

JAN BERGSTRA. *Dynamic recursion theory.*

We add a dynamic feature to recursion theory in order to increase its applicability to the theory of computing devices.

Therefore we formally define processes over a given language. Then we take "process" instead of "function" as the primitive for our recursion theory. A notion of relative recursion is developed and a degree structure D_0^{\leq} is defined on the set of processes. Looking at ordinary degree theory as a classification of sets of information from an algorithmic point of view, we may see D_0^{\leq} as a classification of memory structures in very much the same way. A calculus of operators is developed in style of recursion on continuous functionals of type two. We study D_0^{\leq} in some detail proving e.g. the existence of minimal degrees.

A subrecursive version $D_0^{\leq c}$ of D_0^{\leq} is defined. $D_0^{\leq c}$ contains computable processes which are related by means of finite operator call automata. It turns out to have an interesting structure. We find the existence of a maximal degree and minimal degrees. Furthermore we distinguish and relate several classes of degrees e.g. linear degrees, monotonic degrees, information degrees, degrees admitting initialisation and reducible degrees. Many open problems are formulated.

Then we discuss a version of dynamic recursion theory in which time plays an explicit role. This leads to recursion on autonomous processes, which can be defined surprisingly easily given the previous definitions.

Finally we show how to translate some notions of abstract complexity theory into this framework. We explain the use of our DRT in the clarification of the logical meaning of notions like datastructure (nondeterministic) datatype and (relative) implementability.

EGON BÖRGER and HANS KLEINE BÜNING. *The reachability problem for Petri nets and decision problems for Skolem arithmetic.*

We show that the decision problem for the class C_0 of all closed universal Horn formulae in prenex conjunctive normal form of extended Skolem arithmetic without equality (i.e. first-order formulae built up from the multiplication sign, constants for the natural numbers and free occurring symbols) is polynomially equivalent to the reachability problem for Petri nets—and therefore recursive following Sacerdote and Tenney (1977)—if restricted to the class of formulae with (a) only monadic predicate symbols, with (b) only binary disjunctions in the quantifier free matrix and (c) without terms containing a variable more than once. We show that this result is optimal in the sense that leaving out one of the restrictions (a) to (c) yields classes of formulae whose decision problem can assume any prescribed recursively enumerable complexity in terms of many-one degrees of unsolvability. As a corollary we obtain a lower bound for the complexity of any decision procedure for C_0 , namely that the decision problem for the subclass S_0 of all formulas in C_0 of form

$$\bigwedge_x (Qa \wedge \neg Qb \bigwedge_{i \leq r} (Qc_i x \rightarrow Qd_i x))$$

where Q denotes a monadic predicate symbol and a, b, c_i, d_i, r are positive natural numbers—is polynomially equivalent to that of C_0 and requires exponential space.

PETER CLOTE. *Recursion theoretic analysis of the barrier theorem.*

In [3] Jockusch considered the recursion theoretic version of Ramsey's theorem: given a recursive partition $G: [\mathbb{N}]^n \rightarrow m$ for $n, m \geq 2$ what can be said about the complexity of the infinite homogeneous set? In the positive direction Jockusch showed that there is always an infinite homogeneous Π_1^0 set. In the negative direction he constructed a recursive partition $G_n: [\mathbb{N}]^n \rightarrow 2$ for each $n \geq 2$ without any infinite homogeneous Σ_1^0 set. Schematically

$$\mathbb{N} \rightarrow (\Pi_1^0)_m^n, \quad \mathbb{N} \nrightarrow (\Sigma_1^0)_2^n.$$

Here we consider the recursion theoretic version of a generalization of Ramsey's theorem formulated by Nash-Williams. A barrier $B \subseteq [\mathbb{N}]^{<\omega}$ is a set of increasing tuples such that

- (1) the base of $B = \cup B$ is infinite;
- (2) $X \subseteq \cup B, X$ infinite, then there is an $s \in B$ where s is an initial segment of X (when listed in increasing order);
- (3) $s, t \in B$ then $s \not\mid t$ i.e. as sets $s \not\subseteq t$ and $t \not\subseteq s$.