is the precession frequency of a slightly displaced straight vortex line around the center of the Bose-Einstein condensate.

It is crucial for our purposes to realize that, due to the Euler dynamics of the vortex, the coordinates $\hat{x}_n$ and $\hat{y}_n$ are canonically conjugate variables, and therefore they obey the Heisenberg uncertainty relation $[\hat{x}_n, \hat{y}_n] = iR^2/2N$. This means that the position of the vortex is always smeared out by quantum fluctuations. For a vortex in a lattice, these fluctuations can become large due to the reduced particle numbers in every layer, as opposed to the total number of particles, and the radial spreading of the pancake-shaped condensate wave function as the lattice depth is increased.

The kelvon dispersion relation was determined by taking into account only the lowest-order expansion of the effective potential experienced by the vortex. However, expanding up to fourth order in the vortex displacements results in an interaction $(V_0/2)\sum_n \hat{v}_n^\dagger\hat{v}_n^\dagger\hat{v}_n\hat{v}_n$ for the kelvons. In the regime where the vortex is stable, $V_0 < 0$ and this interaction is attractive. The physics of the vortex line is thus described by a one-dimensional Bose-Hubbard model with a negative interaction strength. The corresponding Hamiltonian is given by

$$\hat{H} = \sum_n [\hbar\omega_K(0) + J_V]\hat{v}_n^\dagger\hat{v}_n - \frac{J_V}{2}\sum_{\langle n,m\rangle}\hat{v}_m^\dagger\hat{v}_n + \frac{V_0}{2}\sum_n \hat{v}_n^\dagger\hat{v}_n^\dagger\hat{v}_n\hat{v}_n, \qquad (3)$$

where $\langle n,m\rangle$ denotes the nearest-neighbor layers.

We calculate the parameters of this Bose-Hubbard model using a variational ansatz that we used in our earlier work [24], where the density of the condensate wave function in each layer was proportional to $\exp[-(x^2+y^2)/R^2]$. In this way we find that the strength of the nearest-neighbor coupling is given by

$$J_V = \frac{\hbar\omega_r}{4\pi^2}\Gamma\left[0, \frac{l_r^4}{R^4}\right]\left(\frac{\omega_L\lambda}{\omega_r l_r}\right)^2\left(\frac{\pi^2}{4} - 1\right)\exp\left(-\frac{\lambda^2 m\omega_L}{4\hbar}\right)$$

and the interaction strength is given by

$$V_0 = \frac{2\hbar\omega_r(l_r/R)^2\Gamma[0,l_r^4/R^4] - 3\hbar\omega_r(l_r/R)^2 - 4\hbar\Omega}{4N},$$

where $\omega_L = \sqrt{8\pi^2 V_L/m\lambda^2}$ is the oscillator frequency of the optical lattice, $l_r = \sqrt{\hbar/m\omega_r}$, and $\Gamma[a,z]$ is the incomplete gamma function [25]. Furthermore, a straight and slightly

displaced vortex precesses around the condensate center with the frequency

$$\omega_K(k=0) = (\omega_r l_r^2/2R^2)(1 - \Gamma[0,l_r^4/R^4]) + \Omega. \qquad (4)$$

The precession frequency of the vortex changes from negative to positive at the critical rotation frequency

$$\Omega_c = (\omega_r l_r^2/2R^2)(\Gamma[0,l_r^4/R^4] - 1). \qquad (5)$$

When $\Omega > \Omega_c$, the vortex is locally energetically stable. The condensate size R is given by the solution of the transcendental equation $(l_r/R)^4[1 - \gamma + 2Na\sqrt{m\omega_L/2\pi\hbar} - 4\ln(l_r/R)] = 1$, where γ is the Euler-Mascheroni constant and a is the three-dimensional scattering length. Without the kelvon interaction, the ground state of the vortex corresponds to a vortex line along the symmetry axis of the Bose-Einstein condensate with a quantum-mechanical uncertainty such that $\langle\hat{x}_n^2\rangle = \langle\hat{y}_n^2\rangle$. An attractive interaction will lead to squeezing with $\langle\hat{x}_n^2\rangle \neq \langle\hat{y}_n^2\rangle$, as we show next.

III. VORTEX SQUEEZING

First we have to identify the correct order parameter for the vortex line. Since $\Omega > \Omega_c$, there occurs no Bose-Einstein condensation of the kelvons, and the appropriate order parameter for the vortex line is not $\langle\hat{v}_n\rangle$, which would signal the above-mentioned energetic instability and the tendency of the vortex to move away from the symmetry axis. However, the correct order parameter can be identified with $\Delta = V_0\langle\hat{v}_n\hat{v}_n\rangle$. To arrive at the associated mean-field theory, we quadratically expand the Hamiltonian around this order parameter and obtain, apart from a constant [26],

$$\hat{H} = \sum_n [\hbar\omega_K(0) + J_V]\hat{v}_n^\dagger\hat{v}_n - \frac{J_V}{2}\sum_{\langle n,m\rangle}\hat{v}_m^\dagger\hat{v}_n$$
$$+ \sum_n \left[\frac{\Delta}{2}\hat{v}_n^\dagger\hat{v}_n^\dagger + \frac{\Delta^*}{2}\hat{v}_n\hat{v}_n\right]. \qquad (6)$$

We diagonalize this result by means of a Bogoliubov transformation. It is most convenient to work in momentum space where we define new creation and annihilation operators with the help of the Bogoliubov amplitudes u_k and v_k as $\hat{b}_k = u_k\hat{v}_k^\dagger - v_k\hat{v}_{-k}$ and $\hat{b}_k^\dagger = u_k^*\hat{v}_k - v_k^*\hat{v}_{-k}^\dagger$. Furthermore, to ensure bosonic commutation relations we must have $|u_k|^2 - |v_k|^2 = 1$. The Hamiltonian in Eq. (6) is then diagonalized with the choice of either

$$|v_k|^2 = \frac{|\Delta|^2}{[E(k) + \hbar\omega_K(k)]^2 - |\Delta|^2} \qquad (7)$$

or

$$|u_k|^2 = \frac{[E(k) + \hbar\omega_K(k)]^2}{[E(k) + \hbar\omega_K(k)]^2 - |\Delta|^2}, \qquad (8)$$

where $E(k) = \sqrt{\hbar\omega_K(k)^2 - |\Delta|^2}$ is the dispersion of the Bogoliubov quasiparticles.

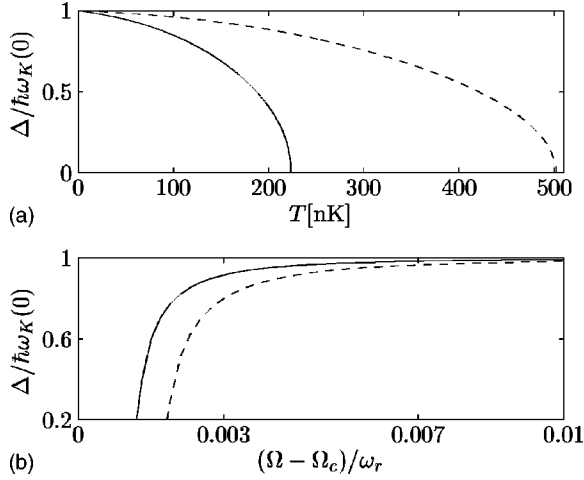Requiring self-consistency of the approach leads to a gap equation

FIG. 1. The solution of the gap equation as a function of temperature and rotation frequency. (a) Shows the order parameter as a function of temperature for two different rotation frequencies. The solid line represents $\Omega - \Omega_c = 0.01 \times \omega_r$ and the dashed line represents $\Omega - \Omega_c = 0.02 \times \omega_r$. The result for the order parameter is scaled to the precession frequency of the vortex. (b) Shows the order parameter as a function of rotation frequency at two different temperatures. The solid line represents a temperature $T = 10$ nK and the dashed line represents the temperature $T = 20$ nK. The curves are calculated for the parameters $\lambda = 795$ nm, $\omega_r = 2\pi \times 100$ Hz, $N = 1000$, $N_s = 51$, and $V_L = 15 E_r$, where $E_r = 2\pi^2 \hbar^2 / m\lambda^2$ is the recoil energy of the atom after the absorption of a photon from the laser beam. These parameters are representative of current experimental capabilities. We also use this same set of parameters in the other figures.

$$\frac{1}{V_0} = -\frac{1}{N_s} \sum_k \frac{1 + 2N_k}{2E(k)}, \qquad (9)$$

where $N_k = 1/(e^{\beta E(k)} - 1)$ is the Bose distribution. We solve the gap equation for the order parameter, analogous to the BCS theory of superconductors [27]. In Fig. 1 we present the behavior of the order parameter as a function of temperature and rotation frequency for typical parameter values. We see from Fig. 1 that there is a critical temperature below which the order parameter becomes nonzero and then increases monotonically with decreasing temperature. Furthermore, the absolute value of the order parameter at zero temperature is usually close to the precession frequency. The deviation becomes large only very close to the critical rotation frequency Ω_c.

The complex order parameter $\Delta = |\Delta| e^{i\phi}$ has an interesting physical interpretation in terms of the quantum-mechanical uncertainty of the vortex position. It turns out that in a coordinate system rotated by an angle θ, we have

$$\langle \hat{y}_n^2 \rangle - \langle \hat{x}_n^2 \rangle = \frac{|\Delta| R^2 \cos(\phi - 2\theta)}{|V_0| N N_s}, \qquad (10)$$

where ϕ is the phase of the order parameter. This implies that a nonzero value of the order parameter is reflected in the squeezing of the vortex position distribution. The main axis of the uncertainty ellipse is at an angle $\theta = \phi/2$ with the x
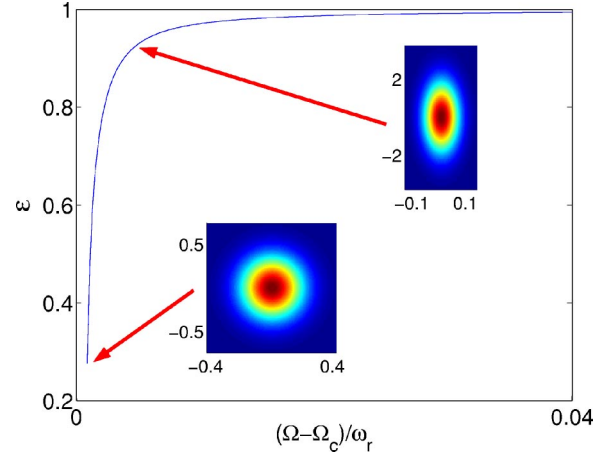

FIG. 2. (Color online) The squeezing of the quantum-mechanical position uncertainty ellipse. The figure shows the relative squeezing $\varepsilon = \langle \hat{y}^2 - \hat{x}^2 \rangle / \langle \hat{x}^2 + \hat{y}^2 \rangle$ of the uncertainty ellipse of the vortex position as a function of rotation frequency at a low temperature of 5 nK. Subplots show samples of the Gaussian probability distributions for the vortex position for two different rotation frequencies. In the subplots, we use the trap length $l_r = \sqrt{\hbar/m\omega_r}$ as a unit of length. In order to ease visual inspection, we use different aspect ratios for the axes of the subplots. For the parameters used, the coherence length at the center of the condensate is about $0.34 l_r$.

axis. In equilibrium, the uncertainty ellipse of the vortex position distribution is independent of the layer index n. Therefore, the measurement of the vortex positions in different layers samples the same distribution and provides a signature for the expected transition into a squeezed state. In Fig. 2, we plot the relative squeezing as a function of the rotation frequency at a fixed temperature. As can be seen from this figure, the size of the uncertainty ellipse can easily be several times the radial trap length, and it also becomes very strongly deformed as the rotation frequency is increased. Furthermore, the long axis of the uncertainty ellipse can be much larger than the vortex core size. The results in Fig. 2 were obtained with only one set of typical parameters. By a careful choice of, for example, lattice depth, number of particles, number of layers, or temperature, a considerable degree of tunability is possible. For example, a shallower lattice implies a larger relative squeezing.

IV. PHASE FLUCTUATIONS

For our theory to be valid, constraints have to be set for the temperature. Since we assume a pure condensate, we are ignoring the influence of the noncondensate atoms on the vortex line. Therefore, the temperature should be much lower than the critical temperature for Bose-Einstein condensation in the layers. This condition is relatively easy to satisfy. However, a more stringent condition is set by the phase fluctuations of the order parameter. In an infinite one-dimensional system, phase fluctuations destroy the long-range order. In the finite system we are considering here, phase fluctuations can only be excited if the temperature is high enough [28–30]. In order to calculate the energy cost for a phase gradient of the order parameter, we must deter-

mine the associated stiffness or superfluid density $\rho_S(T)$. Since the lattice breaks the Galilean invariance, this calculation is not entirely standard.

The superfluid density is defined by the system's response to an imposed phase gradient [31]. Therefore, we want to study the long-wavelength effects of the transformation $\hat{v}_n \to \hat{v}_n e^{i\phi_n/2}$. The phase gradients enter only in the nearest-neighbor coupling and result in an interaction energy

$$\hat{H}_I = -\frac{J_V}{2}\sum_{\langle n,m\rangle} \hat{v}_m^\dagger \hat{v}_n [i(\phi_n - \phi_m)/2 - (\phi_n - \phi_m)^2/8],$$

which can be considered as a disturbance in the long-wavelength limit. With this interaction energy, we calculate the contribution to the free energy of the phase gradients in second-order perturbation theory. Up to second order, the effective imaginary time action for the phase fluctuations is

$$S_\phi = \int_0^{\hbar\beta} d\tau \langle H_I(\tau)\rangle - \frac{1}{2\hbar}\int_0^{\hbar\beta}\int_0^{\hbar\beta} d\tau\, d\tau' \langle H_I(\tau)H_I(\tau')\rangle.$$

Furthermore, it is sufficient to keep only the terms up to second order in the phase differences between layers. In this way, we find the action for the phase gradients as

$$S_\phi = \int_0^{\hbar\beta} d\tau \frac{J_V}{8N_s}\sum_{k'}\{\cos(k'\lambda/2)[|u_{k'}|^2 N_{k'} + |v_{k'}|^2 (N_{k'}+1)]$$

$$- J_V\beta\sin^2(k'\lambda/2)N_{k'}(N_{k'}+1)\}\sum_k k^2 \phi_k \phi_{-k}. \qquad (11)$$

This result should be compared with the standard (Galilean invariant) Landau [27] result

$$S_L = \int_0^{\hbar\beta} d\tau \int dz \frac{\hbar^2}{2m_\Delta}\rho_s(T)\langle\nabla\phi\rangle^2, \qquad (12)$$

where the superfluid density at temperature T is

$$\rho_s(T) = \frac{1}{N_s\lambda}\sum_k \left[|u_k|^2 N_k + |v_k|^2(N_k+1) - \frac{\hbar^2\beta}{m_\Delta}k^2 N_k(N_k+1)\right]. \qquad (13)$$

When we take the lattice spacing to zero while keeping $J_V\lambda^2$ constant, we can see that Eq. (11) recovers the standard result when the mass of the pairing field is defined as $m_\Delta = 2m_K$, where

$$m_K = \frac{4\hbar^2}{J_V\lambda^2} \qquad (14)$$

is the effective mass of the kelvons. Furthermore, from this we see that the superfluid density in our case is defined by

$$\rho_s(T) = \frac{1}{N_s\lambda}\sum_k \{\cos(k\lambda/2)[|u_k|^2 N_k + |v_k|^2(N_k+1)]$$

$$- J_V\beta\sin^2(k\lambda/2)N_k(N_k+1)\}. \qquad (15)$$

Equating the energy cost due to a $4\pi/N_s\lambda$ phase gradient with the thermal energy gives us an estimate for the temperature scale $T_\phi = J_V\rho_S(T)\pi^2\lambda/2N_s k_B$ of the phase fluctuations.
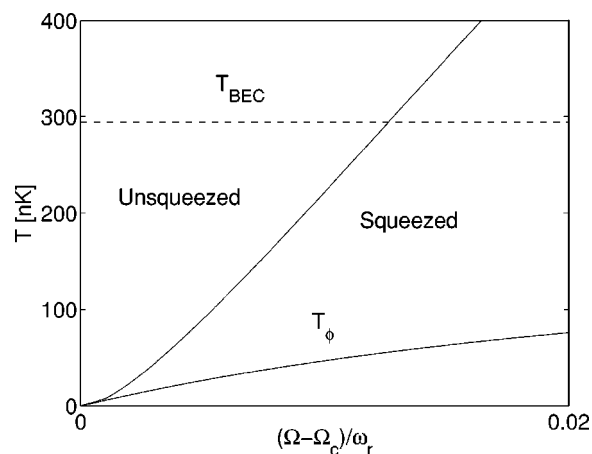

FIG. 3. The phase diagram for the vortex squeezing transition. The uppermost solid line is the critical temperature for the squeezing transition. The assumption of a pure condensate implies temperatures well below the critical temperature of the Bose-Einstein condensation indicated by the dashed line. Phase fluctuations can be ignored well below the lowest line.

In order to avoid phase fluctuations, we should be well below this temperature scale.

At temperatures higher than T_ϕ, the vortex can still be squeezed, but the phase fluctuations result in a different main axis of the vortex position uncertainty ellipse in different layers. In Fig. 3 we plot the critical temperature for Bose-Einstein condensation, the critical temperature for the transition into a squeezed vortex, and the temperature scale of the phase fluctuations for typical parameter values. This figure indicates that the vortex spontaneously squeezes at a temperature that is easily accessible experimentally. Below this temperature there is a region in which the vortex is squeezed, but with fluctuating main axes. At even lower temperatures, the phase fluctuations become negligible and long-range order over the size of the system is established.

V. SUMMARY AND CONCLUSIONS

We studied the equilibrium squeezing of the vortex line in an optical lattice and predicted that strong squeezing is indeed possible in the experimentally realistic parameter regime. Although the kelvon interaction induced by the vortex displacement is the most important one, other mechanisms for kelvon interactions also exist. In principle, the kelvons are also coupled to the collective modes of the Bose-Einstein condensate, for example, to the quadrupole mode [24,32]. Such processes typically induce an effective attractive kelvon interaction, and thus renormalize the value of the interaction strength, but do not make it positive. Therefore, we expect that the squeezing transition is robust with respect to coupling to the collective modes of the condensate.

ACKNOWLEDGMENTS

[1] R. J. Donnelly, *Quantized Vortices in Helium II* (Cambridge University Press, Cambridge, 1991).

[2] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).

[3] M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **83**, 2498 (1999).

[4] K. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000).

[5] J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle, Science **292**, 476 (2001).

[6] E. Hodby, G. Hechenblaikner, S. A. Hopkins, O. M. Maragó, and C. J. Foot, Phys. Rev. Lett. **88**, 010405 (2002).

[7] D. A. Butts and D. S. Rokshar, Nature (London) **397**, 327 (1999).

[8] J. E. Williams and M. J. Holland, Nature (London) **401**, 568 (1999).

[9] R. J. Dodd, K. Burnett, M. Edwards, and C. W. Clark, Phys. Rev. A **56**, 587 (1997).

[10] F. Zambelli and S. Stringari, Phys. Rev. Lett. **81**, 1754 (1998).

[11] A. L. Fetter and A. A. Svidzinsky, J. Phys.: Condens. Matter **13**, R135 (2001), and references therein.

[12] F. S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, and M. Inguscio, Science **293**, 843 (2001).

[13] S. Burger, F. S. Cataliotti, C. Fort, F. Minardi, M. Inguscio, M. L. Chiofalo, and M. P. Tosi, Phys. Rev. Lett. **86**, 4447 (2001).

[14] C. Orzel, A. K. Tuchman, M. L. Fenselau, M. Yasuda, and M. A. Kasevich, Science **291**, 2386 (2001).

[15] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature (London) **415**, 39 (2002).

[16] M. Greiner, O. Mandel, T. W. Hänsch, and I. Bloch, Nature (London) **419**, 51 (2002).

[17] D. Jaksch, C. Bruder, J. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).

[18] D. van Oosten, P. van der Straten, and H. T. C. Stoof, Phys. Rev. A **63**, 053601 (2001).

[19] E. Cornell (private communication).

[20] A. Fetter, Phys. Rev. **162**, 143 (1967).

[21] A. Fetter, Phys. Rev. **163**, 390 (1967).

[22] V. Bretin, P. Rosenbusch, F. Chevy, G. V. Shlyapnikov, and J. Dalibard, Phys. Rev. Lett. **90**, 100403 (2003).

[23] T. Mizushima, M. Ichioka, and K. Machida, Phys. Rev. Lett. **90**, 180401 (2003).

[24] J.-P. Martikainen and H. T. C. Stoof, Phys. Rev. Lett. **91**, 240403 (2003).

[25] Alternatively, the incomplete gamma function can be expressed in terms of the perhaps more familiar exponential integral as $\Gamma[1-p,z]=z^{1-p}E_p(z)$.

[26] H. T. C. Stoof, Phys. Rev. A **49**, 3824 (1994).

[27] J. R. Schrieffer, *Theory of Superfluidity* (Benjamin, New York, 1964).

[28] D. S. Petrov, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **85**, 3745 (2000).

[29] J. O. Andersen, U. A. Khawaja, and H. T. C. Stoof, Phys. Rev. Lett. **88**, 070407 (2002).

[30] U. A. Khawaja, J. O. Andersen, N. P. Proukakis, and H. T. C. Stoof, Phys. Rev. A **66**, 013615 (2002).

[31] A. M. Rey, K. Burnett, R. Roth, M. Edwards, C. J. Williams, and C. W. Clark, J. Phys. B **36**, 825 (2003).

[32] J.-P. Martikainen and H. T. C. Stoof, Phys. Rev. A **69**, 053617 (2004).