Noisy dynamics of a vortex in a partially Bose-Einstein condensed gas

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We study the dynamics of a straight vortex line in a partially Bose-Einstein condensed atomic gas. Using a variational approach to the stochastic field equation that describes the dynamics of the condensate at nonzero temperature, we derive the stochastic equations of motion for the position of the vortex core. Using these results, we calculate the time it takes the vortex to spiral out of the condensate. Due to the fact that we include thermal fluctuations in our description, this lifetime of the vortex is finite even if its initial position is in the center of the condensate.

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I. INTRODUCTION

Contrary to classical fluids, superfluids support rotation only through quantized vortices. Since quantized vortices are therefore one of the hallmarks of superfluidity, the experimental and theoretical study of these topological excitations has attracted a great deal of attention in the field of Bose-Einstein condensed gases. Following their first experimental observation [1], they have now been observed and experimentally studied by various groups [2–4].

Theoretically, the dynamics of a single vortex line in a Bose-Einstein condensate has been studied extensively in the zero-temperature limit [5]. In the absence of external rotation, the vortex is predicted to precess around the center of the condensate, which has indeed been observed experimentally by Anderson *et al.* [6]. However, in this experiment it is also observed that the distance of the vortex core to the center of the condensate increases with time, i.e., the vortex spirals out of the condensate. This observation is a sign of the presence of dissipation and thus cannot be explained on the basis of a zero-temperature approach. To understand it, we have to include the effects of the noncondensed component of the gas. Although the effects of the noncondensed thermal cloud on the equilibrium properties of rotating Bose gases have been investigated [7–10], the nonequilibrium dynamics of a single vortex at nonzero temperatures has attracted relatively little attention.

It is the purpose of this paper to study the effects of the thermal cloud on the motion of a single vortex in a Bose-Einstein condensate. The starting point of our study is the stochastic Gross-Pitaevskii equation derived by one of us [11,12]. This equation generalizes the usual Gross-Pitaevskii equation, which provides an accurate description of the dynamics of the condensate at zero temperature, to nonzero temperatures by the inclusion of a dissipative term that describes the growth or decay of the condensate. Moreover, thermal fluctuations are included by an additive noise term that is related to the growth or decay of the condensate by means of a fluctuation-dissipation theorem. Although this equation can be solved numerically [13], we intend to capture as much of the physics as possible by using a variational approach to stochastic field equations that was developed by two of us [14].

To make the paper more self-contained, we briefly discuss the zero-temperature dynamics of a vortex in a Bose-Einstein condensate in Sec. II. In Sec. III we then derive the stochastic equations of motion that describe the motion of the vortex core at a nonzero temperature. In Sec. IV we use these equations to derive an equation for the average distance of the vortex to the center of the condensate and use the latter result to calculate the lifetime of the vortex. We compare our results with the available theoretical results [15,16]. Unfortunately, there does not exist a detailed experimental study of the lifetime of the vortex. We end in Sec. V with our conclusions.

II. VORTEX DYNAMICS AT ZERO TEMPERATURE

The dynamics of a Bose-Einstein condensate is, at zero temperature, well described by the Gross-Pitaevskii equation

$$i\hbar\frac{\partial\,\phi(\mathbf{x},t)}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+V^{\mathrm{ext}}(\mathbf{x})+T^{2\mathrm{B}}|\phi(\mathbf{x},t)|^2\right]\phi(\mathbf{x},t),\tag{1}$$

for its macroscopic wave function $\phi(\mathbf{x},t)$. Here m is the atomic mass, and $T^{2\mathrm{B}}=4\pi a\hbar^2/m$ is the two-body transition (T) matrix at zero energy, with $a>0$ the s-wave scattering length of the atoms of interest. We take the external trapping potential of the form

$$V^{\mathrm{ext}}(\mathbf{x})=\frac{1}{2}m[\omega^2(x^2+y^2)+\omega_z^2z^2],\tag{2}$$

with $\omega_z \gg \omega$, which implies that we have a pancake-shaped condensate. The reason for choosing this geometry is that it allows us to neglect the curvature of the vortex line, provided that the vortex is close to the center of the condensate.

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To study the dynamics of a vortex at zero temperature we use the variational *ansatz*

$$\phi(\mathbf{x},t) = \frac{\sqrt{N_c}}{\sqrt{\pi^{3/2}q^2q_z[q^2+u_x^2(t)+u_y^2(t)]}} \times\{[x-u_x(t)]+i[y-u_y(t)]\}\exp\left\{-\frac{x^2+y^2}{2q^2}-\frac{z^2}{2q_z^2}\right\}, \tag{3}$$

where N_c is the number of condensate atoms. This *ansatz* has the same form as the exact wave function for a noninteracting Bose-Einstein condensate with a vortex along the z axis. We treat the width of the condensate in the radial and axial directions, denoted by q and q_z, respectively, as time-independent variational parameters. The pancake-shaped geometry implies that $q_z \ll q$. The coordinates of the vortex in the x-y plane, denoted by $u_i(t)$, are taken to be time-dependent variational parameters. For the case that the vortex is close to the center of the condensate, we expect that the variational *ansatz* in Eq. (3) offers a quantitatively good description of the wave function of the condensate in the weakly interacting limit that is determined by the condition

$$\frac{N_c a}{\sqrt{\dfrac{\hbar}{m(\omega^2\omega_z)^{1/3}}}} \ll 1, \tag{4}$$

with N_c the number of atoms in the condensate.

To determine the equations for the variational parameters we note that the Gross-Pitaevskii equation follows from a variation of the action

$$S[\phi^*,\phi] = \int dt \int d\mathbf{x}\left\{\frac{i\hbar}{2}\left[\phi^*(\mathbf{x},t)\frac{\partial\phi(\mathbf{x},t)}{\partial t} - \phi(\mathbf{x},t)\frac{\partial\phi^*(\mathbf{x},t)}{\partial t}\right] + \phi^*(\mathbf{x},t)\left[\frac{\hbar^2\nabla^2}{2m} - V^{\text{ext}}(\mathbf{x}) - \frac{T^{2\text{B}}}{2}|\phi(\mathbf{x},t)|^2\right]\phi(\mathbf{x},t)\right\}. \tag{5}$$

After substitution of the *ansatz* in Eq. (3) into this action we find the result

$$S[\mathbf{u},q] = \int dt\left\{\frac{N_c\hbar}{q^2}[u_y(t)\dot{u}_x(t) - u_x(t)\dot{u}_y(t) + \omega_p(q,N_c)u_y^2(t) + \omega_p(q,N_c)u_x^2(t)] - N_cV(q,q_z,N_c) + O(u_i^3)\right\}, \tag{6}$$

with the precession frequency of the vortex equal to

$$\omega_p(q,q_z,N_c) = \frac{\hbar}{2mq^2} + \frac{m\omega^2q^2}{2\hbar} - \frac{aN_c\hbar}{\sqrt{2\pi}mq^2q_z}, \tag{7}$$

and the potential, which represents the total energy of the condensate per particle, given by

$$V(q,q_z,N_c) = \frac{\hbar^2}{mq^2} + \frac{\hbar^2}{4mq_z^2} + m\omega^2q^2 + \frac{1}{4}m\omega_z q_z^2 + \frac{aN_c\hbar^2}{2\sqrt{2\pi}mq^2q_z}. \tag{8}$$

As a result we observe that the equilibrium widths of the condensate in radial and axial direction, denoted from now on again by q and q_z, respectively, are determined by minimizing the potential $V(q,q_z,N_c)$ for a given number of condensate atoms. By varying the action with respect to the position of the vortex we find that the motion of the vortex is determined by the equations

$$\dot{u}_x(t) = -\omega_p(N_c)u_y(t), \quad \dot{u}_y(t) = \omega_p(N_c)u_x(t), \tag{9}$$

which imply that the vortex precesses around the center of the condensate with precession frequency $\omega_p(N_c)$, which is equal to $\omega_p(q,q_z,N_c)$ evaluated at the equilibrium values of q and q_z.

Although the above variational analysis has already provided us with a simple description of the precession of the vortex around the center of the condensate, it fails to account for the experimental obervation that the vortex spirals out of the condensate in the absence of rotation [6]. Contrary, the vortex remains, according to the equations of motion in Eq. (9), at a fixed distance from the condensate center. In particular, this implies that if the initial position of the vortex is the center of the condensate, it will remain there forever. This discrepancy between theory and experiment is resolved by including the effects of thermal fluctuations, as we will see in the next sections.

III. VORTEX DYNAMICS AT NONZERO TEMPERATURE

The dynamics of a Bose-Einstein condensate is at nonzero temperatures, i.e., in the presence of a thermal cloud at temperature $T=1/(k_B\beta)$ and with a chemical potential μ, determined by the stochastic Gross-Pitaevskii equation [11–14]

$$i\hbar\frac{\partial\phi(\mathbf{x},t)}{\partial t} = \left(1+\frac{\beta}{4}\hbar\Sigma^{\text{K}}(\mathbf{x},t)\right)\left\{-\frac{\hbar^2\nabla^2}{2m} + V^{\text{ext}}(\mathbf{x}) - \mu + T^{2\text{B}}|\phi(\mathbf{x},t)|^2\right\}\phi(\mathbf{x},t) + \eta(\mathbf{x},t). \tag{10}$$

This Langevin field equation quite generally generalizes the Gross-Pitaevskii equation to nonzero temperatures, and includes both dissipation, i.e., decay or growth of the Bose-Einstein condensate, and (thermal) fluctuations. The dissipation and the noise arise physically due to collisions between noncondensed atoms in which one of the atoms is scattered into the condensate, and the time-reversed process. The complex Gaussian noise in the stochastic Gross-Pitaevskii equation is completely determined by the correlations

$$\langle\eta^*(\mathbf{x},t)\eta(\mathbf{x}',t')\rangle = \frac{i\hbar^2}{2}\Sigma^{\text{K}}(\mathbf{x},t)\delta(t-t')\delta(\mathbf{x}-\mathbf{x}'), \tag{11}$$

where the strength of the noise is determined by the so-called Keldysh self-energy, given by

$$\hbar\Sigma^{\rm K}(\mathbf{x},t) = -4\pi i (T^{2\rm B})^2 \int \frac{d\mathbf{k}_1}{(2\pi)^3}\int \frac{d\mathbf{k}_2}{(2\pi)^3}\int \frac{d\mathbf{k}_3}{(2\pi)^3}(2\pi)^3 \times \delta(\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\,\delta(\epsilon_1-\epsilon_2-\epsilon_3) \times [N_1(1+N_2)(1+N_3)+(1+N_1)N_2N_3]. \quad (12)$$

In this expression, N_i is the Bose-distribution function of the thermal cloud, evaluated at an energy of a thermal particle, which is in the Hartree-Fock approximation given by

$$\epsilon_i = \frac{\hbar^2\mathbf{k}_i^2}{2m} + V^{\rm ext}(\mathbf{x}) + 2T^{2\rm B}|\langle\phi(\mathbf{x},t)\rangle|^2. \quad (13)$$

This approximation is valid in the regime where the temperature is large compared to the zero-point energy of the external trapping potential but smaller than the critical temperature. Note also that we have taken the thermal cloud to be in equilibrium, although this can easily be generalized.

In the first approximation we neglect the inhomogeneity of the thermal cloud over the size of the condensate. In a good approximation, the Keldysh self-energy then turns out to be given by [17]

$$\hbar\Sigma^{\rm K} = \frac{-48ima^2(k_{\rm B}T)^2}{\pi\hbar^2}, \quad (14)$$

which we will use from now on. Note that since the Keldysh self-energy determines both the dissipation and the strength of the fluctuations, the stochastic Gross-Pitaevskii equation can be shown to automatically fulfill the fluctuation-dissipation theorem which ensures that the condensate relaxes to the correct equilibrium distribution.

The (Wigner) probability distribution for the condensate wave function, which results from the stochastic field equation in Eq. (10), can be written as the functional integral

$$P[\phi,\phi^*;t] = \int_{\phi(\mathbf{x},t)=\phi(\mathbf{x})}^{\phi^*(\mathbf{x},t)=\phi^*(\mathbf{x})} d[\phi^*]d[\phi]\exp\left\{\frac{i}{\hbar}S^{\rm eff}[\phi^*,\phi]\right\}, \quad (15)$$

with the effective action given by

$$S^{\rm eff}[\phi^*,\phi] = \int_{t_0}^{t} dt' \int d\mathbf{x}\frac{2}{\hbar\Sigma^{\rm K}}\left|\left(i\hbar\frac{\partial}{\partial t'} + \left\{1+\frac{\beta}{4}\hbar\Sigma^{\rm K}\right\} \times\left[\frac{\hbar^2\nabla^2}{2m} - V^{\rm ext}(\mathbf{x}) + \mu - T^{2\rm B}|\phi(\mathbf{x},t')|^2\right]\right)\phi(\mathbf{x},t')\right|^2. \quad (16)$$

To determine the dynamics of the vortex at nonzero temperatures, we have to slightly generalize the *ansatz* in Eq. (3) to allow also for fluctuations in the number of condensate atoms and the global phase of the condensate wave function [14]. Hence we now use the *ansatz*

$$\phi(\mathbf{x},t) = \frac{\sqrt{N_{\rm c}(t)}e^{i\theta_0(t)}}{\sqrt{\pi^{3/2}q^2q_z[q^2+u_x^2(t)+u_y^2(t)]}}\{[x-u_x(t)] + i[y-u_y(t)]\}\exp\left\{-\frac{x^2+y^2}{2q^2}-\frac{z^2}{2q_z^2}\right\}. \quad (17)$$

For the moment we consider the case $T^{2\rm B}=0$, but keep the Keldysh self-energy nonzero. This implies that we are dealing with a noninteracting Bose gas in contact with a heat bath. Subsitution of the above *ansatz* into the action leads to

$$S^{\rm eff}[N_{\rm c},\theta_0,\mathbf{u}] = \int_{t_0}^{t} dt'\frac{2}{\hbar\Sigma^{\rm K}}\Bigg\{N_{\rm c}(t')\left[\hbar\frac{d\theta_0(t')}{dt'}+\mu_{\rm c}(t') + \frac{\hbar}{q^2}[u_x(t')\dot u_y(t') - u_y(t')\dot u_x(t')] - \mu\right]^2 + \frac{\hbar^2}{4N_{\rm c}(t')}\left[\frac{dN_{\rm c}(t')}{dt'} + \frac{\beta}{2}i\Sigma^{\rm K}[\mu_{\rm c}(t') - \mu]N_{\rm c}(t')\right]^2 + \frac{\hbar^2N_{\rm c}(t')}{q^2}[\dot u_x(t') + \omega_{\rm p}(q,0)u_y(t') - \gamma u_x(t')]^2 + \frac{\hbar^2N_{\rm c}(t')}{q^2}[\dot u_y(t') - \omega_{\rm p}(q,0)u_x(t') - \gamma u_y(t')]^2 + O(u_i^3)\Bigg\}, \quad (18)$$

where we have again taken for the time-independent variational parameters q and q_z the values obtained by minimizing the potential in Eq. (8). The damping rate γ is given by

$$\gamma = \frac{\beta\hbar^2 i\Sigma^{\rm K}}{8mq^2}, \quad (19)$$

and the condensate chemical potential reads

$$\mu_{\rm c}(t) = \frac{\partial(N_{\rm c}V(q,q_z,0))}{\partial N_{\rm c}} + \frac{\hbar^2}{2mq^4}[u_x^2(t)+u_y^2(t)]. \quad (20)$$

From the action for the variational parameters we are able to deduce the stochastic rate equation for the number of atoms and the stochastic equations of motion for the global phase and for the position of the vortex. First, the stochastic rate equation for the number of atoms is given by [14]

$$\frac{dN_{\rm c}(t)}{dt} = -\frac{\beta}{2}i\Sigma^{\rm K}[\mu_{\rm c}(t)-\mu]N_{\rm c}(t) + 2\sqrt{N_{\rm c}(t)}\,\eta(t), \quad (21)$$

with the correlations of the Gaussian noise given by

$$\langle\eta(t')\eta(t)\rangle = \frac{i\Sigma^{\rm K}}{4}\delta(t-t'). \quad (22)$$

Second, the stochastic equation of motion for $\theta_0(t)$ is given by [14]

$$\hbar \frac{d\theta_0(t)}{dt} = \mu - \mu_c(t) - \frac{\hbar}{q^2}[u_x(t)\dot{u}_y(t) - u_y(t)\dot{u}_x(t)] + \frac{\nu(t)}{\sqrt{N_c(t)}}, \tag{23}$$

where the correlation of the Gaussian noise are given by

$$\langle \nu(t')\nu(t)\rangle = \frac{i\hbar^2\Sigma^K}{4}\delta(t-t'). \tag{24}$$

Since we are mostly interested in the stochastic equations of motion for the position of the vortex, we neglect from now on the fluctuations in the number of condensate atoms and the global phase of the condensate. The Langevin equations for the position of the vortex core are, from the action in Eq. (18), seen to be given by

$$\dot{u}_x(t) = -\omega_p(0)u_y(t) + \gamma u_x(t) + \eta_x(t),$$

$$\dot{u}_y(t) = \omega_p(0)u_x(t) + \gamma u_y(t) + \eta_y(t), \tag{25}$$

where the Gaussian noise is completely determined by

$$\langle \eta_i(t)\eta_j(t')\rangle = \frac{q^2 i\Sigma^K}{4N_c}\delta_{ij}\delta(t-t') \equiv \sigma\delta_{ij}\delta(t-t'). \tag{26}$$

The stochastic equations for the position of the vortex core in Eq. (25) have two extra terms with respect to the zero-temperature result in Eq. (9). First, the stochastic equations have a damping term proportional to the damping rate γ. As we will see, this term leads to exponentional increase in the distance between the vortex position and the center of the condensate. Secondly, the stochastic equations have an additive noise term which represents the effect of thermal fluctuations on the position of the vortex core. If these noise terms would not have been present, the lifetime of the vortex would again be infinite if the initial position of the vortex is the center of the condensate. Physically, the vortex is "kicked" out of the center of the condensate due to thermal fluctuations, which are represented by the Gaussian noise terms in Eq. (25).

Although we have, in the first instance, derived the stochastic equation for the position of the vortex core by assuming that we are dealing with a Bose-Einstein condensate of noninteracting atoms, we can now consider the interacting case by replacing the precession frequency in $\omega_p(0)$ by $\omega_p(N_c)$. The Fokker-Planck equation, which determines the probability distribution for the position of the vortex, is then given by

$$\frac{\partial P[u_x,u_y,t]}{\partial t} = \left\{ \frac{\partial}{\partial u_x}[\omega_p(N_c)u_y - \gamma u_x] + \frac{\partial}{\partial u_y}[-\omega_p(N_c)u_x - \gamma u_y] + \frac{\sigma}{2}\left[\frac{\partial^2}{\partial u_x^2} + \frac{\partial^2}{\partial u_y^2}\right]\right\} P[u_x,u_y,t]. \tag{27}$$

In the next section, we use this equation to determine the average distance of the vortex to the center of the condensate. This result is then used to obtain the lifetime of the vortex.

IV. LIFETIME OF THE VORTEX

The average distance of the vortex to the center of the condensate is given by

$$r(t) \equiv \langle \sqrt{u_x^2 + u_y^2}\rangle(t) \equiv \int du_x du_y \sqrt{u_x^2 + u_y^2}\, P[u_x,u_y,t]. \tag{28}$$

With the Fokker-Planck equation for the position of the vortex core in Eq. (27) we find, with the use of a partial integration, that $r(t)$ obeys the equation of motion

$$\frac{dr(t)}{dt} = \gamma r(t) + \frac{\sigma}{2r(t)}. \tag{29}$$

The general solution of this equation is given by

$$r(t) = \frac{\sqrt{[2\gamma r^2(0) + \sigma]e^{2\gamma t} - \sigma}}{\sqrt{2\gamma}}. \tag{30}$$

Note that for small times t and distance $r(t)$ we have that $r(t) \propto \sqrt{t}$. This may be understood from the fact that, if we neglect the contribution of the precession frequency $\omega_p(N_c)$ and the damping rate γ to the Fokker-Planck equation in Eq. (27), the vortex simply undergoes Brownian motion. For large times t and distance $r(t)$ we have that $r(t) \propto e^{\gamma t}$, as is easily seen from Eqs. (29) and (30).

For a given initial position $r(0) = r_{min}$, the vortex lifetime is defined as the time it takes the vortex to reach the edge of the condensate at r_{max}. Of course, for our Gaussian *ansatz* the edge of the condensate is not defined uniquely, but we could, for instance, take $r_{max} = q$. In the Thomas-Fermi limit we have $r_{max} = R$, with R the Thomas-Fermi radius. The lifetime of the vortex is in first instance given by

$$\tau = \frac{1}{2\gamma}\ln\left[\frac{r_{max}^2 + \sigma/(2\gamma)}{r_{min}^2 + \sigma/(2\gamma)}\right]. \tag{31}$$

Let us first discuss the case where we neglect the contribution of the noise due to thermal fluctuations, i.e., we take $\sigma = 0$. The lifetime of the vortex is then given by

$$\tau_0 = \frac{\pi}{6}\left(\frac{q}{a}\right)^2 \frac{\hbar}{(k_B T)}\ln\left(\frac{r_{max}}{r_{min}}\right). \tag{32}$$

This result is very similar to the result obtained by Fedichev and Shlyapnikov [15], who indeed neglect thermal fluctuations. Schmidt *et al.* [16], who include fluctuations, numerically find a similar result. In particular, the dependence on the ratio r_{max}/r_{min} is identical, and we observe that, in the absence of thermal fluctuations, the lifetime of the vortex diverges as $r_{min} \to 0$. Interestingly, although we consider the weakly interacting limit, as opposed to Fedichev and Shlyapnikov who consider the Thomas-Fermi limit, the temperature dependence of the prefactor of the lifetime in Eq. (32) is the same in both results. This is even more surprising because of the fact that the dissipation of the vortex considered by Fedichev and Shlyapnikov is physically due to scattering of quasiparticles of the vortex core. This mechanism of decay is physically quite distinct from the collisional effects we consider here.

Including thermal fluctuations, the vortex lifetime is finite even if its initial position is the center of the condensate. In particular, for that case we have that

$$\tau = \frac{\pi}{12}\left(\frac{q}{a}\right)^2 \frac{\hbar}{(k_B T)} \ln\left[1 + \frac{N_c \hbar^2 r_{\max}^2}{m q^4} \frac{1}{(k_B T)}\right]. \tag{33}$$

We observe that the leading low-temperature dependence of the lifetime of the vortex is $T^{-1} \ln[N_c \hbar\omega/(k_B T)]$, whereas in the case where we neglected the thermal fluctuations we had that the lifetime was proportional to T^{-1}.

V. CONCLUSIONS

We have studied, by means of a variational analysis, the motion of a vortex in a Bose-Einstein condensate. The main results were the stochastic equations of motion for the position of the vortex and their corresponding Fokker-Planck equation, which enabled us to analytically calculate the lifetime of the vortex. We have compared our results for the latter with the available theoretical results [15,16]. It is surprising that, although the collisional decay mechanism that is considered here is physically quite different from the decay mechanism discussed by Fedichev and Shlyapnikov [15], we nevertheless find the same temperature dependence if we neglect the thermal fluctuations. Including thermal fluctuations leads to a different temperature dependence. Note also that, because we consider condensate dissipation due to collisions, the lifetime of the vortex is inversely proportional to the square of the scattering length. The result for the lifetime found by Fedichev and Shlyapnikov is proportional to $a^{-4/5}$. It would therefore be very interesting to experimentally measure the dependence on the temperature and the scattering length of the vortex lifetime.

Our present analysis was limited to the case where the overall density profile is close to a Gaussian, i.e., the so-called weakly interacting limit. In principle, our treatment can be generalized to the case where the condensate is described by a Thomas-Fermi density profile. However, in this limit technical problems occur due to the fact that we need to incorporate the density profile of the core of the vortex. This strongly limits the feasibility of analytical results. We expect, however, that the main difference between the weakly interacting limit and the Thomas-Fermi limit is the difference in the precession frequency of the vortex, which can easily be found from a zero-temperature approach [5]. Therefore, we believe that the lifetime results presented here provide also in the Thomas-Fermi limit a more than qualitative understanding of the dynamics of a vortex at nonzero temperatures.

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