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Part S1 - Quantum Fluids and Solids: Bose-Einstein condensation

Superfluid properties of atomic ^{6}Li in a magnetic trap

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Recently we showed that a homogeneous gas of spin-polarized ^{6}Li atoms can become superfluid at experimentally attainable temperatures and densities. Moreover we showed that the lifetime of the gas can be sufficiently long to perform experiments. The critical temperature depends on the densities of the two hyperfine states involved. Here we present a theory for the superfluid state for arbitrary ratios of these densities.

1. INTRODUCTION

Last year's achievement of Bose-Einstein condensation in weakly interacting atomic gases has been a major step forward in the field of atomic physics. In the experiments involved, dilute gases of ^{87}Rb [1], ^{7}Li [2] and ^{23}Na [3] were magnetically trapped and cooled in such a way that BEC could be observed. Now the experimental techniques and equipment are available, it has become possible to make a detailed study of the quantum degeneracy effects in these and other weakly interacting atomic gases. In particular, it would be interesting to trap not only bosonic particles, but also for example fermionic atoms as ^{6}Li. A combination of theoretical calculations and experimental results predicted a large and negative s-wave scattering length for ^{6}Li. This makes this gas a good candidate for observing a BCS transition [4] to a superfluid state at a reasonable temperature. In a recent publication [5] we indeed calculated T_c and showed that a homogeneous gas of spin polarized ^{6}Li becomes superfluid at temperatures of the order of those achieved in the BEC experiments with ^{7}Li. The main focus of this paper will be to present also a theory for the superfluid state of this gas.

2. THEORY

The system under consideration consists of a dilute, homogeneous gas of ^{6}Li atoms. The atoms are electron spin polarized and populate only two (of the three) nuclear spin states, with densities which we denote as $n_\uparrow$, respectively $n_\downarrow$. The reason for the presence of two spin states is that fermions in equal spin states cannot, due to the Pauli principle, interact via s-wave scattering and we expect at most an extremely low transition temperature to a superfluid state.

The hamiltonian of the gas is given by

$$H = \int d\vec{x} \Bigg\{ \sum_{\alpha=\uparrow,\downarrow} \psi^\dagger_\alpha(\vec{x}) \left(-\frac{\hbar^2\nabla^2}{2m} - \mu_\alpha \right) \psi_\alpha(\vec{x}) + + \frac{1}{2} \sum_{\alpha=\uparrow,\downarrow} V_0\, \psi^\dagger_\alpha(\vec{x}) \psi^\dagger_{-\alpha}(\vec{x}) \psi_{-\alpha}(\vec{x}) \psi_\alpha(\vec{x}) \Bigg\}, \quad (1)$$

where μ_α is the chemical potential of spin state $|\alpha\rangle$ and the interparticle (triplet) potential is approximated by a momentum independent local potential of strength $V_0 = \int d\vec{x} V(\vec{x})$. The density of particles in state $|\alpha\rangle$ is given by $n_\alpha = \langle \psi^\dagger_\alpha(\vec{x}) \psi_\alpha(\vec{x}) \rangle$. Assuming a nonzero value of the BCS order parameter

$$\Delta_0 = V_0 \langle \psi_\uparrow(\vec{x}) \psi_\downarrow(\vec{x}) \rangle, \quad (2)$$

the mean-field hamiltonian becomes quadratic in the particle fields ψ_α and can be solved exactly. However, to incorporate all two-body scattering processes correctly, the chemical potential for each spin state must be renormalized to $\mu'_\alpha = \mu_\alpha - T^{2B} n_{-\alpha}$, where $T^{2B} = \frac{4\pi a \hbar^2}{m}$ is the two-body scattering matrix and a the (negative) s-wave scattering length. Note that a Cooper pair consists of two atoms in different spin states. Applying the usual Bogoliubov transformation [4] yields a diagonal effective hamiltonian with eigenvalues given by:

$$\hbar\omega_{k,\alpha} = m_\alpha \delta\epsilon_F + \sqrt{\xi_k^2 + \Delta_0^2}, \quad (3)$$

where $m_\alpha = \pm\frac{1}{2}$ for $\alpha = \uparrow, \downarrow$ respectively. Furthermore $\delta\epsilon_F = \mu'_\uparrow - \mu'_\downarrow$ gives the difference in the two Fermi levels and $\xi_k = \frac{\hbar^2 k^2}{2m} - \epsilon_F$ gives the free particle energy relative to the average Fermi level $\epsilon_F = \frac{\mu'_\uparrow + \mu'_\downarrow}{2}$. The densities satisfy

$$n_\alpha = \sum_{\vec{k}} u_k^2 N(\hbar\omega_{k,\alpha}) + v_k^2 (1 - N(\hbar\omega_{k,-\alpha})), \quad (4)$$

where $u_k^2 + v_k^2 = 1$, $u_k^2 = \frac{1}{2}(1 + \xi_k / \sqrt{\xi_k^2 + \Delta_0^2})$, and the factor $N(x) = (e^{\beta x} + 1)^{-1}$ denotes the Fermi distribution. For fixed n_α, Eq.(4) determines the chemical potentials μ'_α, but it can be shown that for the conditions of interest it is a good approximation to take μ'_α equal to the Fermi energy $\epsilon_{F,\alpha} = (3\pi n_\alpha / \sqrt{2})^{2/3} \hbar^2 / m$ at zero temperature.

The BCS order parameter has to be determined selfconsistently from Eq.(2). This leads to the following 'gap-equation' (where we already corrected for the divergence appearing in Eq.(2) due to the momentum independent approximation of the triplet potential):

$$\frac{1}{V} \sum_{\vec{k}} \left\{ \frac{1 - N(\hbar\omega_{k,\uparrow}) - N(\hbar\omega_{k,\downarrow})}{2\sqrt{\xi_k^2 + \Delta_0^2}} - \frac{1}{2\xi_k} \right\} = -\frac{1}{T^{2B}}. \quad (5)$$

The important point is that the ratio of densities $n_\uparrow / n_\downarrow$ (and therefore $\mu'_\uparrow / \mu'_\downarrow$) need not be equal to 1. However, the most favorable ratio is 1, for this gives the highest value of T_c for a fixed total density, as can be calculated from Eq.(5) in the limit $\Delta_0 \to 0$ [5], i.e. formation of Cooper pairs is most easy if both spin states have the same Fermi energy ($\delta\epsilon_F = 0$).

3. RESULTS AND DISCUSSION

To gain a better understanding of the consequence of having nonequal densities, we plot the dispersion relations given by Eq.(3). First we consider the case $\Delta_0 = 0$, i.e. $T \geq T_c$ (solid lines in Fig.(1)). For $k > k_F = \sqrt{2m\epsilon_F}/\hbar$, we find just the normal free particle dispersions relative to the respective Fermi levels $\hbar\omega_{k,\alpha} = \epsilon_k - \mu'_\alpha$, whereas for $k < k_F$ the particle curves (thin solid lines) are reflected with respect to the k-axis to get below the Fermi level the dispersions for holes. Note that the Bogoliubov transformation couples particles in state $|\alpha\rangle$ with holes in state $|-\alpha\rangle$. This implies that there is only one Fermi energy $\epsilon_F = (\mu'_\uparrow + \mu'_\downarrow)/2$, above respectively below which we have particles c.q. holes. So, if $\mu'_\uparrow > \mu'_\downarrow$, the state $|\uparrow\rangle$ contains particles above ϵ_F, and the state $|\downarrow\rangle$ is partially filled with holes below ϵ_F even if $T = 0$. This can also be seen from the figure where part of the dispersion $\hbar\omega_{k,\uparrow}$ is negative, implying occupation of states. The consequence is that even for $\Delta_0 = 0$, the creation of a particle-hole pair (a particle with spin $\downarrow$ and a hole with spin $\uparrow$) at ϵ_F, exhibits a gap of $\delta\epsilon_F$.

For $\Delta_0 \neq 0$, i.e. $T < T_c$, we find similar curves, except for the fact that we now see a smoothening and shift in the dispersion relations (dashed lines in Fig.(1)). The particle dispersions now exhibit a gap

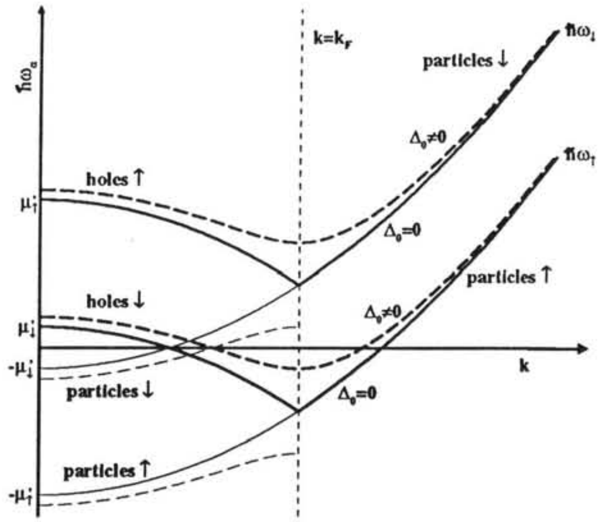

Figure 1: Bogoliubov dispersion $\hbar\omega_\alpha$ for $\Delta_0 = 0$ (fat solid lines) and $\Delta_0 \neq 0$ (fat dashed lines), if $\mu'_\uparrow > \mu'_\downarrow$. The thin lines give the particle dispersions for $k < k_F$.

at $k = k_F$. The fact that this gap appears for both spin states at the same energy is maybe surprising at first instance, but indicates that ϵ_F is indeed the effective Fermi level of the total system.

Summarizing, we have discussed some aspects of the theory of the superfluid state of spin-polarized atomic ^{6}Li. An important extension of this theory is to include also the effects of an inhomogeneous magnetic trap beyond the local-density approximation. Work in this direction is in progress.

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